

Evaluate The Indefinite Integral $\int x \cos(x) dx$ By Using Integration By Parts.

Evaluate The Indefinite Integral $\int x \cos(x) dx$ By Using Integration By Parts. This fundamental calculus problem involves finding the indefinite integral of the function $x \cos(x)$ with respect to x . The integral $\int x \cos(x) dx$ appears commonly in calculus, especially when dealing with products of algebraic and trigonometric functions. Solving this integral efficiently requires understanding and applying the technique of integration by parts—a powerful method derived from the product rule of differentiation. In this comprehensive article, we will explore the step-by-step process of evaluating $\int x \cos(x) dx$ using integration by parts, discuss the underlying principles, and highlight tips to master this technique for similar integrals.

Understanding Integration By Parts

Before diving into the specific problem, it's essential to grasp the core concept of integration by parts. This technique is based on the product rule of differentiation:

$$\frac{d}{dx}[u \cdot v] = u' \cdot v + u \cdot v'$$

Rearranged to solve integrals, the formula for integration by parts is:

$$\int u \, dv = uv - \int v \, du$$

where:

- $u = u(x)$ is a function chosen to differentiate,
- $dv = dv(x)$ is a function chosen to integrate.

Applying this formula transforms a complex integral into simpler parts, often making the problem more manageable.

Step-by-Step Solution of $\int x \cos(x) \, dx$ Using Integration By Parts

To evaluate $\int x \cos(x) \, dx$, follow the systematic approach:

1. Choosing u and dv

The key to effective integration by parts is selecting the appropriate u and dv :

- Set $u = x$ because differentiating x simplifies it.
- Set $dv = \cos(x) \, dx$ because integrating $\cos(x)$ is straightforward.

2. Compute du and v

- Differentiating u :

$$\begin{aligned} & \left[\right. \\ & du = dx \\ & \left. \right] \end{aligned}$$

- Integrating dv :

$$\begin{aligned} & \left[\right. \\ & v = \int \cos(x) \, dx = \sin(x) \\ & \left. \right] \end{aligned}$$

3. Apply the Integration By Parts Formula

Using:

$$\begin{aligned} & \left[\right. \\ & \int u \, dv = uv - \int v \, du \\ & \left. \right] \end{aligned}$$

we substitute:

$$\begin{aligned} & \left[\right. \\ & \int x \cos(x) \, dx = x \sin(x) - \int \sin(x) \, dx \\ & \left. \right] \end{aligned}$$

4. Evaluate the Remaining Integral

The remaining integral:

$$\int \sin(x) \, dx = -\cos(x) + C$$

Thus, the complete solution is:

$$\boxed{\int x \cos(x) \, dx = x \sin(x) + \cos(x) + C}$$

where C is the constant of integration.

Key Points to Remember When Using Integration By Parts

When applying integration by parts, keep in mind the following:

- Selection of u and dv : Choose u to simplify after differentiation and dv to be easily integrable.
- LIATE Rule: A helpful heuristic for selecting u :
 - Logarithmic functions
 - Inverse trigonometric functions
 - Algebraic functions (like x)
 - Trigonometric functions
 - Exponential functions
- Repeat if Needed: Sometimes, the integral requires applying integration by parts multiple times.

Applications of Integration By Parts Beyond $\int x \cos(x) \, dx$

Integral

Integration by parts is invaluable in various fields:

- Physics: Calculating work, energy, and electromagnetic problems.
- Engineering: Signal processing and system analysis.
- Mathematics: Solving integrals involving products of functions, such as exponential and polynomial functions.
- Probability: Deriving expected values involving products.

Understanding how to manipulate integrals like $\int x \cos(x) \, dx$ prepares you for tackling more complex problems across scientific disciplines.

Common Mistakes to Avoid

When performing integration by parts, be cautious of:

- Incorrectly choosing u and dv : Poor choices can complicate the integral.
- Forgetting the constant of integration C : Always include C in indefinite integrals.
- Miscalculating derivatives or integrals: Double-check derivatives and integrals during each step.
- Overlooking the need for multiple iterations: Some integrals require repeated application of the technique.

Practice Problems to Strengthen Your Skills

To enhance your mastery of integration by parts, try solving these problems:

- $\int x^2 \sin(x) \, dx$
- $\int e^x \cos(x) \, dx$
- $\int \ln(x) \, dx$ (Hint: Use integration by parts with $u = \ln(x)$)
- $\int x e^{2x} \, dx$
- $\int x \tan^{-1}(x) \, dx$

Consistent practice will help you recognize patterns and apply integration by parts more efficiently.

