# Determine Whether The Series Converges Or Diverges. [infinity] 6 N2 7 N = 1

## Determine Whether The Series Converges Or Diverges. [infinity] 6 N2 7 N = 1

Understanding the behavior of infinite series is a fundamental aspect of calculus and mathematical analysis. When confronted with a series such as the one described—"6 N2 7 N = 1"—the primary goal is to determine whether the series converges (approaches a finite sum) or diverges (grows without bound or oscillates indefinitely). This article offers an in-depth exploration of the methods used to analyze such series, clarify their structure, and apply appropriate tests to reach a definitive conclusion.

## Deciphering the Series: Clarifying the Notation

Before delving into convergence tests, it is essential to interpret the given series notation. The original expression appears to be:

6 N2 7 N = 1

which seems to be a misinterpretation or typographical error. A likely intended form might be:

∑_{n=1}^∞ (6n^2 + 7n)

This form is a common type of series involving polynomial terms. Alternatively, it could be:

∑_{n=1}^∞ \frac{6n^2 + 7n}{1}

but the division by 1 does not affect the sum. Given the context, the most plausible series for analysis is:

Series A:
\[
\sum_{n=1}^\infty (6n^2 + 7n)
\]

or, in a more general form:

\[
\sum_{n=1}^\infty a_n

\]

where:

\[
a_n = 6n^2 + 7n
\]

Note: If the original notation intended a different series, such as involving factorials, geometric components, or other functions, please specify. For now, we proceed under the assumption that the series in question is:

\[
\sum_{n=1}^\infty (6n^2 + 7n)
\]

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# Analyzing the Series Structure

## Type of Series

The series:

\[
\sum_{n=1}^\infty (6n^2 + 7n)
\]

is a sum of polynomial terms, which are standard examples of divergent series because their terms do not tend to zero as \( n \to \infty \).

## Behavior of the Terms

To determine convergence, examine the behavior of the individual terms:

\[
a_n = 6n^2 + 7n
\]

As \( n \to \infty \):

\[
a_n \sim 6n^2
\]

which grows without bound. Since the terms do not approach zero, the series cannot converge.

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# Convergence Tests and Their Application

## Test 1: The Divergence (nth-Term) Test

Principle:
If $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum a_n$ diverges.

Application:
Calculate $\lim_{n \to \infty} a_n$:

$$
\lim_{n \to \infty} (6n^2 + 7n) = \infty
$$

Because the limit is infinity (not zero), the series diverges by the divergence test.

Conclusion:
The series diverges.

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## Test 2: Comparison with Known Divergent Series

Since the terms $a_n$ grow approximately like $n^2$, compare with the p-series:

$$
\sum_{n=1}^\infty \frac{1}{n^p}
$$

which converges if and only if $p > 1$.

In this case, the terms grow faster than $1/n^p$ for any $p \leq 2$, hence the original series diverges.

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# Alternative Approach: Summations of Polynomial Series

Even without the divergence test, summations involving polynomial terms are well-understood. The sum of the first $N$ terms:

$$
S_N = \sum_{n=1}^N (6n^2 + 7n)
$$

can be expressed using known formulas:

$$
\sum_{n=1}^N n = \frac{N(N+1)}{2}
$$

$$
\sum_{n=1}^N n^2 = \frac{N(N+1)(2N+1)}{6}
$$

Therefore,

$$
S_N = 6 \sum_{n=1}^N n^2 + 7 \sum_{n=1}^N n
$$

$$
S_N = 6 \times \frac{N(N+1)(2N+1)}{6} + 7 \times \frac{N(N+1)}{2}
$$

Simplify:

$$
S_N = N(N+1)(2N+1) + \frac{7N(N+1)}{2}
$$

As $N \to \infty$, $S_N \to \infty$ because the leading term is proportional to $N^3$. This confirms divergence of the infinite series.

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# Special Cases and Variations

If the series involved terms with decreasing magnitude, such as:

$$
\sum_{n=1}^\infty \frac{6n^2 + 7n}{n^k}
$$

\]

for some \(k > 0\), the analysis would differ. For example:

\[
\sum_{n=1}^\infty \frac{6n^2 + 7n}{n^3} = \sum_{n=1}^\infty \left(6 \frac{n^2}{n^3} + 7 \frac{n}{n^3}\right) = \sum_{n=1}^\infty \left(\frac{6}{n} + \frac{7}{n^2}\right)
\]

which converges when \(k > 1\) due to the p-series test.

However, in the original case, the terms grow unbounded, so the series diverges.

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# Summary and Final Conclusion

Based on the analysis above, the key points are:

- The terms of the series \(a_n = 6n^2 + 7n\) do not tend to zero but instead grow without bound as \(n \to \infty\).
- The divergence test confirms that the series diverges because \(\lim_{n \to \infty} a_n \neq 0\).
- Summation formulas for polynomial sequences reveal that the partial sums tend to infinity as \(N \to \infty\), further confirming divergence.
- Any series whose terms do not approach zero cannot converge.

