2p - 5q= 83p - 7q = 11Solve The Simultaneous Equation Using Elimination Method

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Understanding how to solve simultaneous equations is an essential skill in algebra, particularly for students and professionals dealing with mathematical modeling, engineering problems, and various real-world applications. The elimination method is one of the most effective strategies for solving systems of linear equations, especially when the coefficients are arranged in such a way that variables can be eliminated straightforwardly.

In this article, we will explore the process of solving the following system of equations using the elimination method:

$$
\begin{cases}
2p - 5q = 11 \quad \text{(Equation 1)} \\
83p - 7q = 11 \quad \text{(Equation 2)}
\end{cases}
$$

We will delve into the detailed steps involved, discuss the underlying principles, and provide tips to ensure accurate solutions. Whether you're a student preparing for exams or a professional refining your algebra skills, this comprehensive guide aims to clarify the process and enhance your understanding of solving simultaneous equations with the elimination method.

Understanding the System of Equations

Before jumping into the solution, it's crucial to comprehend what a system of simultaneous equations represents. These are two or more equations with multiple variables, where the solution is the set of variable values satisfying all equations simultaneously.

In our case, the system involves two equations with two variables, p and q:

1. $2p - 5q = 11$
2. $83p - 7q = 11$

The goal is to find the values of p and q that satisfy both equations at once.

Why Use the Elimination Method?

The elimination method is particularly advantageous because it allows us to eliminate one variable by adding or subtracting the equations after suitable multiplication. This simplifies the system to a single-variable equation, making it easier to solve.

Advantages of the elimination method include:
- Systematic approach for solving systems with two variables.
- Useful when coefficients of one variable are already opposites or can be made so through multiplication.
- Efficient for large systems when combined with substitution.

When to prefer elimination:
- When coefficients of one variable are the same or can be made the same (or opposites).
- When the equations are arranged conveniently for elimination.

Step-by-Step Solution Using the Elimination Method

Let's now walk through the process of solving the system:

\[
\begin{cases}
2p - 5q = 11 \quad \text{(Equation 1)} \\
83p - 7q = 11 \quad \text{(Equation 2)}
\end{cases}
\]

Step 1: Prepare the equations for elimination

Our goal is to eliminate either \(p \) or \(q \). To do this efficiently, we need the coefficients of one variable to be equal (or opposites). Let's evaluate both options.

- Coefficients of \(p \): 2 and 83
- Coefficients of \(q \): -5 and -7

Since the coefficients of \(p \) are 2 and 83, their least common multiple (LCM) is \(2 \times 83 = 166 \).

Similarly, for \(q \), the coefficients are -5 and -7, with an LCM of 35.

Choosing to eliminate \(p \) involves making the coefficients of \(p \) equal. To do this, multiply Equation 1 by 83 and Equation 2 by 2:

- Multiply Equation 1 by 83:

\[

$$
83 \times (2p - 5q) = 83 \times 11
$$
$$
166p - 415q = 913
$$

- Multiply Equation 2 by 2:

$$
2 \times (83p - 7q) = 2 \times 11
$$
$$
166p - 14q = 22
$$

Now, both equations have the same coefficient for p (166), which allows for elimination.

Step 2: Subtract the equations to eliminate p

Subtract the second new equation from the first:

$$
(166p - 415q) - (166p - 14q) = 913 - 22
$$

Simplify:

$$
166p - 415q - 166p + 14q = 891
$$

$$
(166p - 166p) + (-415q + 14q) = 891
$$

$$
0 - 401q = 891
$$

Step 3: Solve for q

$$
-401q = 891
$$

$$
q = - \frac{891}{401}
$$

This fraction cannot be simplified further, so:

$$
q = - \frac{891}{401}
$$

Step 4: Substitute q back into one of the original equations to find p

Let's substitute into Equation 1:

$$
2p - 5q = 11
$$

Plugging in q:

$$
2p - 5 \left(- \frac{891}{401} \right) = 11
$$

Simplify:

$$
2p + \frac{4455}{401} = 11
$$

Express 11 as a fraction with denominator 401:

$$
2p + \frac{4455}{401} = \frac{11 \times 401}{401} = \frac{4411}{401}
$$

Subtract $\frac{4455}{401}$ from both sides:

$$
2p = \frac{4411}{401} - \frac{4455}{401} = \frac{4411 - 4455}{401} = \frac{-44}{401}
$$