Summary: Mastering the Integral of $\int x \cos(x) \, dx$ with Integration By Parts

In conclusion, evaluating $\int x \cos(x) \, dx$ using integration by parts involves a strategic selection of u and dv , followed by differentiation and integration to simplify the problem. The key steps are:

- Choosing $u = x$ and $dv = \cos(x) \, dx$
- Calculating $du = dx$ and $v = \sin(x)$
- Applying the formula to obtain:

$$\int x \sin(x) - \int \sin(x) \, dx$$

- Solving the remaining integral:

$$- \cos(x) + C$$

- Final answer:

$$\int x \sin(x) + \cos(x) + C$$

This method exemplifies the power and elegance of integration by parts, especially for integrals involving products of algebraic and trigonometric functions. With consistent practice and understanding of the underlying principles, you can confidently tackle a wide range of similar integrals with efficiency and precision.

Additional Resources for Learning Integration Techniques

For further study and mastery of integration by parts and related calculus topics, consider exploring:

- Textbooks: "Calculus: Early Transcendentals" by James Stewart
- Online Courses: Khan Academy's Calculus Series
- Mathematical Software: WolframAlpha, GeoGebra for visualizing integrals

- Practice Websites: Paul's Online Math Notes, Brilliant.org

Mastering indefinite integrals like $\int x \cos(x) \, dx$ enhances your mathematical toolkit, opening doors to advanced topics in calculus, differential equations, and applied mathematics.

In summary, the process of evaluating $\int x \cos(x) \, dx$ via integration by parts is a fundamental skill that exemplifies the power of calculus techniques. Understanding each step, recognizing patterns, and applying the method systematically will solidify your proficiency in integral calculus and prepare you for more complex mathematical challenges.

Frequently Asked Questions

How do you set up the integral of $x \cos(x) \, dx$ for integration by parts?

You identify parts of the integral as u and dv . Typically, let $u = x$ (which simplifies upon differentiation) and $dv = \cos(x) \, dx$ (which is straightforward to integrate).

What are the derivatives and integrals used in integrating $x \cos(x) \, dx$ by parts?

Derivatives: $du = dx$; Integrals: $v = \int \cos(x) \, dx = \sin(x)$.

What is the formula for integration by parts used in solving $\int x \cos(x) \, dx$?

The formula is $\int u \, dv = uv - \int v \, du$. Applying this to $x \cos(x) \, dx$ gives $\int x \cos(x) \, dx = x \sin(x) - \int \sin(x) \, dx$.

What is the step-by-step solution for $\int x \cos(x) \, dx$ using integration by parts?

Let $u = x \Rightarrow du = dx$; $dv = \cos(x) \, dx \Rightarrow v = \sin(x)$. Then, $\int x \cos(x) \, dx = x \sin(x) - \int \sin(x) \, dx = x \sin(x) + \cos(x) + C$.

Why is integration by parts suitable for evaluating $\int x \cos(x) \, dx$?

Because the integrand is a product of two functions where one simplifies upon differentiation (x), and the other is easily integrable ($\cos(x)$), making integration by parts an efficient method.

Can the result of $\int x \cos(x) dx$ be verified by differentiation?

Yes. Differentiating the result $x \sin(x) + \cos(x)$ yields the original integrand, confirming the correctness.

What is the indefinite integral of $x \cos(x) dx$ after applying integration by parts?

The indefinite integral is $x \sin(x) + \cos(x) + C$.

Are there alternative methods to evaluate $\int x \cos(x) dx$ besides integration by parts?

While other methods like tabular integration are possible, integration by parts is the most straightforward approach for this integral.

What common mistakes should be avoided when solving $\int x \cos(x) dx$ via integration by parts?

Avoid forgetting to include the constant of integration, mixing up u and dv , or making errors in differentiating or integrating the parts.

How does the choice of u and dv affect the efficiency of integration by parts in this problem?

Choosing $u = x$ (which simplifies upon differentiation) and $dv = \cos(x) dx$ makes the integral straightforward, demonstrating an optimal choice that simplifies calculations.

Additional Resources

Evaluation of the Indefinite Integral $\int x \cos(x) dx$ Using Integration by Parts

Understanding how to evaluate integrals involving products of functions is a foundational skill in calculus, essential for students and professionals working in mathematics, engineering, physics, and related fields. Among the various techniques available, integration by parts stands out as one of the most versatile and powerful methods, especially suited for integrals where the integrand is a product of polynomial and transcendental functions. This article provides a comprehensive, step-by-step analysis of how to evaluate the indefinite integral $\int x \cos(x) dx$ using the method of integration by parts, emphasizing both theoretical understanding and practical execution.

Introduction to Integration by Parts

Integration by parts is a technique derived from the product rule of differentiation. It allows us to transform the integral of a product of functions into simpler components, often making the integral more manageable. The formula is expressed as:

$$\int u \, dv = uv - \int v \, du$$

where:

- $u = u(x)$ is a differentiable function,
- $dv = dv(x)$ is an integrable differential.

The primary goal in applying integration by parts is to select u and dv such that:

1. u simplifies upon differentiation (du becomes simpler).
2. dv is easily integrable to obtain v .

