Describe The Nature Of The Roots For The Equation 32x^(2)-12x+5= One Real Root

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Understanding the nature of the roots of a quadratic equation is a fundamental aspect of algebra, especially when analyzing the behavior of quadratic functions. In this comprehensive article, we will explore the specific quadratic equation:

$$32x^2 - 12x + 5 = 0$$

and examine why this equation has exactly one real root. We will discuss the concepts of discriminant, the quadratic formula, and the conditions under which quadratic equations have one real root, two real roots, or complex roots. By the end of this article, you will have a clear understanding of the mathematical principles that determine the nature of roots for this quadratic equation.

Understanding Quadratic Equations and Their Roots

A quadratic equation is a second-degree polynomial equation of the form:

$$ax^2 + bx + c = 0$$

where:
- $a \neq 0$,
- b and c are real numbers.

The solutions or roots of a quadratic equation are the values of x that satisfy the equation. These roots can be real or complex, depending on the value of the discriminant.

The Discriminant and Its Role

The discriminant, denoted by Δ (delta), is a key component in analyzing the roots of a quadratic equation. It is calculated as:

$$\Delta = b^2 - 4ac$$

The value of the discriminant determines the nature of the roots:

- If $\Delta > 0$: The quadratic has two distinct real roots.
- If $\Delta = 0$: The quadratic has exactly one real root (a repeated root).
- If $\Delta < 0$: The quadratic has two complex conjugate roots (no real roots).

Given this, to analyze the roots of the specific quadratic equation $32x^2 - 12x + 5 = 0$, we need to compute its discriminant.

Calculating the Discriminant for $32x^2 - 12x + 5 = 0$

Let's identify the coefficients:

- $a = 32$
- $b = -12$
- $c = 5$

Using the discriminant formula:

$$\Delta = b^2 - 4ac$$

Substitute the known values:

$$\Delta = (-12)^2 - 4 \times 32 \times 5$$

Calculate step by step:

1. $(-12)^2 = 144$
2. $4 \times 32 = 128$
3. $128 \times 5 = 640$

Thus,

$$\Delta = 144 - 640 = -496$$

Since the discriminant $\Delta = -496$ is less than zero, this indicates that the quadratic equation has no real roots; instead, it has two complex conjugate roots.

Reconciling with the Statement: "One Real Root"

The initial statement claims that the quadratic equation has one real root. However, our calculation shows that the discriminant is negative, which implies no real roots.

This discrepancy suggests the need to analyze further. Possible reasons include:

- The statement might be erroneous or based on an approximation.
- The question might be considering the case of a double root at a specific point under a different condition.
- Alternatively, the equation might have been misinterpreted or mistyped.

To clarify, let's review the different scenarios for quadratic roots and see under what conditions the equation might have exactly one real root.

When Does a Quadratic Have Exactly One Real Root?

A quadratic equation has exactly one real root when the discriminant is zero:

$$\Delta = 0$$

In such a case, the quadratic has a repeated root, also called a double root. The root can be found using the quadratic formula:

$$x = \frac{-b}{2a}$$

Let's check if this applies to our equation.

Solving for the Roots of $32x^2 - 12x + 5 = 0$

Applying the quadratic formula:

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Substitute the known values:

$$x = \frac{-(-12) \pm \sqrt{-496}}{2 \times 32}$$
$$x = \frac{12 \pm \sqrt{-496}}{64}$$

Since $\sqrt{-496}$ involves an imaginary number:

$$\sqrt{-496} = i \sqrt{496}$$

The roots are:

$$x = \frac{12 \pm i \sqrt{496}}{64}$$

This confirms the roots are complex conjugates, not real.

Conclusion: Roots of the Given Equation

Based on our calculations:

- The discriminant $\Delta = -496$ is negative.
- Therefore, the quadratic $32x^2 - 12x + 5 = 0$ has no real roots.
- It has two complex conjugate roots.

This analysis contradicts the initial statement that the equation has one real root.

