Choose The Function That Includes The Given Points. (0,2),(5,13),(2,8),(4,10)

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When working with a set of points such as (0,2), (5,13), (2,8), and (4,10), one common challenge is to determine the function that passes through all these points. Whether you're tackling a math problem in school, developing a data model, or analyzing patterns, selecting the correct function is essential. This article provides a comprehensive guide to help you choose the appropriate function that includes the given points, exploring different types of functions, methods of analysis, and practical steps to find the best fit.

Understanding the Importance of Choosing the Correct Function

Before delving into the specifics, it's crucial to understand why selecting the right function matters. The right function accurately models the data points, enabling predictions, insights, and further analysis. An incorrect function can lead to errors, misinterpretations, or ineffective solutions.

Types of Functions to Consider

When given a set of points, several types of functions can be considered for inclusion. The most common options include:

Linear Functions

- Represented as y = mx + b
- Suitable if the points form approximately a straight line
- Quick to analyze and fit

Quadratic Functions

- Represented as y = ax^2 + bx + c
- Suitable if points suggest a parabola
- Useful for modeling acceleration or curvature

Higher-Degree Polynomial Functions

- For more complex data patterns
- Can exactly fit a set of points but may lead to overfitting

Piecewise Functions

- Different functions apply over different intervals
- Useful if data suggests multiple behaviors

Other Functions (Exponential, Logarithmic, etc.)

- Considered if data exhibits exponential growth/decay or logarithmic patterns

Step-by-Step Approach to Find the Suitable Function

To determine the function that includes the points (0,2), (5,13), (2,8), and (4,10), follow these steps:

1. Plot the Points

- Visualize the points on a coordinate plane
- Observe the overall pattern, slope, and curvature

2. Check for Linearity

- Calculate the slopes between pairs of points
- If slopes are consistent, a linear function may suffice
- For the given points:

  - Slope between (0,2) and (2,8): (8-2)/(2-0) = 6/2 = 3
  - Slope between (2,8) and (4,10): (10-8)/(4-2) = 2/2 = 1
  - Slope between (0,2) and (5,13): (13-2)/(5-0) = 11/5 = 2.2

- Slopes are inconsistent, so a linear function is unlikely

3. Consider Quadratic or Polynomial Functions

- Since linear fit doesn’t seem appropriate, check quadratic or higher-degree

polynomial
- Use methods like polynomial interpolation or regression to find the coefficients

4. Use Polynomial Interpolation

- Polynomial interpolation finds a polynomial passing exactly through all points
- For four points, a cubic polynomial (degree 3) will fit perfectly

5. Formulate and Solve the System of Equations

- Assume the polynomial: $y = ax^3 + bx^2 + cx + d$
- Plug in each point to generate equations:

For (0,2):
- $2 = a0 + b0 + c0 + d \rightarrow d = 2$

For (2,8):
- $8 = a(2)^3 + b(2)^2 + c2 + d$
- $8 = 8a + 4b + 2c + 2$

For (4,10):
- $10 = a(4)^3 + b(4)^2 + c4 + d$
- $10 = 64a + 16b + 4c + 2$

For (5,13):
- $13 = a(5)^3 + b(5)^2 + c5 + d$
- $13 = 125a + 25b + 5c + 2$

- Simplify and solve this system for a, b, c

Calculating the Polynomial Coefficients

Let's proceed step-by-step:

1. From the first point:
- $d = 2$

2. Substitute d into the other equations:

Equation 1 (from (2,8)):
$8 = 8a + 4b + 2c + 2$
$\rightarrow 8a + 4b + 2c = 6$

Equation 2 (from (4,10)):
$10 = 64a + 16b + 4c + 2$

→ $64a + 16b + 4c = 8$

Equation 3 (from (5,13)):
$13 = 125a + 25b + 5c + 2$
→ $125a + 25b + 5c = 11$

3. Now, solve the system:

Divide equations to simplify:

Equation A:
$8a + 4b + 2c = 6$

Equation B:
$64a + 16b + 4c = 8$

Equation C:
$125a + 25b + 5c = 11$

Divide Equation A by 2:
$4a + 2b + c = 3$

Divide Equation B by 4:
$16a + 4b + c = 2$

Divide Equation C by 5:
$25a + 5b + c = 2.2$

Now, subtract equations to find a, b, c:

Subtract the first from the second:
$(16a - 4a) + (4b - 2b) + (c - c) = 2 - 3$
→ $12a + 2b = -1$

Subtract the second from the third:
$(25a - 16a) + (5b - 4b) + (c - c) = 2.2 - 2$
→ $9a + b = 0.2$

Now, solve these two equations:

Equation 1:
$12a + 2b = -1$
→ $6a + b = -0.5$

Equation 2:
$9a + b = 0.2$

Subtract Equation 1 from Equation 2:
$(9a - 6a) + (b - b) = 0.2 - (-0.5)$
→ $3a = 0.7$
→ $a = 0.2333$ (approximate)

Substitute a back into $6a + b = -0.5$:
$6 \cdot 0.2333 + b = -0.5$
$1.4 + b = -0.5$
$\rightarrow b = -1.9$

Finally, find c using $4a + 2b + c = 3$:
$4 \cdot 0.2333 + 2 \cdot (-1.9) + c = 3$
$0.9333 - 3.8 + c = 3$
$c = 3 - 0.9333 + 3.8$
$c \approx 5.8667$

Resulting polynomial:

$y \approx 0.2333x^3 - 1.9x^2 + 5.8667x + 2$

This polynomial passes through all four points.

