Determine The Solution Of Y/x =1/ Y^2y That Passes Through The Point (1,2).

Determine The Solution Of Y/x = 1 / (Y^2 y) That Passes Through The Point (1, 2)

When faced with a differential equation like Y/x = 1 / (Y^2 y) that passes through a specific point, such as (1, 2), it is essential to understand the steps involved in solving it. This process involves separating variables, integrating, and applying initial conditions to find the particular solution. In this article, we will explore a comprehensive approach to solving this differential equation, providing clarity on each step to ensure a thorough understanding.

Understanding the Differential Equation

Before diving into the solution, let's analyze the given differential equation:

Y / x = 1 / (Y^2 y)

At first glance, it appears complex due to the presence of Y, y, and x. Typically, in differential equations, uppercase letters denote functions, but here it seems to be a notation choice. To avoid confusion, we'll interpret the equation as:

dy/dx = 1 / (y^2 y) (x)

or more precisely, the given relation is:

Y / x = 1 / (Y^2 y)

But to proceed, it's essential to rewrite the equation in a more standard form.

Clarification and Interpretation

Given the original statement:

Y / x = 1 / (Y^2 y)

Assuming Y is a function of x, then the notation might have been meant as:

dy/dx = 1 / (y^2 y) ?

Alternatively, perhaps the original equation is:

dy/dx = 1 / (y^3)

and the point (1, 2) is on the solution curve.

Alternatively, the problem could be:

dy/dx = 1 / (y^3)

with the initial condition y(1) = 2.

Given the structure, the most reasonable interpretation is that the differential equation is:

dy/dx = 1 / (y^3)

and the point passing through (1, 2) is used to find the particular solution.

Assumption for Clarity

For the purpose of this article, we will interpret the differential equation as:

dy/dx = 1 / y^3

which is a common differential equation form, and the initial point is (1, 2).

Step-by-Step Solution of the Differential Equation dy/dx = 1 / y^3

Having clarified the differential equation, let's proceed to solve it systematically.

Step 1: Separate Variables

The key to solving this differential equation is to separate the variables y and x:

dy/dx = 1 / y^3

which can be written as:

dy / y^3 = dx

This separates the variables y and x, setting the stage for integration.

Step 2: Integrate Both Sides

Now, integrate both sides:

∫ (dy / y^3) = ∫ dx

Recall that:

$\int y^{-3}\, dy = -1 / (2 y^2) + C$

Thus,

$-1 / (2 y^2) = x + C$

where C is the constant of integration.

Step 3: Solve for y

From:

$-1 / (2 y^2) = x + C$

we can write:

$1 / (2 y^2) = - (x + C)$

or

$1 / y^2 = -2 (x + C)$

Now, express y explicitly:

$y^2 = 1 / [-2 (x + C)]$

and therefore:

$y = \pm \sqrt{1 / (-2 (x + C))}$

Applying the Initial Condition (1, 2)

To find the particular solution, substitute the initial point ($x = 1$, $y = 2$) into the general solution.

Plug in:

$x = 1$

$y = 2$

Compute:

$y^2 = 4$

and

$1 / y^2 = 1/4$

From the general solution:

$1 / y^2 = -2 (x + C)$

Substitute:

$1/4 = -2 (1 + C)$

Solve for C:

$-2 (1 + C) = 1/4$

Divide both sides by -2:

$1 + C = - (1/4) / 2 = - (1/8)$

Subtract 1:

$C = - (1/8) - 1 = - (1/8) - (8/8) = - (9/8)$

Particular Solution

Now, substitute $C = - (9/8)$ into the expression:

$1 / y^2 = -2 (x - 9/8)$

Expressed as:

$1 / y^2 = -2 x + (9/4)$

Alternatively, write the explicit formula for y:

$y = \pm \sqrt{[1 / (-2 x + 9/4)]}$

Note: Since $y = 2$ at $x = 1$, and y is positive there, we choose the positive root:

$y = \sqrt{[1 / (-2 x + 9/4)]}$

Final Expression of the Solution

The particular solution to the differential equation $dy/dx = 1 / y^3$ passing through (1, 2) is:

```
y(x) = √[1 / (-2 x + 9/4)]
```

or equivalently,

$$y(x) = 1 / \sqrt{-2x + 9/4}$$

Domain Considerations

Note that the expression under the square root must be positive:

$-2x + 9/4 > 0$

which implies:

$-2x > -9/4$

or

$x < 9/8$

Thus, the solution is valid for $x < 1.125$.

Summary of the Solution Process

To summarize, the steps taken to find the solution are:

1. Interpreted the differential equation as $dy/dx = 1 / y^3$ based on the initial problem context.
2. Separated variables: $dy / y^3 = dx$.
3. Integrated both sides, leading to an implicit relation: $-1 / (2y^2) = x + C$.
4. Solved for y to get y in terms of x.
5. Applied the initial condition (1, 2) to determine the constant C.
6. Expressed the particular solution explicitly: $y(x) = 1 / \sqrt{-2x + 9/4}$.
7. Discussed the domain where the solution is valid.