Analyzing the Integral $\int x \cos(x) \, dx$

The integral in question involves a product of:

- A polynomial function x ,
- A trigonometric function $\cos(x)$.

This setup suggests an ideal candidate for integration by parts because:

- Differentiating x simplifies it (reduces polynomial degree),
- Integrating $\cos(x)$ is straightforward.

Key considerations:

- Choosing $u = x$ because differentiating reduces it to 1.
- Choosing $dv = \cos(x) \, dx$ because integrating cosine is direct.

Step-by-Step Evaluation

Step 1: Assign u and dv

Based on the above reasoning:

$$\begin{aligned} & \left[\begin{aligned} u &= x \quad \rightarrow \quad du = dx \\ dv &= \cos(x), \quad dx \quad \rightarrow \quad v = \int \cos(x), \quad dx = \sin(x) \end{aligned} \right. \end{aligned}$$

Step 2: Apply the Integration by Parts Formula

Using the formula:

$$\left[\int x \cos(x), \quad dx = uv - \int v, \quad du \right]$$

Substituting the identified u, v, du :

$$\left[\int x \cos(x), \quad dx = x \sin(x) - \int \sin(x), \quad dx \right]$$

Step 3: Simplify the Remaining Integral

The remaining integral is straightforward:

$$\left[\int \sin(x), \quad dx = -\cos(x) + C \right]$$

where C is the constant of integration.

Step 4: Write the Final Solution

Putting it all together:

$$\boxed{\int x \cos(x) \, dx = x \sin(x) + \cos(x) + C}$$

This expression represents the indefinite integral of $(x \cos(x))$.

Additional Insights and Variations

While the above procedure is standard, understanding the nuances and potential variations enhances mastery:

Why Integration by Parts Works Here

- It leverages the reduction of polynomial degree upon differentiation.
- It simplifies the integrand to a basic trigonometric integral.
- The method is iterative; sometimes, multiple integrations by parts are needed, but in this case, one step suffices.

Alternative Approaches

Though integration by parts is the most straightforward, other methods include:

- Tabular integration: Useful for repeated parts, more relevant for higher powers.
- Series expansion: For complex functions, but overkill here.
- Numerical methods: Not suitable for indefinite integrals but useful for definite integrals.

General Tips for Applying Integration by Parts Effectively

- Identify the parts: Think about which component simplifies upon differentiation and which is easiest to integrate.
- Use LIATE rule: Logarithmic, Inverse trigonometric, Algebraic, Trigonometric, Exponential functions—helps in choosing u .
- Check for simplification: After substitution, ensure the resulting integral is simpler.
- Be prepared for iterative steps: Sometimes, multiple integrations by parts are needed.

Practical Applications of the Result

The integral's solution is more than a mathematical curiosity; it appears in:

- Physics: In calculating work, torque, or oscillatory motion where polynomials multiply trigonometric functions.
- Engineering: Signal processing, control systems, and Fourier analysis often require such integrals.
- Mathematics: In solving differential equations and evaluating Fourier transforms.

The explicit form $\int x \cos(x) \, dx = x \sin(x) + \cos(x) + C$ is instrumental in modeling phenomena involving weighted oscillations and polynomial interactions.

Conclusion

The indefinite integral $\int x \cos(x) \, dx$ exemplifies the elegance and utility of the integration by parts technique. By carefully choosing $u = x$ and $dv = \cos(x)$, the integral simplifies to a manageable form that leads to a straightforward solution. This process underscores the importance of strategic function selection and understanding the underlying principles of calculus.

Mastering such integrals enhances problem-solving capabilities across disciplines, providing a solid foundation for tackling more complex integrals involving products of functions. Whether in theoretical research or practical applications, the integration by parts method remains a cornerstone technique, and this example offers a clear, detailed illustration of its power and versatility.

In summary:

- Integration by parts transforms the integral of a product into simpler parts.
- For $\int x \cos(x) \, dx$, choosing $u = x$ and $dv = \cos(x) \, dx$ is optimal.
- The integral evaluates to $x \sin(x) + \cos(x) + C$.
- Understanding the rationale behind choosing parts and performing the steps meticulously ensures accurate and efficient computation.

Embrace the method, and complex integrals become approachable tools in your mathematical toolkit!

Evaluate The Indefinite Integral $\int x \cos x \, dx$ By Using Integration By Parts

Evaluate The Indefinite Integral $\int x \cos x \, dx$ By Using Integration By Parts

Related Articles

- [Plz Help I Only Need This Question Who Ever Get Its Right Will Get 77 Point](#)
- [According To Charles Cooley, There Can Be No Sense Of Self Without Society.](#)
- [A New Assistant Can _____ Within A Month.a. Be Foundb. Is Foundc. Findsd. Find](#)

[Back to Home](#)