Divide both sides by 2:

$$
p = \frac{-44}{401} \times \frac{1}{2} = - \frac{44}{802} = - \frac{22}{401}
$$

Final solution:

$$
\boxed{
p = - \frac{22}{401}, \quad q = - \frac{891}{401}
}
$$

Verifying the Solution

Verification is a crucial step to ensure the calculated values satisfy both original equations.

Check in Equation 2:

$$
83p - 7q = 11
$$

Substitute $p = - \frac{22}{401}$ and $q = - \frac{891}{401}$:

$$
83 \times \left(- \frac{22}{401} \right) - 7 \times \left(- \frac{891}{401} \right) = 11
$$

Calculate each term:

$$
- \frac{83 \times 22}{401} + \frac{7 \times 891}{401} = 11
$$

$$
- \frac{1826}{401} + \frac{6237}{401} = 11
$$

Combine:

$$
\frac{-1826 + 6237}{401} = 11
$$

$$
\frac{4411}{401} = 11
$$

Express 11 as $\frac{4411}{401}$:

$$
\frac{4411}{401} = \frac{4411}{401}
$$

which confirms the solution is correct.

Key Tips for Solving Simultaneous Equations Using Elimination

- Always aim to make the coefficients of one variable equal or opposites before elimination.
- Multiply equations by suitable numbers to achieve this.
- Be cautious with signs when adding or subtracting equations.
- Simplify fractions where possible to keep calculations manageable.
- Verify solutions by substituting back into original equations.

Additional Examples and Practice Problems

To deepen understanding, practice with these problems:

1. Solve the system:
\[
\begin{cases}
3x + 4y = 10 \\
6x - 2y = 4
\end{cases}
\]
2. Solve for \(a \) and \(b \):
\[
\begin{cases}
7a + 2b = 20 \\
14a - 4b = 10
\end{cases}
\]

Applying the elimination method to these problems will reinforce the concepts discussed.

Conclusion

Solving simultaneous equations is a fundamental skill in algebra that forms the basis for more advanced mathematical topics. The elimination method provides a systematic and reliable approach to find solutions efficiently, especially when coefficients align favorably.

In the example of solving:

\[
\begin{cases}
2p - 5q = 11 \\

83p - 7q = 11
\end{cases}
\]

we demonstrated how to manipulate the equations to eliminate one variable, solve for the other, and verify the solutions for accuracy. Remember, practice is key to mastering this method, and understanding the underlying principles will empower you to tackle more complex systems confidently.

By mastering the elimination method, you enhance your problem-solving toolkit, enabling you to approach a wide array of mathematical challenges with confidence and precision

Frequently Asked Questions

How do you approach solving the simultaneous equations $2p - 5q = 83$ and $11 = 83p - 7q$ using the elimination method?

First, write both equations clearly. Then, eliminate one variable by multiplying equations to align coefficients, subtract to eliminate that variable, and solve for the remaining variable. Finally, substitute back to find the other variable.

What steps are involved in using the elimination method for solving the equations $2p - 5q = 83$ and $11 = 83p - 7q$?

Steps include: (1) Rearrange equations if necessary, (2) multiply to match coefficients of one variable, (3) subtract equations to eliminate that variable, (4) solve for the remaining variable, and (5) substitute back to find the other variable.

Can you demonstrate solving $2p - 5q = 83$ and $11 = 83p - 7q$ using elimination?

Yes. First, rewrite the second as $83p - 7q = 11$. Then, multiply the first by 83 to align p coefficients: $(2p - 5q) 83$. Use these to eliminate p and solve for q, then substitute back to find p.

What is the solution to the system $2p - 5q = 83$ and $11 = 83p - 7q$ using elimination?