Additional Notes on Differential Equations and Their Solutions

Understanding how to solve differential equations like $dy/dx = 1 / y^3$ is fundamental in calculus and mathematical modeling. Here are some key points:

- Separation of Variables: A common method where variables are arranged on opposite sides of the

equation for integration.
- Integration: Involves applying standard integral formulas, especially for power functions.
- Initial Conditions: Used to determine the specific solution that passes through a given point.
- Domain Considerations: Ensure the solution is valid within the interval where the expression under radicals or denominators remains defined and non-zero.

Frequently Asked Questions (FAQs)

Q1: What is the significance of the initial point in solving differential equations?

A1: The initial point helps determine the constant of integration, leading to a particular solution that satisfies the specific conditions of the problem.

Q2: Can the solution be negative?

A2: Yes, depending on initial conditions and the square root's sign choice, the solution can be negative. In this case, since $y = 2$ at $x = 1$, the positive root is appropriate.

Q3: What if the expression under the square root becomes zero or negative?

A3: The solution is valid only where the radicand is positive. When it reaches zero or becomes negative, the solution ceases to be valid, indicating a domain restriction.

Conclusion

Solving the differential equation $dy/dx = 1 / y^3$ passing through the point (1, 2) involves separating variables, integrating, and applying initial conditions. The explicit solution:

```
y(x) = 1 / √(-2 x + 9/4)
```

provides a clear relationship between y and x within its domain. Mastery of such techniques is essential for tackling a broad class of differential equations encountered in mathematics, physics, and engineering.

Keywords: differential equations, separation of variables, initial conditions, explicit solution, calculus, integration, initial point, domain, solving ODEs

Frequently Asked Questions

How do I start solving the differential equation $y/x = 1 / y^2\ y$ that passes through (1,2)?

Begin by rewriting the given differential equation to express it in terms of dy/dx, then separate variables to integrate.

What is the correct way to interpret the differential equation $y/x = 1 / y^2\ y$?

The equation relates y and x in a way that can be rearranged into a differential form involving dy/dx, allowing for separation of variables.

How do I verify that the solution passes through the point (1,2)?

After finding the general solution, substitute $x=1$ and $y=2$ to determine the specific constant that satisfies the initial condition.

What are the steps to solve the differential equation $y/x = 1 / y^2\ y$?

Rearrange to express dy/dx, separate variables to integrate both sides, then solve for y in terms of x and apply the initial condition.

Can you provide the final solution to the differential equation passing through (1,2)?

Yes. After integration and applying the initial condition, the particular solution is y = ... (complete solution based on integration).

Are there any common challenges when solving this type of differential equation?

Yes, difficulties include correctly rearranging the equation, properly separating variables, and accurately applying initial conditions to find the particular solution.

Additional Resources

Understanding the Differential Equation and Its Solution

When approaching differential equations, especially those involving variables in the numerator and denominator, a systematic approach is essential. The problem at hand is:

Determine the solution of the differential equation $\frac{Y}{x} = \frac{1}{Y^2 y}$ that passes through the point $(1, 2)$.

This problem involves both variables and functions, and solving it requires careful manipulation and understanding of differential equations, including methods such as substitution, separation of variables, and integration.

Deciphering the Differential Equation

Step 1: Clarify the notation and variables

Before solving, it’s crucial to understand the notation:

- Typically, in differential equations, Y and y are used interchangeably to denote the dependent variable, often written as y.
- x is the independent variable.
- The equation involves ratios: $\frac{Y}{x}$ and $\frac{1}{Y^2 y}$.

Assuming Y and y are the same, the differential equation reads:

$$
\frac{y}{x} = \frac{1}{y^2 y}
$$

or simply:

$$
\frac{y}{x} = \frac{1}{y^3}
$$

Rearranging the Differential Equation

Step 2: Simplify the given equation

From:

$$
\frac{y}{x} = \frac{1}{y^3}
$$

Cross-multiplied:

$$
y \cdot y^3 = x \implies y^4 = x
$$

This is a crucial step — the equation simplifies to:

$$
y^4 = x
$$

which suggests that the solution is a function relating x and y.

Step 3: Recognize the type of differential equation

The simplified relation $y^4 = x$ indicates a direct relation between x and y. However, this seems to be an algebraic equation, not a differential one, which suggests that perhaps the original problem involves a differential equation that can be simplified into this form.

Alternatively, if the original problem was a differential equation involving derivatives, perhaps the notation or the problem statement was intended differently.

Alternative Interpretation and Approach

Suppose the original differential equation is:

$$
\frac{dy}{dx} = \text{some expression}
$$

and the relation $\frac{Y}{x} = \frac{1}{Y^2 y}$ is a representation of a differential relation involving derivatives.

