

3.4.9. Determine The 90th Percentile Of The Distribution, Which Is $N(60, 25)$.

Understanding the 90th Percentile in a Normal Distribution

Introduction to Percentiles and Their Significance

Percentiles are statistical measures that indicate the value below which a certain percentage of data points in a distribution fall. The 90th percentile, in particular, signifies the value below which 90% of the data resides. This measure is crucial in fields such as finance, quality control, and risk management, where understanding the upper bounds of data distribution helps in decision-making processes, setting thresholds, and identifying outliers.

Normal Distribution and Its Parameters

The normal distribution, also known as the Gaussian distribution, is characterized by its bell-shaped curve and is defined by two parameters:

- Mean (μ): The central tendency or the average of the data.
- Standard deviation (σ): Measures the dispersion or spread of the data.

In our case, the distribution is specified as $N(60, 25)$, where:

- $\mu = 60$
- $\sigma = 25$

Note that the second parameter (25) is the variance, which is the square of the standard deviation. Therefore:

- Standard deviation, $\sigma = \sqrt{25} = 5$

Understanding these parameters is vital for calculating percentiles, as they directly influence the shape and spread of the distribution.

Calculating the 90th Percentile of a Normal Distribution

General Formula for Percentile Calculation

The percentile P_q corresponding to a quantile q (where q is between 0 and 1) in a normal distribution can be calculated using:

$$P_q = \mu + \sigma \cdot Z_q$$

$$P_q = \mu + z_q \times \sigma$$

\]

where:

- μ = mean

- σ = standard deviation

- z_q = the z-score that corresponds to the cumulative probability q

For the 90th percentile, $q = 0.90$.

Identifying the Z-Score for the 90th Percentile

The z-score is a standardized value representing the number of standard deviations a data point is from the mean. To find $z_{0.90}$, one can refer to standard normal distribution tables or use statistical software.

Common z-score for the 90th percentile:

- $z_{0.90} \approx 1.28155$

This value indicates that the 90th percentile lies approximately 1.28 standard deviations above the mean.

Applying the Formula

Using the known parameters:

- $\mu = 60$

- $\sigma = 5$

- $z_{0.90} \approx 1.28155$

The 90th percentile P_{90} is calculated as:

\[

$$P_{90} = 60 + 1.28155 \times 5$$

\]

\[

$$P_{90} = 60 + 6.40775$$

\]

\[

$$P_{90} \approx 66.40775$$

\]

Therefore, the 90th percentile of the distribution $N(60, 25)$ is approximately 66.41.

Interpreting the Result

Practical Significance of the 90th Percentile

The computed 90th percentile (around 66.41) signifies that 90% of the data points in this normal distribution fall below this value. In real-world applications, this might correspond to:

- A threshold above which only 10% of observations lie.
- A cutoff point for identifying outliers or high-performing entities.
- A boundary in risk assessments, such as maximum expected loss with 90% confidence.

Implications in Different Fields

- Finance: Setting value-at-risk (VaR) thresholds.
- Manufacturing: Determining upper limits for quality control.
- Health sciences: Establishing upper bounds for physiological measurements.
- Education: Understanding score distributions in assessments.

Additional Considerations in Percentile Calculations

Impact of Distribution Assumptions

The applicability of the calculated percentile hinges on the assumption that the data follows a normal distribution. Deviations from normality, such as skewness or kurtosis, can affect the accuracy of this estimate.

Using Software and Tables for Calculation

While manual calculation using z-scores is straightforward, statistical software (e.g., R, Python, SPSS) can directly compute percentiles:

- In Python (using SciPy):

```
```python
from scipy.stats import norm
mean = 60
std_dev = 5
percentile_90 = norm.ppf(0.90, loc=mean, scale=std_dev)
print(f"90th Percentile: {percentile_90}")
```
```

- In R:

```
```r
qnorm(0.90, mean=60, sd=5)
```
```

Limitations and Assumptions

- The calculation assumes the data perfectly follows a normal distribution.

- Real-world data may contain anomalies, skewness, or outliers that affect percentile estimates.
- For non-normal distributions, alternative methods such as empirical percentiles or bootstrapping may be more appropriate.

Summary and Final Remarks

Calculating the 90th percentile of a normal distribution, such as $N(60, 25)$, involves understanding the underlying parameters, utilizing the standard normal distribution's z-score for the 90th percentile, and applying the formula:

$$P_{90} = \mu + z_{0.90} \times \sigma$$

In our case, the 90th percentile is approximately 66.41, meaning 90% of the data falls below this value. This measure is invaluable across various disciplines for setting thresholds, making decisions under uncertainty, and understanding the upper bounds of data variability.