The solution is $p = 1$ and $q = -16$. By multiplying and subtracting equations appropriately, you eliminate one variable and solve for the other, then substitute to find the remaining variable.

Why is the elimination method effective for solving equations like $2p - 5q = 83$ and $11 = 83p - 7q$?

Because it allows you to systematically eliminate one variable by combining equations, simplifying the process of solving for the remaining variable without substitution.

Are there any tips for correctly applying the elimination method to such equations?

Yes, ensure coefficients are aligned by multiplying equations as needed, perform operations carefully to avoid sign errors, and always check solutions by substituting back into original equations.

How do I verify the solutions obtained from the elimination method in these equations?

Substitute the found values of p and q back into both original equations to confirm that both sides are equal, ensuring the solution is correct.

Is it necessary to rearrange equations before applying elimination in 2p - 5q = 83 and 11 = 83p - 7q?

Rearranging can make the process easier, such as writing all equations with variables on one side and constants on the other, but it's not always necessary. The key is aligning coefficients for elimination.

What challenges might I face when solving these equations using elimination, and how can I overcome them?

Challenges include sign errors and incorrect coefficient alignment. Overcome them by carefully multiplying equations, double-checking calculations, and verifying solutions by substitution.

Additional Resources

2p - 5q = 83p - 7q = 11Solve The Simultaneous Equation Using Elimination Method

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Introduction

Simultaneous equations are fundamental in mathematics, especially in algebra, where two or more equations are solved together to find common solutions that satisfy all equations simultaneously. They are pivotal across numerous disciplines, including engineering, physics, economics, and computer science, offering a structured way to model real-world problems involving multiple unknowns.

The particular problem under discussion involves a set of equations:

> 2p - 5q = 83
> 83p - 7q = 11

The challenge lies in solving these equations efficiently using the elimination method, a popular algebraic technique designed to eliminate one variable, making it easier to solve for the remaining variable.

This article provides a comprehensive, step-by-step analysis of how to approach and solve this set of simultaneous equations, emphasizing the elimination method. It aims to clarify the process, highlight common pitfalls, and underscore the significance of algebraic manipulation in problem-solving.

Understanding the Equations

Breaking Down the Given Equations

The equations are:

1. $2p - 5q = 83$ (Equation 1)
2. $83p - 7q = 11$ (Equation 2)

At first glance, these equations involve two variables, p and q, and are linear in nature. The goal is to find the values of p and q that satisfy both equations simultaneously.

Key observations:

- The coefficients of p and q differ significantly between the two equations.
- The constants on the right sides (83 and 11) are quite different, which suggests the solutions might involve larger or smaller numbers.

Significance of the Coefficients and Constants

The coefficients determine how the variables are scaled in each equation. For elimination, these coefficients should be manipulated so that the coefficients of one variable become equal (or opposites) in magnitude, allowing for elimination through addition or subtraction.

The constants serve as the “targets” for the equations once variables are eliminated, leading to a single-variable equation that can be solved directly.

The Elimination Method: Conceptual Overview

What is the Elimination Method?

The elimination method involves manipulating the equations such that one variable cancels out when the equations are added or subtracted. This is achieved by aligning the coefficients of one variable, often through multiplication, to make them equal in magnitude but opposite in sign.

Key steps in elimination:

1. Identify the variable to eliminate.
2. Multiply equations by suitable factors to align coefficients.
3. Add or subtract the equations to eliminate the chosen variable.
4. Solve for the remaining variable.
5. Substitute back to find the eliminated variable.

Why Use the Elimination Method?

- It simplifies solving systems when coefficients are easily manipulated.
- It reduces complexity compared to substitution, especially with larger coefficients.
- It's systematic and straightforward once the coefficients are aligned.

Step-by-Step Solution Using Elimination

Step 1: Prepare the Equations for Elimination

Our target is to eliminate either p or q. For clarity, let's choose to eliminate q, as it appears with coefficients -5 and -7.

Equation 1:
2p - 5q = 83

Equation 2:
83p - 7q = 11

To eliminate q, we need the coefficients of q to be equal in magnitude but opposite in sign.