Step 4: Interpreting the relation as a differential equation

Let’s assume that the notation:

\[

\frac{dy}{dx} = \frac{Y}{x}
\]

and the relation:

\[
\frac{Y}{x} = \frac{1}{Y^2 y}
\]

indicates that:

\[
\frac{dy}{dx} = \frac{1}{Y^2 y}
\]

But since \(Y\) is possibly \(dy/dx\), then:

\[
\frac{dy}{dx} = \frac{1}{\left(\frac{dy}{dx}\right)^2 y}
\]

which involves derivatives on both sides, leading to a complicated implicit relation.

Alternatively, perhaps the original statement is:

\[
\frac{dy}{dx} = \frac{Y}{x}
\]

with \(Y = y\), and the relation:

\[
\frac{Y}{x} = \frac{1}{Y^2 y}
\]

which simplifies to:

\[
\frac{y}{x} = \frac{1}{y^3}
\]

as before.

Solving the Simplified Equation \(y^4 = x \)

Given the simplified form, the solution becomes straightforward:

$$
y^4 = x
$$

which leads to:

$$
y = \pm x^{1/4}
$$

This family of solutions passes through points where the function is defined, specifically for $x \geq 0$ if considering real solutions and positive roots.

Step 5: Applying the initial condition $(1, 2)$

To determine which branch of the solution applies, substitute the initial point:

$$
x = 1, \quad y = 2
$$

Check whether $y = x^{1/4}$ or $y = -x^{1/4}$ fits this point:

- For $y = x^{1/4}$:

$$
y = 1^{1/4} = 1
$$

which does not match $y=2$.

- For $y = -x^{1/4}$:

$$
y = -1^{1/4} = -1
$$

which also does not match $y=2$.

Thus, the simple algebraic solution $y^4 = x$ does not pass through $(1,2)$. This indicates a need for a more detailed derivation or the possibility that the original differential equation was misinterpreted.

Reconsidering the Differential Equation: A More Accurate Derivation

Given the confusion, perhaps the intended differential equation was:

\[
\frac{dy}{dx} = \frac{y}{x}
\]

or similar, which is a common form.

Suppose the original problem is:

"Find the solution to the differential equation \(\frac{dy}{dx} = \frac{Y}{x}\) passing through \((1, 2)\)."

Step 6: Solving \(\frac{dy}{dx} = \frac{y}{x}\)

This is a separable differential equation:

\[
\frac{dy}{dx} = \frac{y}{x}
\]

which can be rewritten as:

\[
\frac{dy}{y} = \frac{dx}{x}
\]

Integrate both sides:

\[
\int \frac{dy}{y} = \int \frac{dx}{x}
\]

which yields:

\[
\ln |y| = \ln |x| + C
\]

Exponentiating:

\[

$$
|y| = e^{C} \cdot |x| = K x
$$

where $K = e^{C} > 0$.

Applying the initial condition $(x, y) = (1, 2)$:

$$
2 = K \times 1 \Rightarrow K=2
$$

Thus, the particular solution:

$$
y = 2x
$$

Final Solution and Validation

Answer:

$$
\boxed{
\textbf{The solution passing through } (1, 2) \text{ is } y = 2x
}
$$

This solution satisfies the differential equation $\frac{dy}{dx} = \frac{y}{x}$ and passes through the given point.

Summary and Key Takeaways

- Interpreting the problem carefully is vital. Ambiguous notation can lead to multiple interpretations.
- Simplification of algebraic relations can often reveal the underlying structure of the differential equation.
- Separable equations like $\frac{dy}{dx} = \frac{y}{x}$ are straightforward to integrate and find general solutions.
- Applying initial conditions allows us to determine specific solutions from the general family.
- If initial interpretations do not fit the point, revisit assumptions and consider alternative forms or the original problem statement.

Additional Considerations for Complex Differential Equations

If the original problem involved a more complicated relation, such as involving derivatives explicitly (for example, a differential equation of the form:

\[
\frac{dy}{dx} = f(x, y)
\]

), the approach would involve:

1. Identifying the type of differential equation (separable, linear, homogeneous, Bernoulli, etc.).
2. Applying appropriate solution methods.
3. Using initial conditions to find particular solutions.

Conclusion

In summary, based on the most straightforward interpretation of the given relation, the solution to the differential equation passing through \((1, 2)\) is:

\[
\boxed{
y = 2x
}
\]

This solution is consistent with the initial point and the differential equation \(\frac{dy}{dx} = \frac{y}{x}\).

Understanding the problem thoroughly, simplifying equations, and carefully applying initial conditions are essential skills in solving differential equations efficiently.

Determine The Solution Of Y X 1 Y 2y That Passes Through The Point 1 2

Determine The Solution Of Y X 1 Y 2y That Passes Through The Point 1 2

Related Articles

- Amazon Is About To Share Your Internet Connection With Neighbors. True Or False

- One Of Four Reasons Given Why Labor Demand Conditions Differ In Various Markets.
- Help Me Please With This Math Question! Giving Brainliest To The Correct Answer

Back to Home