Verifying the Function

Ensure the function correctly includes all points:

- At x=0:
$y \approx 0 + 0 + 0 + 2 = 2$ ✓

- At x=2:
$y \approx 0.2333(8) - 1.9(4) + 5.8667(2) + 2$
$\approx 1.8664 - 7.6 + 11.7334 + 2 \approx 8.0$ ✓

- At x=4:
$y \approx 0.2333(64) - 1.9(16) + 5.8667(4) + 2$
$\approx 14.9312 - 30.4 + 23.4668 + 2 \approx 10.0$ ✓

- At x=5:
$y \approx 0.2333(125) - 1.9(25) + 5.8667(5) + 2$
$\approx 29.1625 - 47.5 + 29.3335 + 2 \approx 13.0$ ✓

The function accurately models the points provided.

Conclusion: How to Choose the Best Function

Choosing the right function depends on the data pattern and the purpose of analysis:

- If points align linearly, a linear function is sufficient.

- If points follow a curve, quadratic

Frequently Asked Questions

How can I determine the function that passes through the points (0,2), (5,13), (2,8), and (4,10)?

You can use methods like polynomial interpolation or systems of equations to find a function, such as a polynomial, that exactly fits all the given points.

What type of function is most suitable for fitting the points (0,2), (5,13), (2,8), and (4,10)?

A quadratic or cubic polynomial is often suitable, but the degree depends on the number of points and the desired fit; in this case, a cubic polynomial can exactly pass through four points.

Can I use linear functions to include all four points (0,2), (5,13), (2,8), and (4,10)?

No, because four points generally cannot all lie on a single straight line unless they are collinear, which is not the case here.

How do I set up equations to find a polynomial function passing through the points (0,2), (5,13), (2,8), and (4,10)?

Assign the polynomial coefficients and substitute each point into the polynomial equation, creating a system of equations to solve for the coefficients.

What is the general approach to choose the correct function from multiple options that include these points?

Test each candidate function by substituting the points into the function to see if they satisfy the equations, selecting the one that passes through all points.

Is it possible to find a unique quadratic function passing through all four points (0,2), (5,13), (2,8), (4,10)?

No, because four points generally require a cubic polynomial for a unique fit; a quadratic may only pass through three of them unless the points are specially aligned.

What tools or software can help me find the function passing through these points?

Graphing calculators, algebra software like WolframAlpha, Desmos, or programming languages with math libraries (Python, MATLAB) can be used to interpolate or fit the data.

How do I verify if a proposed function includes the points (0,2), (5,13), (2,8), and (4,10)?

Substitute each point into the function and check if the equation holds true for all points; if yes, the function includes all points.

What is an example of a function that passes through the points (0,2), (5,13), (2,8), and (4,10)?

A cubic polynomial such as $f(x) = a x^3 + b x^2 + c x + d$ can be found by solving the system of equations derived from the points; for example, one possible function is $f(x) = 2 + 0.8 x + 0.4 x^2 - 0.1 x^3$ (coefficients determined through calculation).

Additional Resources

Choosing the Correct Function to Pass Through Given Points: An Expert Analysis

When confronted with a set of points in a coordinate plane, a fundamental challenge often arises: determining the function that precisely passes through all these points. This problem appears frequently in various fields such as data interpolation, polynomial fitting, computer graphics, and scientific modeling. The core question is: Which type of function—linear, polynomial, or piecewise—best fits the given data?

In this article, we delve into the process of selecting an appropriate function for the points (0, 2), (5, 13), (2, 8), and (4, 10). We will explore different methods, analyze their properties, and recommend the most suitable approach based on the data characteristics.

Understanding the Problem: The Given Points

Before selecting a function, it’s crucial to examine the data points carefully:

Point	Coordinates
A	(0, 2)
B	(5, 13)
C	(2, 8)
D	(4, 10)

These four points are scattered across the coordinate plane, with no immediately obvious pattern. Our goal is to find a single function $f(x)$ such that:

$$
f(0) = 2, \quad f(2) = 8, \quad f(4) = 10, \quad f(5) = 13
$$

Approaches to Function Selection

Choosing the right function involves balancing complexity, accuracy, and applicability. The major approaches include:

- Linear Interpolation
- Polynomial Interpolation
- Piecewise Polynomial (Spline) Interpolation
- Other Function Types (e.g., exponential, logarithmic)

Each approach has its advantages and limitations, which we will analyze thoroughly.

Linear Interpolation: The Simplest Approach

What is Linear Interpolation?

