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Differentiation is a fundamental concept in calculus that involves finding the rate at which a function changes with respect to a variable, typically x. When working with algebraic expressions involving fractions and multiple terms, it’s often beneficial to differentiate each term separately before combining the results. This approach simplifies the process and helps prevent mistakes, especially in more complex functions. In this article, we will explore the differentiation of the function $3x - \frac{2}{x^2}$ step by step, emphasizing the importance of differentiating each term individually before combining the derivatives. This method not only clarifies the process but also reinforces key calculus principles.

Understanding the Function

Before diving into the differentiation process, it’s essential to understand the structure of the given function:

$$
f(x) = 3x - \frac{2}{x^2}
$$

This function consists of two terms:
- A linear term $3x$
- A rational term $- \frac{2}{x^2}$

The goal is to find the derivative $f'(x)$, which indicates how $f(x)$ changes with respect to x.

Why Differentiate Each Term Separately?

Differentiating each term separately is a standard technique in calculus because:

1. **Simplicity:** It simplifies the process, especially when dealing with sums or differences of functions.

2. **Clarity:** It makes the derivation easier to follow, reducing the likelihood of errors.

3. **Modularity:** It allows you to handle complex functions by breaking them down into manageable parts.

4. **Application of Rules:** It facilitates the use of fundamental differentiation rules such as the power rule, constant rule, and the quotient rule or negative power rule.

In the case of $3x - \frac{2}{x^2}$, differentiating each term individually before combining yields a clear, step-by-step derivation.

Step-by-Step Differentiation Process

Let’s proceed with the differentiation of $f(x) = 3x - \frac{2}{x^2}$. We will differentiate each term separately.

Step 1: Differentiate the First Term $3x$

- Recognize that $3x$ is a simple linear function.
- Apply the power rule: $\frac{d}{dx} [ax] = a$, where a is a constant.
- Alternatively, express $3x$ as $3x^1$ and differentiate using the power rule:

$$
\frac{d}{dx} [3x^1] = 3 \times 1 \times x^{1-1} = 3 \times x^{0} = 3
$$

- Result:

$$
\boxed{\frac{d}{dx} [3x] = 3}
$$

Step 2: Differentiate the Second Term $- \frac{2}{x^2}$

- Rewrite $- \frac{2}{x^2}$ as $-2x^{-2}$, which makes differentiation straightforward using the power rule.

$$
f(x) = -2x^{-2}
$$

- Apply the power rule:

$$
\frac{d}{dx} [-2x^{-2}] = -2 \times (-2) \times x^{-2 - 1} = (-2) \times (-2) \times x^{-3}
$$

- Simplify:

$$
(-2) \times (-2) = 4
$$

- So,

$$
\frac{d}{dx} \left[- \frac{2}{x^2} \right] = 4x^{-3}
$$

- Rewrite back in fractional form:

$$
4x^{-3} = \frac{4}{x^3}
$$

- Result:

$$
\boxed{\frac{d}{dx} \left[- \frac{2}{x^2} \right] = \frac{4}{x^3}}
$$

Combining the Derivatives

Having differentiated each term individually, we now combine the results to obtain the derivative of the entire function:

$$
f'(x) = \frac{d}{dx} [3x] - \frac{d}{dx} \left[\frac{2}{x^2} \right]
$$

Substituting the derivatives from above:

$$
f'(x) = 3 + \frac{4}{x^3}
$$

This is the derivative of the original function, expressed both in simplified form and fractional notation.

Final Expression of the Derivative

The derivative of $f(x) = 3x - \frac{2}{x^2}$ is:

$$
\boxed{f'(x) = 3 + \frac{4}{x^3}}
$$

Alternatively, you can write it as:

$$
f'(x) = 3 + 4x^{-3}
$$

Both forms are correct; choosing between them depends on the context or preference for fractional or exponential notation.

Additional Insights and Applications

Understanding how to differentiate functions like $3x - \frac{2}{x^2}$ is vital in many mathematical and applied contexts, such as physics, engineering, economics, and data analysis.

Applications of Differentiation in Real-World Scenarios

- **Physics:** Calculating velocity and acceleration when position functions involve rational expressions.
- **Economics:** Marginal cost or revenue functions that include rational terms.
- **Engineering:** Analyzing systems with parameters modeled by functions involving powers and reciprocals.