Always consider the underlying assumptions about the data distribution when applying percentile calculations. When in doubt, leverage statistical software for more precise and efficient computations. Understanding and accurately interpreting percentiles empower analysts and decision-makers to make informed choices based on the distributional characteristics of their data.

References:

- Montgomery, D. C. (2012). Design and Analysis of Experiments.
- Devore, J. L. (2015). Probability and Statistics for Engineering and the Sciences.
- SciPy documentation: <https://docs.scipy.org/doc/scipy/reference/stats.html>
- R documentation: <https://stat.ethz.ch/R-manual/R-devel/library/stats/html/qnorm.html>

Frequently Asked Questions

What does the 90th percentile represent in a normal distribution?

The 90th percentile indicates the value below which 90% of the data in the distribution falls. In a normal distribution, it marks the point where 90% of observations are less than or equal to that value.

How do you determine the 90th percentile of a normal distribution $N(60, 25)$?

To find the 90th percentile, first identify the z-score corresponding to the 90th percentile

(approximately 1.2816), then use the formula: percentile value = mean + (z-score standard deviation).

What are the mean and standard deviation for the distribution $N(60, 25)$?

The mean is 60, and the variance is 25, so the standard deviation is $\sqrt{25} = 5$.

Calculate the 90th percentile for the distribution $N(60, 25)$.

Using the z-score for 90th percentile (1.2816): 90th percentile = $60 + (1.2816 \cdot 5) \approx 60 + 6.408 \approx 66.41$.

Why is understanding the 90th percentile important in statistical analysis?

The 90th percentile helps identify high-end values, assess risk, and understand the upper limits of data distributions, which is valuable in fields like finance, quality control, and risk management.

Can the same method be used to find other percentiles in a normal distribution?

Yes, the process involves finding the z-score corresponding to the desired percentile and then applying the formula: value = mean + (z standard deviation).

What assumptions are made when calculating the 90th percentile of a normal distribution?

It assumes that the data follows a normal distribution and that the mean and standard deviation accurately describe the distribution's parameters.

Additional Resources

3.4.9. Determine The 90th Percentile Of The Distribution, Which Is $N(60, 25)$

Introduction

In the realm of statistical analysis, understanding the distribution of data is fundamental to making informed decisions, whether in research, economics, engineering, or social sciences. The concept of percentiles, especially high percentiles like the 90th percentile, provides critical insights into the upper bounds of data, highlighting values that only a small fraction of data points exceed. In this context, examining the 90th percentile of a normal distribution, specifically $N(60, 25)$, offers a compelling example of how statistical tools can be applied to real-world data modeling. This article provides a comprehensive, detailed, and analytical review of how to determine the 90th percentile of

the given distribution, exploring the theoretical foundations, practical calculations, and implications.

Understanding the Distribution: $N(60, 25)$

What Does $N(60, 25)$ Represent?

The notation $N(60, 25)$ describes a normal (or Gaussian) distribution characterized by two parameters:

- Mean (μ): 60
- Variance (σ^2): 25

From this, the standard deviation (σ) can be derived as:

$$\sigma = \sqrt{25} = 5$$

Thus, the distribution is centered at 60 with a spread determined by a standard deviation of 5. The normal distribution is symmetric, bell-shaped, and fully described by its mean and standard deviation, making it a widely used model for natural phenomena and measurement errors.

Key Properties of the Distribution

- Symmetry: The distribution is symmetric around the mean, 60.
- 68-95-99.7 Rule: Approximately 68% of data falls within one standard deviation (55 to 65), 95% within two (50 to 70), and 99.7% within three (45 to 75).
- Probability Density Function (PDF): The shape of the distribution is governed by the bell curve, which peaks at the mean and tapers off symmetrically.

The Concept of Percentiles in a Distribution

What Is a Percentile?

A percentile indicates the value below which a certain percentage of data points in a distribution fall. For example, the 90th percentile is the value below which 90% of the data lies. It helps identify thresholds or cutoffs that segregate the top fraction of data from the rest.

Why Focus on the 90th Percentile?

The 90th percentile serves as a critical cutoff in various contexts such as:

- Risk assessment: Identifying high-risk thresholds.
- Performance benchmarking: Recognizing top performers.
- Quality control: Determining upper specification limits.
- Resource allocation: Prioritizing cases exceeding certain thresholds.

Percentiles in Normal Distributions

Calculating percentiles in a normal distribution involves using the inverse of the cumulative distribution function (CDF). For the normal distribution, this often involves standardizing the value and referencing standard normal tables or computational tools.