Final Verdict:
The series \(\sum_{n=1}^\infty (6n^2 + 7n)\) diverges.

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# Key Takeaways and Tips for Series Analysis

- Always check the limit of the general term \(a_n\) as \(n\) approaches infinity.
- Polynomial or exponential growth in terms generally leads to divergence.
- For series with decreasing terms, the p-series test is invaluable.
- Use summation formulas to analyze the behavior of partial sums.
- Recognize that divergence due to the terms not approaching zero is the most straightforward conclusion.

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# Conclusion

Determining whether a series converges or diverges hinges primarily on the behavior of its terms at infinity. For the series in question—assuming it is of the form $\sum_{n=1}^\infty (6n^2 + 7n)$—the divergence is clear: the terms grow without bound, and their limit does not tend to zero. Consequently, the series diverges, emphasizing the importance of the divergence test as a primary tool in the analysis of infinite series.

Understanding these fundamental principles equips mathematicians and students with the ability to analyze a wide range of series, from simple polynomial sums to complex exponential and ratio tests. Recognizing the growth patterns of terms and applying the correct convergence criteria are essential skills in higher mathematics.

# Frequently Asked Questions

## How can I determine if the series $\sum (6/n^2)$ converges or diverges?

Since $\sum (6/n^2)$ is a p-series with $p=2 > 1$, it converges by the p-series test.

## Does the series $\sum (6/n^2)$ converge or diverge as n approaches infinity?

It converges because the general term $6/n^2$ approaches 0 and the series is a p-series with $p=2 > 1$.

## What test should I use to check the convergence of $\sum (6/n^2)$?

The p-series test is appropriate here, which confirms convergence for $p > 1$.

## Is the series $\sum (6/n^2)$ absolutely convergent?

Yes, since all terms are positive and the series converges, it is absolutely convergent.

## How does the comparison test help in determining the convergence of $\sum (6/n^2)$?

You can compare it to a convergent p-series like $\sum (1/n^2)$. Since $6/n^2$ is less than a constant multiple of $1/n^2$, the series converges.

## What is the sum of the series $\sum (6/n^2)$ from n=1 to infinity?

The sum is 6 times the value of the Riemann zeta function at 2, which is $\zeta(2) = \pi^2/6$, so the sum equals $\pi^2$.

## Can the series $\sum (6/n^2)$ diverge? Why or why not?

No, it cannot diverge because it is a p-series with $p=2 > 1$, which guarantees convergence.

## What is the limit of the nth term $6/n^2$ as n approaches infinity?

The limit is 0, which is necessary for the series to converge.

## How does the divergence test apply to the series $\sum (6/n^2)$?

Since the limit of $6/n^2$ as $n\to\infty$ is 0, the divergence test is inconclusive, but further tests show the series converges.

## Is the series $\sum (6/n^2)$ a convergent p-series? Why?

Yes, because it is a p-series with $p=2 > 1$, which guarantees convergence.

# Additional Resources

Determine Whether The Series Converges Or Diverges. [infinity] 6 N2 7 N = 1: A Comprehensive Guide to Series Convergence and Divergence

When analyzing infinite series, one of the fundamental questions mathematicians and students alike ask is: Does this series converge or diverge? The phrase determine whether the series converges or diverges encapsulates a core aspect of mathematical analysis and calculus, especially when dealing with series involving sequences like $N^2$, linear terms, or more complex expressions. In this guide, we will explore the principles, methods, and step-by-step approaches to analyze the convergence or divergence of a typical series, with a focus on understanding the behavior of series such as the one hinted at in the expression "infinity 6 N2 7 N = 1"—which appears to resemble a series involving terms with $N^2$ and linear N components.

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Understanding the Series

Before diving into convergence tests, it's essential to clarify the structure of the series in question. Based on the given expression, it appears we're examining a series of the form:

$$
\sum_{N=1}^{\infty} \frac{6N^2 + 7N}{1}
$$

However, since the expression is somewhat ambiguous, let's interpret it more systematically:

Possible Series Forms

1. Series with quadratic and linear terms:

$$
\sum_{N=1}^{\infty} \frac{6N^2 + 7N}{\text{some denominator}}
$$

2. Series involving constant terms:

For example, perhaps the series is:

$$
\sum_{N=1}^{\infty} \frac{6N^2 + 7N}{N}
$$

or

$$
\sum_{N=1}^{\infty} \frac{6N^2 + 7N}{N^2}
$$

3. Series with denominators involving N, such as:

$$
\sum_{N=1}^{\infty} \frac{6N^2 + 7N}{N^k}
$$

Given the limited information, the most common scenario in such series involves polynomial expressions over powers of N, which are classic objects of analysis in convergence tests.

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Fundamental Concepts of Series Convergence

What does it mean for a series to converge?
A series $\sum_{N=1}^{\infty} a_N$ converges if the sequence of its partial sums:

$$
S_N = \sum_{n=1}^N a_n
$$

approaches a finite limit as $N \to \infty$. If the partial sums tend to infinity or oscillate indefinitely, the series diverges.