Key Differentiation Rules Used

1. **Power rule:** $\frac{d}{dx} [x^n] = n x^{n-1}$

2. **Constant multiple rule:** $\frac{d}{dx} [a \times f(x)] = a \times \frac{d}{dx} [f(x)]$
3. **Negative and fractional powers:** Handling x^{-n} and $x^{m/n}$ forms.

Conclusion

Differentiating a function step by step before combining the results is an essential skill in calculus. It ensures clarity, accuracy, and a better understanding of how each component contributes to the overall derivative. For the function $3x - \frac{2}{x^2}$, differentiating each term separately yields a straightforward and correct derivative:

$$
f'(x) = 3 + \frac{4}{x^3}
$$

Mastering this approach equips students and professionals with the tools to handle more complex derivatives with confidence. Remember, always analyze the structure of your functions carefully, apply the appropriate rules, and combine the derivatives systematically for the best results.

Keywords: Differentiation, calculus, derivative of $3x - \frac{2}{x^2}$, step-by-step differentiation, power rule, rational functions, calculus tips, derivative rules

Frequently Asked Questions

How do I differentiate the function f(x) = (3x - 2) / x^2 using the quotient rule?

To differentiate f(x) = (3x - 2) / x^2, first identify numerator u = 3x - 2 and denominator v = x^2. Then, find u' = 3 and v' = 2x. Apply the quotient rule: f'(x) = (u'v - uv') / v^2 = (3 x^2 - (3x - 2) 2x) / x^4. Next, simplify numerator step by step before combining terms.

Can I differentiate 3x - 2 / x^2 by first separating the terms?

Yes, you can rewrite the function as (3x / x^2) - (2 / x^2), which simplifies to 3 / x - 2 / x^2. Then, differentiate each term separately using power rule: d/dx [3 / x] = -3 / x^2 and d/dx [-2 / x^2] = 4 / x^3. This method simplifies the differentiation process.

What is the step-by-step differentiation process for $f(x) = (3x - 2) / x^2$?

Step 1: Identify $u = 3x - 2$ and $v = x^2$. Step 2: Compute $u' = 3$ and $v' = 2x$. Step 3: Apply the quotient rule: $f'(x) = (u'v - uv') / v^2$. Step 4: Substitute: $(3 x^2 - (3x - 2) 2x) / x^4$. Step 5: Expand numerator: $3x^2 - (6x^2 - 4x)$. Step 6: Simplify numerator: $3x^2 - 6x^2 + 4x = -3x^2 + 4x$. Step 7: Write final derivative: $(-3x^2 + 4x) / x^4$.

How do I simplify the derivative after applying the quotient rule to $(3x - 2)/x^2$?

After applying the quotient rule, you get the numerator as $-3x^2 + 4x$. Factor numerator if needed, or divide each term by x^4 to simplify: $(-3x^2 + 4x) / x^4 = -3x^2 / x^4 + 4x / x^4 = -3 / x^2 + 4 / x^3$. This provides the simplified derivative.

Is it better to differentiate $(3x - 2)/x^2$ directly or by rewriting as separate terms?

Rewriting as separate terms, $(3 / x - 2 / x^2)$, is often easier because you can directly apply the power rule to each term, reducing chances of errors and simplifying the differentiation process.

What is the derivative of $f(x) = (3x - 2) / x^2$ at a specific point, say $x=1$?

First, differentiate $f(x)$ as shown: $f'(x) = (-3 / x^2 + 4 / x^3)$. At $x=1$, substitute: $f'(1) = -3 / 1^2 + 4 / 1^3 = -3 + 4 = 1$. Therefore, the derivative at $x=1$ is 1.

How does the power rule apply to differentiating functions like $(3x - 2)/x^2$?

By rewriting the function as separate terms with negative exponents, such as $3x\, x^{-2} - 2\, x^{-2}$, you can apply the power rule directly: $d/dx[x^{n}] = n\, x^{n-1}$. This makes differentiation straightforward after rewriting.

What are common mistakes to avoid when differentiating $(3x - 2)/x^2$?

Common mistakes include forgetting to apply the quotient rule properly, neglecting to simplify before differentiating, or misapplying the power rule by not rewriting the function as separate terms. Always verify each step and consider rewriting the function to simplify differentiation.

