

Evaluate The Integral. (use C For The Constant Of Integration.) $18dx / 2x+xx$

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Introduction to Integral Evaluation

Integrals are fundamental in calculus, providing a way to determine the accumulation of quantities, areas under curves, and solutions to differential equations. When evaluating an indefinite integral, we often include a constant of integration (denoted as C) because indefinite integrals represent a family of functions. In this article, we will explore the process of evaluating the integral:

$$\int \frac{18}{2x + xx} dx$$

with detailed steps, explanations, and insights to help you understand the method and application of integration techniques.

Understanding the Integral Expression

Before diving into the solution, it's essential to analyze and simplify the integrand:

$$\frac{18}{2x + xx}$$

Step 1: Simplify the Denominator

Notice that the denominator contains two terms:

$$2x + xx$$

which can be rewritten as:

$$2x + x^2$$

because $(xx = x^2)$.

Step 2: Factor the Denominator

The quadratic expression $(2x + x^2)$ can be factored:

$$x^2 + 2x = x(x + 2)$$

This simplifies the integral to:

$$\int \frac{18}{x(x+2)} dx$$

Applying Partial Fraction Decomposition

To evaluate the integral:

$$\int \frac{18}{x(x+2)} dx$$

we'll use partial fraction decomposition, which is a technique for integrating rational functions where the numerator degree is less than the denominator degree.

Step 1: Set Up Partial Fractions

Express the integrand as:

$$\frac{18}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

where A and B are constants to be determined.

Step 2: Find Constants A and B

Multiplying both sides by $x(x+2)$:

$$18 = A(x+2) + Bx$$

which simplifies to:

$$18 = Ax + 2A + Bx$$

Group like terms:

$$18 = (A+B)x + 2A$$

To find A and B , equate coefficients:

- Coefficient of x :

$$A + B = 0$$

- Constant term:

$$\begin{aligned} &2A = 18 \\ &\end{aligned}$$

Solve the second equation:

$$\begin{aligned} &2A = 18 \rightarrow A = 9 \\ &\end{aligned}$$

Substitute $(A = 9)$ into the first equation:

$$\begin{aligned} &9 + B = 0 \rightarrow B = -9 \\ &\end{aligned}$$

Step 3: Rewrite the Integral

Now, the integral becomes:

$$\int \left(\frac{9}{x} - \frac{9}{x + 2} \right) dx$$

which can be split into two separate integrals:

$$9 \int \frac{1}{x} dx - 9 \int \frac{1}{x + 2} dx$$

Evaluating the Integrals

Step 1: Basic Logarithmic Integrals

Recall that:

$$\int \frac{1}{x} dx = \ln |x| + C$$

and

$$\int \frac{1}{x + a} dx = \ln |x + a| + C$$

Applying these:

$$9 \ln |x| - 9 \ln |x + 2| + C$$

where (C) is the constant of integration.

Step 2: Simplify the Expression

Using properties of logarithms:

$$\ln a - \ln b = \ln \left(\frac{a}{b} \right)$$

we can write the indefinite integral as:

$$9 \ln \left| \frac{x}{x+2} \right| + C$$

Final Answer

Thus, the evaluated indefinite integral is:

$$\boxed{\int \frac{18}{2x+xx} dx = 9 \ln \left| \frac{x}{x+2} \right| + C}$$

where (C) is the constant of integration.

Additional Tips for Evaluating Similar Integrals

- Always simplify the integrand before applying advanced techniques.
- Factor denominators to identify potential partial fractions.
- Determine constants carefully when using partial fractions.
- Remember the basic integrals involving $(1/x)$ and $(1/(x+a))$.
- Use properties of logarithms to simplify the final expression.
- Include the constant of integration (C) for indefinite integrals to represent the family of solutions.

Common Mistakes to Avoid

- Forgetting to factor the denominator properly.
- Incorrectly solving for constants (A) and (B) .
- Overlooking absolute value signs in logarithmic expressions.
- Not including the constant (C) at the end.
- Assuming the integrand is straightforward without simplifying first.