- Coefficient of q in Eq 1: -5
- Coefficient of q in Eq 2: -7

Find the least common multiple (LCM) of 5 and 7, which is 35.

- Multiply Eq 1 by 7:
(7) (2p - 5q) = 7 83

$\Rightarrow 14p - 35q = 581$

- Multiply Eq 2 by 5:

$(5)(83p - 7q) = 5\ 11$

$\Rightarrow 415p - 35q = 55$

Now, both equations have the same coefficient for q (-35q), enabling us to eliminate q by subtraction.

Step 2: Subtract Equations to Eliminate q

Subtract the second scaled equation from the first:

$(14p - 35q) - (415p - 35q) = 581 - 55$

Simplify:

$14p - 35q - 415p + 35q = 526$

Notice that -35q and +35q cancel out:

$(14p - 415p) = 526$

$-401p = 526$

Step 3: Solve for p

Dividing both sides by -401:

$p = 526 / -401$

$p = -526 / 401$

This fraction can be left as is, or converted to a decimal for practical purposes:

$p \approx -1.312$

Step 4: Substitute p Back to Find q

Now, substitute $p \approx -1.312$ into one of the original equations to solve for q.

Using Equation 1:

$2p - 5q = 83$

Plugging in p:

2(-1.312) - 5q = 83

-2.624 - 5q = 83

Now, solve for q:

-5q = 83 + 2.624

-5q = 85.624

q = -85.624 / 5

q ≈ -17.125

Final Solution and Interpretation

The approximate solutions are:

- p ≈ -1.312
- q ≈ -17.125

Expressed as exact fractions:

- p = -526/401
- q = -85.624/5 (or as fraction, q = -85.624/5, which simplifies to the decimal form unless precise fractions are desired).

Significance of the solutions:

These values satisfy both original equations, verified by substitution. The solutions reveal the relationship between p and q in the context of the problem.

Verification of the Solutions

Check in Equation 2:

83p - 7q = 11

Substitute p and q:

$83 (-526/401) - 7 (-85.624/5)$

Calculate each term:

$83 (-526/401) = (83 -526) / 401 = (-43658) / 401 \approx -108.8$

$7 (-85.624/5) = 7 (-17.125) = -119.875$

Now, sum:

$-108.8 - (-119.875) = -108.8 + 119.875 = 11.075$

Close to 11, with minor discrepancies due to rounding.

Check in Equation 1:

$2p - 5q = 83$

$2 (-526/401) = -1052/401 \approx -2.624$

$-5 (-85.624/5) = -5 (-17.125) = 85.625$

Sum:

$-2.624 + 85.625 \approx 83.001$

Again, very close to 83, confirming the approximate solutions are consistent within rounding errors.

Implications and Broader Context

Mathematical Significance:
The process illustrates the power of algebraic manipulation techniques like elimination to solve systems efficiently. Not only does it facilitate finding solutions, but it also enhances understanding of linear relationships and algebraic structure.

Real-World Applications:
Such systems are prevalent in economics (supply and demand models), physics (motion equations), and engineering (circuit analysis). The capacity to solve simultaneous equations accurately impacts decision-making, design, and analysis across sectors.

Educational Value:
Mastering the elimination method prepares students for complex problem-solving, providing a foundation for advanced topics like matrices and linear algebra.

Conclusion

The problem involving:

> $2p - 5q = 83$
> $83p - 7q = 11$

demonstrates the practical application of the elimination method to solve simultaneous equations. By methodically manipulating coefficients, eliminating variables, and solving for the unknowns, we obtained approximate solutions consistent with the original equations.

This approach underscores the importance of strategic algebraic manipulation, critical thinking, and verification in mathematical problem-solving. Whether in academics or professional contexts, the elimination method remains an essential tool in the arsenal of algebraic techniques, enabling efficient and accurate solutions to systems of equations.

Ultimately, mastering such methods enhances analytical skills and deepens comprehension of the interconnectedness of mathematical concepts, empowering individuals to approach complex problems with confidence and clarity.

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