Possible Clarifications and Corrections

If the original question intended to explore a different quadratic or a different condition, here are some scenarios:

- The quadratic is $32x^2 - 12x + 5 = 0$: No real roots, as shown.
- The quadratic is $32x^2 - 12x + c = 0$ with some value of c that makes $\Delta = 0$:

Let's find the value of c that makes the discriminant zero:

$$\Delta = b^2 - 4ac = 0$$
$$(-12)^2 - 4 \times 32 \times c = 0$$
$$144 - 128c = 0$$
$$128c = 144$$
$$c = \frac{144}{128} = \frac{9}{8} = 1.125$$

Thus, the quadratic:

$$32x^2 - 12x + \frac{9}{8} = 0$$

has exactly one real root.

In this case:

$$x = \frac{-b}{2a} = \frac{12}{64} = \frac{3}{16}$$

and the root is a repeated root at $x = \frac{3}{16}$.

Summary of Key Concepts

- The discriminant determines the nature of roots.
- For $\Delta > 0$: Two distinct real roots.
- For $\Delta = 0$: One real, repeated root.
- For $\Delta < 0$: Complex conjugate roots.
- The specific quadratic $32x^2 - 12x + 5 = 0$ has discriminant -496, hence no real roots.
- Adjusting the constant term c can produce a quadratic with exactly one real root.

Implications in Algebra and Applications

Understanding the nature of roots is crucial in various fields such as physics, engineering, and finance, where quadratic models are common.

- In physics, the roots of quadratic equations can determine the points where an object reaches certain positions or velocities.
- In engineering, they can define stability conditions in systems.
- In finance, quadratic equations often appear in modeling profit maximization or cost functions.

Knowing how to analyze the discriminant allows professionals and students to quickly assess the behavior of quadratic models.

Final Remarks

In conclusion, the quadratic equation $32x^2 - 12x + 5 = 0$ does not have one real root but rather no real roots, due to its negative discriminant. To have exactly one real root, the constant term would need to be $c = \frac{9}{8}$, leading to a discriminant of zero and a double root at $x = \frac{3}{16}$.

This detailed exploration underscores the importance of discriminant calculations and understanding quadratic roots' nature, enabling accurate analysis of quadratic equations in mathematics and real-world applications.

Frequently Asked Questions

What is the quadratic equation given in the problem?

The quadratic equation is $32x^2 - 12x + 5 = 0$.

What does it mean for a quadratic equation to have one real root?

It means the quadratic has a repeated root, and its graph touches the x-axis at exactly one point, indicating the discriminant is zero.

How do you determine the nature of the roots for the quadratic equation?

By calculating the discriminant, D = b^2 - 4ac. If D > 0, there are two real roots; if D = 0, one real root; if D < 0, no real roots.

What is the value of the discriminant for the equation 32x^2 - 12x + 5 = 0?

The discriminant is D = (-12)^2 - 4 32 5 = 144 - 640 = -496.

Does the discriminant indicate the equation has one real root?

No, since the discriminant is negative (-496), the equation has no real roots; it has two complex roots.

Why does the problem state that the equation has one real root?

Because for the equation to have exactly one real root, the discriminant must be zero. In this case, the discriminant is negative, indicating a contradiction.

Is there a mistake in the problem statement regarding the roots?

Yes, since the discriminant is negative, the quadratic does not have one real root; it actually has complex roots.

How can the roots be described if the discriminant is negative?

The roots are complex conjugates and do not lie on the real number line.

Additional Resources

Describe The Nature Of The Roots For The Equation $32x^2$ - 12x + 5 = 0: An In-Depth Analysis

Understanding the nature of the roots of a quadratic equation is fundamental in algebra, as it provides insights into the solutions without necessarily solving the equation explicitly. When working with the quadratic equation $32x^2$ - 12x + 5 = 0, a key question arises: Does this quadratic have one real root, two real roots, or complex roots? This guide will walk you through the process of analyzing the roots' nature, focusing on the scenario where the quadratic has exactly one real root. We will explore the concepts thoroughly, offering a step-by-step approach, and explaining the mathematical tools involved.

Introduction to Quadratic Equations and Roots

A quadratic equation is a second-degree polynomial expressed in the general form:

\[ax^2 + bx + c = 0 \]

where \(a \neq 0 \). The solutions or roots of this equation are the values of \(x \) that satisfy it.

The roots' nature depends on the discriminant, which is derived from the coefficients a, b, and c. The discriminant D is given by:

$$D = b^2 - 4ac$$

Based on the value of D:

- If $D > 0$, there are two distinct real roots.
- If $D = 0$, there is one real root (also called a repeated or double root).
- If $D < 0$, the roots are complex conjugates (not real).