Additional Resources

Understanding the Differentiation of the Expression $3x - 2/x^2$: A Step-by-Step Guide

When tackling the differentiation of complex algebraic expressions, it’s essential to approach each component systematically. The expression $3x - 2/x^2$ is a combination of a polynomial term and a rational term, making it a perfect candidate for a detailed, step-by-step differentiation process. In this guide, we will explore how to differentiate $3x - 2/x^2$ by first differentiating each term separately before combining the results. This method not only clarifies the process but also reinforces fundamental calculus principles, especially the power rule and the quotient rule.

Why Differentiate Each Term Separately?

Differentiating each term separately, often called term-by-term differentiation, simplifies the process, especially when dealing with sums or differences of functions. Since differentiation is a linear operator, we can differentiate each part independently before combining the derivatives. This approach is particularly useful when:

- The expression contains both polynomial and rational functions.
- It helps avoid confusion between different rules (like the product rule vs. quotient rule).
- It enhances clarity and reduces errors in complex derivatives.

Step-by-Step Differentiation of $3x - 2/x^2$

Let's break down the process into clear, manageable steps, focusing on each term individually.

Step 1: Rewrite the Expression for Clarity

The original expression:

$f(x) = 3x - 2/x^2$

can be rewritten as:

f(x) = 3x - 2x^(-2)

This rewriting makes it easier to apply differentiation rules, especially the power rule.

Step 2: Differentiate Each Term Separately

Term 1: Differentiating 3x

- The term 3x is a simple polynomial.
- Using the power rule:

d/dx [ax^n] = a n x^(n-1)

- Since 3x = 3 x^1:

d/dx [3x] = 3 1 x^(1-1) = 3 x^0 = 3

Result for term 1:

$$d/dx\,[3x] = 3$$

Term 2: Differentiating $-2x^{-2}$

- The term $-2x^{-2}$ involves a negative exponent.
- Applying the power rule:

$$d/dx\,[a x^n] = a\,n\,x^{n-1}$$

- Here, $a = -2$, $n = -2$:

$$d/dx\,[-2x^{-2}] = -2\,(-2)\,x^{-2-1} = (-2)\,(-2)\,x^{-3}$$

$$= 4\,x^{-3}$$

- Rewrite as:

$$4 / x^3$$

Result for term 2:

$$d/dx\,[-2x^{-2}] = 4 / x^3$$

Step 3: Combine the Derivatives

Using the linearity of differentiation, the derivative of the entire function is:

f'(x) = derivative of first term + derivative of second term

So,

$$f'(x) = 3 + 4 / x^3$$

Final Expression of the Derivative

Putting it all together, the derivative of $3x - 2/x^2$ is:

$$f'(x) = 3 + 4 / x^3$$

This expression can be left as is or rewritten with a common denominator for clarity, especially if you need to perform further operations or integrations.

Additional Tips and Insights

1. Why Rewrite as Power Functions?

Rewriting rational functions like $-2/x^2$ as $-2x^{-2}$ simplifies the differentiation process. It allows the direct application of the power rule rather than using the quotient rule, which can be more cumbersome.

2. When to Use the Quotient Rule?

The quotient rule is necessary when differentiating a function expressed as a ratio of two functions, such as $u(x)/v(x)$. Since we've rewritten the original expression into a sum of functions, applying the power rule directly is more efficient here.

3. Confirming the Results

Always double-check your derivatives by:

- Reverting to the original form if needed.
- Ensuring the derivative makes sense dimensionally.
- Using alternative methods, such as the quotient rule, to verify results.

Summary: Differentiating $3x - 2/x^2$ Step-by-Step

Step	Action	Result
1	Rewrite the expression	$f(x) = 3x - 2x^{-2}$
2	Differentiate 3x	3
3	Differentiate $-2x^{-2}$	$4 / x^3$
4	Combine derivatives	$f'(x) = 3 + 4 / x^3$

By following these steps, you can confidently differentiate similar algebraic expressions involving sums, differences, and rational functions.

Final Thoughts

Differentiation is a fundamental skill in calculus, bridging algebra and analysis. Approaching complex expressions like $3x - 2/x^2$ with a clear, step-by-step method ensures accuracy and deepens understanding. Always start by rewriting the function in a form suitable for the power rule or other applicable rules, differentiate each term carefully, and then combine the results. With practice, this method becomes second nature, empowering you to handle even more intricate functions with confidence.

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