Linear interpolation involves connecting each pair of consecutive points with straight lines. The resulting piecewise function is continuous but not differentiable at the points where the lines meet unless the points lie on a common line.

Application to Our Data

Using the points in order (0, 2), (2, 8), (4, 10), (5, 13), the linear interpolation segments are:

- Between (0, 2) and (2, 8):

\[
f(x) = 2 + (8 - 2) \times \frac{x - 0}{2 - 0} = 2 + 3x
\]

- Between (2, 8) and (4, 10):

\[
f(x) = 8 + (10 - 8) \times \frac{x - 2}{4 - 2} = 8 + x
\]

- Between (4, 10) and (5, 13):

\[

f(x) = 10 + (13 - 10) \times \frac{x - 4}{5 - 4} = 10 + 3(x - 4) = 10 + 3x - 12 = (3x - 2)
\]

This piecewise function is:

\[
f(x) =
\begin{cases}
2 + 3x, & 0 \leq x \leq 2 \\
8 + x, & 2 < x \leq 4 \\
10 + 3x - 12 = 3x - 2, & 4 < x \leq 5
\end{cases}
\]

Note that at the junction points, the function is continuous:

- At \(x=2\): \(f(2) = 2 + 3 \times 2 = 8\) and \(8 + 2 = 10\). Wait, this indicates a mismatch; at \(x=2\), the first segment gives 8, but the second segment also should give 8. Let's verify:

Second segment at \(x=2\):

\[
8 + 2 = 10
\]

which does not match the value from the first segment at \(x=2\). There is a discrepancy, indicating the points are not aligned with the linear segments unless carefully designed.

Correction:

To ensure continuity at \(x=2\):

- First segment: \(f(2) = 2 + 3(2) = 8\)
- Second segment should also satisfy \(f(2) = 8\), so:

\[
f(x) = m(x - 2) + 8
\]

Similarly, at \(x=4\):

\[
f(4) = m(4 - 2) + 8 = 2m + 8
\]

But we know \(f(4) = 10\), so:

\[

$$
2m + 8 = 10 \Rightarrow 2m = 2 \Rightarrow m = 1
$$

Thus, the segments are:

- $0 \leq x \leq 2$:

$$
f(x) = 2 + 3x
$$

- $2 \leq x \leq 4$:

$$
f(x) = 8 + 1 \times (x - 2) = x + 6
$$

- $4 \leq x \leq 5$:

$$
f(x) = 10 + 3(x - 4) = 10 + 3x - 12 = 3x - 2
$$

Verify at $x=4$:

$$
f(4) = 4 + 6 = 10
$$

and at $x=5$:

$$
f(5) = 3 \times 5 - 2 = 15 - 2 = 13
$$

which matches the data points.

Advantages:

- Simplicity and easy computation.
- Preserves data points exactly.

Limitations:

- Not smooth at the junction points.
- Can produce unrealistic oscillations if data points are not well-behaved.

Polynomial Interpolation: Crafting a Single Smooth Function

What is Polynomial Interpolation?

Polynomial interpolation constructs a single polynomial $P(x)$ of degree $n-1$ passing exactly through n data points. For four points, a cubic polynomial suffices.

Constructing the Polynomial

Given points (x_i, y_i), we can use the Lagrange interpolation formula:

$$
P(x) = \sum_{i=1}^{n} y_i \cdot L_i(x)
$$

where

$$
L_i(x) = \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}
$$

Applying this to our points:

- $A = (0, 2)$
- $C = (2, 8)$
- $D = (4, 10)$
- $B = (5, 13)$

Order matters; let's list points as:

$$
(x_1, y_1) = (0, 2) \\
(x_2, y_2) = (2, 8) \\
(x_3, y_3) = (4, 10) \\
(x_4, y_4) = (5, 13)
$$

The Lagrange basis polynomials:

\[

L_1(x) = \frac{(x - 2)(x - 4)(x - 5)}{(0 - 2)(0 - 4)(0 - 5)} = \frac{(x - 2)(x - 4)(x - 5)}{(-2)(-4)(-5)} = \frac{(x - 2)(x - 4)(x - 5)}{-40}
\]

Similarly for \(L_2(x)\):

\[
L_2(x) = \frac{(x - 0)(x - 4)(x - 5)}{(2 - 0)(2 - 4)(2 - 5)} = \frac{x(x - 4)(x - 5)}{(2)(-2)(-3)} = \frac{x(x - 4)(x - 5)}{12}
\]

For \(L_3(x)\):

\[
L_3(x) = \frac{(x - 0)(x - 2)(x - 5)}{(4 - 0)(4 - 2)(4 - 5)} = \frac{x(x - 2)(x - 5)}{(4)(2)(-1)} = \frac{x(x - 2)(x - 5)}{-8}
\]

For \(L_4(x)\):

\[
L_4(x) = \frac{(x - 0)(x - 2)(x - 4)}{(5 - 0)(5 - 2)(5 - 4)} = \frac{x(x - 2)(x - 4)}{(5)(3)(1)} = \frac{x(x - 2)(x - 4)}{15}

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