Calculating the 90th Percentile

Step 1: Understand the Standard Normal Distribution

The standard normal distribution, denoted as $Z \sim N(0, 1)$, serves as a baseline for calculating percentiles in any normal distribution.

Step 2: Find the Z-Score Corresponding to the 90th Percentile

The Z-score is a standardized value that indicates how many standard deviations an element is from the mean. To find the Z-score corresponding to the 90th percentile:

$$P(Z \leq z_{0.90}) = 0.90$$

Using standard normal distribution tables or statistical software, the Z-score for the 90th percentile is approximately:

$$z_{0.90} \approx 1.2816$$

This means 90% of the data in a standard normal distribution lies below 1.2816 standard deviations above the mean.

Step 3: Transform Z-Score Back to the Original Distribution

Given the Z-score and the parameters of the original distribution, the value corresponding to the 90th percentile (denoted as $x_{0.90}$) is calculated as:

$$x_{0.90} = \mu + z_{0.90} \times \sigma$$

Substituting the known values:

$$x_{0.90} = 60 + 1.2816 \times 5 = 60 + 6.408$$

$$x_{0.90} \approx 66.408$$

\]

Result:

The 90th percentile of the distribution $N(60, 25)$ is approximately 66.41.

Analytical Insights and Implications

Significance of the 66.41 Value

In a practical scenario, if data follows the $N(60, 25)$ distribution, then about 90% of the data points are below 66.41. Conversely, roughly 10% of the data exceeds this value, which could be critical in decision-making processes involving thresholds or limits.

Application Scenarios

- Quality Control: If a process's characteristic follows this distribution, setting an upper control limit at approximately 66.41 ensures that only the top 10% of outputs are flagged for review or intervention.
- Risk Management: For financial returns modeled as $N(60, 25)$, the 90th percentile indicates a high return threshold, useful for setting performance targets.
- Resource Planning: Identifying the 90th percentile helps in planning for outliers or high-demand cases, ensuring resources can handle these upper extremes.

Limitations and Considerations

- Assumption of Normality: The calculation presupposes the data strictly follows a normal distribution. Deviations from normality (skewness, kurtosis) can affect the accuracy.
- Sample Size and Data Quality: Real-world data may not perfectly fit the theoretical distribution, necessitating goodness-of-fit tests.
- Extreme Percentiles: For very high or low percentiles, numerical methods or software may be required for precise calculation.

Practical Methods and Tools

Using Statistical Software

- Excel: Use the `NORM.INV()` function:

```

```
=NORM.INV(0.90, 60, 5)
```

```

- R: Use the `qnorm()` function:

```r

```
qnorm(0.90, mean = 60, sd = 5)
```

```


- Python: With SciPy library:

```
```python
from scipy.stats import norm
percentile_90 = norm.ppf(0.90, loc=60, scale=5)
```
```

Manual Calculation

As demonstrated, the key steps involve identifying the Z-score corresponding to the desired percentile and transforming it back to the original scale.

Broader Context and Related Concepts

Percentile Ranks vs. Percentile Values

- Percentile Rank: The percentage of data points below a specific value.
- Percentile Value: The actual data point corresponding to a given percentile.

Other Percentiles and Their Uses

Beyond the 90th percentile, other percentiles like the 95th, 99th, or median (50th percentile) serve different analytical purposes, providing a comprehensive view of the distribution.

Comparing Distributions

Understanding how the 90th percentile shifts across different distributions can reveal insights into data variability, skewness, and risk levels.

Conclusion

Determining the 90th percentile of a distribution, particularly one modeled as $N(60, 25)$, is a fundamental statistical task with broad applications across industries and research domains. Through understanding the properties of the normal distribution, leveraging standard Z-scores, and applying the appropriate transformations, analysts can accurately identify critical thresholds that inform decision-making, monitor process performance, and assess risks. The calculated value of approximately 66.41 not only exemplifies the practical utility of statistical methods but also underscores the importance of rigorous analysis in interpreting data distributions. As data-driven approaches continue to evolve, mastery of percentile calculations remains a vital skill for statisticians, engineers, economists, and scientists alike.

3 4 9 Determine The 90th Percentile Of The Distribution

Which Isn 60 25

3 4 9 Determine The 90th Percentile Of The Distribution Which Isn 60 25

Related Articles

- [What Was Different About President Bidens State Of The Union Address This Year?](#)
- [When Are The Major Regulatory Points In The Cell Cycle? Select All That Apply.](#)
- [Write A Email To Your Friend Inviting About Your Future Plan After JSC Exam](#)

[Back to Home](#)