Step-by-Step Analysis of the Equation $32x^2 - 12x + 5 = 0$

Let's analyze the specific quadratic:

$$32x^2 - 12x + 5 = 0$$

Step 1: Identify the coefficients

- $a = 32$
- $b = -12$
- $c = 5$

Step 2: Calculate the discriminant

Applying the formula:

$$D = b^2 - 4ac$$

$$D = (-12)^2 - 4 \times 32 \times 5$$

$$D = 144 - 4 \times 32 \times 5$$

$$D = 144 - 4 \times 160$$

$$D = 144 - 640$$

$$D = -496$$

Step 3: Interpret the discriminant

Since $D = -496$, which is less than zero, the quadratic equation does not have real roots; instead, it has complex conjugate roots.

Important note: The initial problem statement mentions "One Real Root", but based on the discriminant calculation, this particular quadratic does not have a real root. Therefore, to align with the statement, we need to consider the possibility that the problem might be about a different quadratic, or perhaps there's a typo. Alternatively, the goal could be to understand the conditions under which the quadratic has exactly one real root.

When Does a Quadratic Have Exactly One Real Root?

For the quadratic $ax^2 + bx + c = 0$, the condition for exactly one real root is:

$$
D = 0
$$

This means the discriminant must be zero:

$$
b^2 - 4ac = 0
$$

Deriving the Condition for One Real Root

Suppose the quadratic's coefficients are such that the discriminant equals zero:

$$
b^2 = 4ac
$$

This condition implies the quadratic touches the x-axis at exactly one point, resulting in a repeated root.

Adjusting the Equation to Have One Real Root

Given the original quadratic:

$$ 32x^2 - 12x + 5 = 0 $$

we've found $D = -496 \neq 0$, so it doesn't have exactly one real root. To analyze this scenario, let's consider a general quadratic:

$$ a x^2 + b x + c = 0 $$

with the condition:

$$
b^2 - 4 a c = 0
$$

or equivalently,

$$
c = \frac{b^2}{4a}
$$

Using this, we can construct a quadratic that has exactly one real root.

Example:

Let $a = 1$, $b = -4$. Then,

$$
c = \frac{(-4)^2}{4 \times 1} = \frac{16}{4} = 4
$$

The quadratic:

$$
x^2 - 4x + 4 = 0
$$

has discriminant:

$$
D = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0
$$

and a single real root:

$$
x = \frac{-b}{2a} = \frac{4}{2} = 2
$$

Graphical Interpretation of Roots

Graphing quadratic functions provides an intuitive understanding of their roots:

- When $D > 0$: The parabola intersects the x-axis at two points.
- When $D = 0$: The parabola just touches the x-axis at its vertex.
- When $D < 0$: The parabola does not intersect the x-axis; roots are complex.

In the case of a quadratic with one real root, the vertex of the parabola lies exactly on the x-axis.

Summary: Key Takeaways

- The discriminant is the central tool for analyzing the roots' nature.
- For the quadratic $32x^2 - 12x + 5 = 0$, the discriminant $D = -496$ indicates complex roots.
- To have exactly one real root, the discriminant must be zero.
- The condition for one real root is $b^2 = 4ac$.
- Constructing quadratics with this condition allows understanding of the repeated root scenario.

Additional Tips for Analyzing Quadratic Roots

1. Always compute the discriminant first.
2. Check the sign of D to determine the roots' nature.
3. Use the quadratic formula to find roots when needed:
$$
x = \frac{-b \pm \sqrt{D}}{2a}
$$
4. Graph the parabola to visualize roots and vertex position relative to the x-axis.
5. Recognize that changing coefficients affects the discriminant and roots' nature.

Conclusion

Understanding the nature of roots for quadratic equations is a crucial skill that provides immediate insight into the solutions without the need for explicit solving. In the case of $32x^2 - 12x + 5 = 0$, the negative discriminant confirms the roots are complex, not real. However, by adjusting coefficients or analyzing the discriminant condition, one can explore scenarios where the quadratic has exactly one real (double) root. Mastery of discriminant calculations and their implications enables students and professionals alike to analyze quadratic equations efficiently and accurately.

Remember: Whether dealing with real or complex roots, the discriminant remains your most powerful analytical tool.

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