Conclusion

Evaluating integrals like $(\int \frac{18}{2x+xx} dx)$ involves careful algebraic manipulation, partial fraction decomposition, and application of fundamental integral formulas. Recognizing common patterns and properties of logarithms simplifies the process significantly. Mastering these techniques enhances your proficiency in calculus and prepares you to tackle more complex integrals confidently.

References

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning.
- Thomas, George B., and Franklin D. Finney. Calculus and Analytic Geometry. Pearson.
- Paul's Online Math Notes. Integral Calculus.
[<https://tutorial.math.lamar.edu>] (<https://tutorial.math.lamar.edu>)

By understanding and practicing these steps, you'll be well-equipped to evaluate similar integrals and deepen your calculus knowledge.

Frequently Asked Questions

How do you evaluate the integral of $18 \, dx / (2x + x^2)$?

First, rewrite the integral as $18 / (2x + x^2) \, dx$. Factor the denominator to $x(x + 2)$, then use partial fractions to decompose and integrate accordingly.

What is the first step in solving $\int 18 \, dx / (2x + x^2)$?

Factor the denominator to $x(x + 2)$ to prepare for partial fraction decomposition.

How do you set up partial fractions for $18 / (x(x + 2))$?

Express $18 / (x(x + 2))$ as $A / x + B / (x + 2)$, then solve for A and B.

What are the values of A and B in the partial fraction decomposition of $18 / (x(x + 2))$?

Solving the equations, $A = 9$ and $B = 9$, so $18 / (x(x + 2)) = 9 / x + 9 / (x + 2)$.

What is the integral of $9 / x \, dx$?

The integral of $9 / x \, dx$ is $9 \ln |x| + C$.

What is the integral of $9 / (x + 2) \, dx$?

The integral of $9 / (x + 2) \, dx$ is $9 \ln |x + 2| + C$.

Combine the partial integrals to write the final answer.

$\int 18 / (2x + x^2) \, dx = 9 \ln |x| + 9 \ln |x + 2| + C$.

Is there a constant of integration in the solution?

Yes, include C to account for the constant of integration.

What is the simplified form of the integral result?

The simplified form is $9 \ln |x| + 9 \ln |x + 2| + C$, which can also be written as $9 \ln |x(x + 2)| + C$.

How would you verify the correctness of this integral solution?

Differentiate the result to see if it simplifies back to the original integrand, $18 / (2x + x^2)$.

Additional Resources

Evaluate The Integral. (use C For The Constant Of Integration.) $18dx / 2x+xx$

When it comes to calculus, integrals are fundamental tools used to determine the area under a curve, accumulate quantities, or reverse differentiation processes. The integral presented here, $18 dx / (2x + xx)$, offers an interesting challenge due to its algebraic structure involving both linear and quadratic terms. Carefully evaluating this integral not only enhances understanding of integration techniques but also demonstrates the importance of simplifying complex expressions to manageable forms. In this article, we will explore the step-by-step methods to evaluate this integral, clarify the role of the constant of integration, and discuss the broader implications and applications of such integrals.

Understanding the Integral

Before diving into the solution, it's essential to analyze the integral:

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\[
\int \frac{18}{2x + xx} \, dx
\]
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At first glance, the denominator appears to be a combination of terms involving (x) . Recognizing patterns and simplifying the expression are crucial initial steps. The integral involves a rational function with a polynomial in the denominator, which suggests that techniques such as factoring, substitution, or partial fractions might be applicable.

The key challenges include:

- Simplifying the algebraic expression $(2x + xx)$
- Choosing an appropriate substitution to facilitate integration
- Ensuring the constant of integration, (C) , is included at the end

Step 1: Simplify the Denominator

The denominator $(2x + x^2)$ can be rewritten for clarity:

$$2x + x^2 = 2x + x^2$$

Note that (x^2) is simply (x^2) . Therefore, the integral becomes:

$$\int \frac{18}{x^2 + 2x} \, dx$$

This form is more manageable and suggests that factoring or completing the square might be useful.

Step 2: Factor or Complete the Square

Factoring Approach:

Factor the denominator:

$$x^2 + 2x = x(x + 2)$$

Now, the integral is:

$$\int \frac{18}{x(x + 2)} \, dx$$

This form is ideal for partial fraction decomposition, which breaks a complex rational function into simpler fractions.