Why is determining convergence important?
Understanding whether a series converges allows us to assess the behavior of infinite sums, which appears in various areas of mathematics, physics, engineering, and computer science.

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Common Tests for Series Convergence

To determine whether a series converges or diverges, mathematicians have developed several tests. Some of the most widely used include:

1. The Nth-Term Test

- Statement: If $\lim_{N \to \infty} a_N \neq 0$, then the series $\sum a_N$ diverges.
- Use: Quick check—if the terms don't tend to zero, the series cannot converge.
- Limitation: It does not confirm convergence if the limit is zero; further tests are needed.

2. The Comparison Test

- Statement: If $0 \leq a_N \leq b_N$ for all sufficiently large N, and $\sum b_N$ converges, then $\sum a_N$ converges.
- Use: Comparing with known convergent series like p-series or geometric series.

3. The Limit Comparison Test

- Statement: For positive sequences $a_N$ and $b_N$, if $\lim_{N \to \infty} a_N / b_N = c$, where $c$ is finite and positive, then both series either converge or diverge together.

4. The p-Series Test

- Statement: Series of the form $\sum \frac{1}{N^p}$ converge if and only if $p > 1$.

5. The Root Test

- Statement: Consider $\lim_{N \to \infty} \sqrt[N]{|a_N|}$. If this limit is less than 1, the series converges; if greater than 1, it diverges.

6. The Ratio Test

- Statement: Compute $\lim_{N \to \infty} |a_{N+1}/a_N|$. If less than 1, series converges; if greater than 1, diverges.

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Step-by-Step Analysis of the Series

Given the general form involving $N^2$ and $N$, let's analyze a typical example:

$$
\sum_{N=1}^\infty \frac{6N^2 + 7N}{N^k}
$$

where $k$ is a positive real number. The goal is to determine for which values of $k$ this series converges or diverges.

Step 1: Simplify the Terms

Express the general term:

$$
a_N = \frac{6N^2 + 7N}{N^k} = 6N^{2 - k} + 7N^{1 - k}
$$

This decomposition allows us to analyze each component separately.

Step 2: Analyze the Leading Terms

The behavior of the series as $N \to \infty$ hinges on the exponents:

- $6N^{2 - k}$
- $7N^{1 - k}$

Step 3: Apply the p-Series Test

Recall that the series:

$$
\sum_{N=1}^{\infty} N^{-p}
$$

converges if and only if $p > 1$.

Thus:

- The term $6N^{2 - k}$ behaves like a p-series with $p = k - 2$.
- The term $7N^{1 - k}$ behaves like a p-series with $p = k - 1$.

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Convergence Criteria Based on $k$

Let's analyze the convergence based on different ranges of $k$:

Case 1: $k > 3$

- $2 - k < -1$, so $N^{2 - k}$ tends to zero faster than $1/N$.
- Both terms tend to zero sufficiently quickly.
- Conclusion: The series converges for $k > 3$.

Case 2: $k = 3$

- $2 - 3 = -1$, so $6N^{-1}$
- $7N^{-(2)}$
- The dominant term is $6N^{-1}$, which is a harmonic series—diverges.
- Conclusion: Diverges at $k=3$.

Case 3: $2 < k < 3$

- $2 - k \in (-1, 0)$, so $N^{2 - k}$ tends to zero, but more slowly than $1/N$.
- The dominant term is $6N^{2 - k}$, which diverges since the exponent is less than $-1$.
- Conclusion: Series diverges for $2 < k \leq 3$.

Case 4: $k = 2$

- $2 - 2 = 0$, so $6N^{0} = 6$, a constant.
- The series becomes $\sum 6$, which diverges.

Case 5: $k < 2$

- $2 - k > 0$, so the terms grow without bound, leading to divergence.

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Final Summary: When Does the Series Converge?

| Parameter $k$ | Behavior of Series | Converges? |
|---|---|---|
| $k > 3$ | Terms decay faster than $1/N$ | Yes |
| $k = 3$ | Similar to $\frac{1}{N}$ | No |
| $2 < k < 3$ | Terms decay slower than $1/N$ | No |
| $k = 2$ | Terms are constant; series diverges | No |
| $k < 2$ | Terms grow; series diverges | No |

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Practical Approach to Determine Series Convergence

When faced with a series, follow these steps:

1. Identify the general term $a_N$.
Simplify algebraically to understand its behavior as $N \to \infty$.

2. Consider the dominant behavior.
Focus on the highest order polynomial term in numerator and denominator.

3. Apply an appropriate convergence test.
Use comparison, p-series, ratio, or root tests based on the form.

4. Compare with known series.

For example, p-series or geometric series.

5.

## Determine Whether The Series Converges Or Diverges Infinity 6 N2 7 N 1

Determine Whether The Series Converges Or Diverges Infinity 6 N2 7 N 1

## Related Articles


- The Graph Shows The Cube Root Parent Function.Which Statement Is True? A.B.C.D.
- Select The Correct Answer.If F(x) = 2x-x-6 And G(x) = X2 - 4, Find F(x) + G(x).
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