Step 3: Partial Fraction Decomposition

Express the integrand as:

$$\frac{18}{x(x + 2)} = \frac{A}{x} + \frac{B}{x + 2}$$

Multiply both sides by the denominator $(x(x + 2))$:

$$18 = A(x + 2) + Bx$$

Expand the right side:

$$18 = A x + 2A + B x$$

Combine like terms:

$$18 = (A + B) x + 2A$$

This must hold for all x , so equate coefficients:

- Coefficient of x :

$$A + B = 0$$

- Constant term:

$$2A = 18$$

Solve for A :

$$A = \frac{18}{2} = 9$$

Then, find B :

$$A + B = 0 \rightarrow 9 + B = 0 \rightarrow B = -9$$

Thus,

$$\frac{18}{x(x+2)} = \frac{9}{x} - \frac{9}{x+2}$$

Step 4: Integrate Term-by-Term

The integral now simplifies to:

$$\int \left(\frac{9}{x} - \frac{9}{x+2} \right) dx$$

Break into two integrals:

$$\int$$

$$9 \int \frac{1}{x} \, dx - 9 \int \frac{1}{x+2} \, dx$$

Recall the basic integrals:

$$\int \frac{1}{x} \, dx = \ln |x| + C$$

$$\int \frac{1}{x+a} \, dx = \ln |x+a| + C$$

Apply these:

$$9 \ln |x| - 9 \ln |x+2| + C$$

where (C) is the constant of integration.

Final Result and Interpretation

The evaluated integral is:

$$\boxed{\int \frac{18}{2x+xx} \, dx = 9 \ln |x| - 9 \ln |x+2| + C}$$

This expression can be further simplified using properties of logarithms:

$$9 \left(\ln |x| - \ln |x+2| \right) + C = 9 \ln \left| \frac{x}{x+2} \right| + C$$

Role of the Constant of Integration (C) :

It is crucial to include (C) because indefinite integrals represent a family of functions differing by a constant. This constant accounts for the fact that differentiation of any constant yields zero, so the original function could have had any constant added.

Features and Insights

Pros:

- The integral evaluation demonstrates the power of algebraic manipulation combined with partial fractions.

- Recognizing the structure of the denominator simplifies the process significantly.
- The final expression provides a clear relationship between $\frac{1}{x}$ and the integral's value.

Cons:

- Requires familiarity with partial fraction decomposition, which might be challenging for beginners.
- Assumes the denominator can be factored or expressed as a product of linear factors; more complex denominators may require different techniques.
- The process can be lengthy, especially if the algebra becomes more complicated.

Features to Note:

- The integral's solution involves natural logarithms, common in integrating rational functions.
- The absolute value signs indicate the domain restrictions necessary for the logarithm to be defined (i.e., $x \neq 0$ and $x \neq -2$).

Broader Applications of Such Integrals

Integrals of this form appear frequently in various fields:

- **Physics:** Calculating work done or potential functions where force or potential varies inversely with distance.
- **Economics:** Analyzing proportional relationships and growth models.
- **Engineering:** Signal processing and control systems often involve rational function integrations.
- **Mathematics:** Fundamental in solving differential equations with rational functions.

Understanding how to manipulate and evaluate such integrals enhances problem-solving skills across disciplines.

Conclusion

Evaluating the integral $\int \frac{18}{2x + x^2} dx$ illustrates a disciplined approach combining algebraic simplification, partial fraction decomposition, and recognition of standard integral forms. The key steps involve rewriting the denominator as $x(x + 2)$, decomposing into simpler fractions, and integrating logarithmic functions. Including the constant of integration C ensures the solution's completeness, acknowledging the indefinite nature of the integral.

This process exemplifies the importance of algebraic insight in calculus, providing a framework for tackling more complex rational integrals. Mastery of these techniques empowers students and professionals to analyze and interpret a wide array of real-world problems involving integration. Whether

in physics, engineering, or mathematics, the ability to evaluate such integrals is indispensable, and the methods demonstrated here form a cornerstone of integral calculus education.

Evaluate The Integral Use C For The Constant Of Integration **18dx 2x Xx**

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