Find A Plan Containing The Point (1,2,-6) And The Line R^->(t)=4,5,1+t7,7,5.

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Understanding how to find the equation of a plane that contains a specific point and a line is a fundamental concept in analytical geometry. This process involves using vector operations, such as the cross product, and the parametric equations of the line. In this article, we will explore step-by-step how to determine the equation of a plane containing the point (1, 2, -6) and the line R^-→(t) = (4, 5, 1) + t(7, 7, 5). We will also discuss the geometric interpretations, the importance of such problems, and the general methods used in similar scenarios.

Understanding the Problem

Before delving into calculations, it is essential to understand what is being asked:

- Given Point: (1, 2, -6)
- Given Line: R^-→(t) = (4, 5, 1) + t(7, 7, 5)

The goal is to find the equation of a plane that contains both the given point and the entire line.

Key Concepts:

- A plane in three-dimensional space can be described by a point and a normal vector.
- The normal vector is perpendicular to any vector lying within the plane.
- To determine the plane's equation, we need a point on the plane and a normal vector.

Step-by-Step Approach to Find the Plane

The process involves several steps:

1. Identify a Point on the Line
2. Find a Direction Vector of the Line
3. Determine Two Vectors in the Plane
4. Calculate the Normal Vector to the Plane
5. Write the Equation of the Plane

Let's explore each step in detail.

1. Identify a Point on the Line

The line's parametric equations are:

- $x = 4 + 7t$
- $y = 5 + 7t$
- $z = 1 + 5t$

When $t = 0$:

- Point on the line, $R_0 = (4, 5, 1)$

This point is essential as it lies on the line, and hence, on the plane.

2. Find a Direction Vector of the Line

The direction vector of the line, denoted as d, is derived from the parametric equations:

- $d = (7, 7, 5)$

This vector indicates the line's direction in space.

3. Determine Two Vectors in the Plane

Since the plane contains:

- The point (1, 2, -6),
- The entire line, including the point $R_0 = (4, 5, 1)$,

we can find a vector connecting these two points:

- $v_1 = R_0 - (1, 2, -6) = (4 - 1, 5 - 2, 1 - (-6)) = (3, 3, 7)$

Now, v_1 is a vector from the given point to a point on the line, and it lies within the plane.

We also have the line's direction vector $d = (7, 7, 5)$, which also lies in the plane.

In summary:

- Vector v_1 from the point to the line point: (3, 3, 7)
- Direction vector of the line: (7, 7, 5)

4. Calculate the Normal Vector to the Plane

The normal vector n is perpendicular to any vectors lying in the plane. Therefore, it can be obtained by taking the cross product of two vectors in the plane:

- $n = v_1 \times d$

Calculating the cross product:

\[
\begin{aligned}
\mathbf{n} &= \mathbf{v}_1 \times \mathbf{d} \\
&=
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & 3 & 7 \\
7 & 7 & 5 \\
\end{vmatrix}
\end{aligned}
\]

Expanding the determinant:

\[
\mathbf{n} = \mathbf{i}(3 \times 5 - 7 \times 7) - \mathbf{j}(3 \times 5 - 7 \times 7) + \mathbf{k}(3 \times 7 - 3 \times 7)
\]

Let's compute each component:

- For i component:

\[
3 \times 5 - 7 \times 7 = 15 - 49 = -34
\]

- For j component:

\[
-(3 \times 5 - 7 \times 7) = - (15 - 49) = -(-34) = 34
\]

- For k component:

\[
3 \times 7 - 3 \times 7 = 21 - 21 = 0
\]

Therefore:

\[
\boxed{
\mathbf{n} = (-34, 34, 0)

}
\]

We can simplify the normal vector by dividing by 17:

$$
\mathbf{n} = (-2, 2, 0)
$$

This simplified normal vector is easier to work with and is still perpendicular to the plane.

5. Write the Equation of the Plane

The general equation of a plane with normal vector n = (A, B, C) passing through point $P_0 = (x_0, y_0, z_0)$ is:

$$
A(x - x_0) + B(y - y_0) + C(z - z_0) = 0
$$

Using n = (-2, 2, 0) and the point (1, 2, -6):

$$
-2(x - 1) + 2(y - 2) + 0(z + 6) = 0
$$

Simplify:

$$
-2x + 2 + 2y - 4 = 0
$$

$$
-2x + 2y - 2 = 0
$$

Divide through by -2:

$$
x - y + 1 = 0
$$

Final Equation of the Plane:

$$
x - y + 1 = 0
$$

This is the required plane containing the point (1, 2, -6) and the line R^-→(t).

Geometric Interpretation and Validation

Understanding the geometric significance of the above steps enhances comprehension:

- The point (1, 2, -6) is a specific location in space.
- The line $\vec{R}(t)$ provides an entire set of points extending infinitely in both directions, passing through (4, 5, 1) with direction vector (7, 7, 5).
- The plane must contain both the point and the line, so it must include the line's entire span and pass through the given point.

Validation:

- Check if the point (1, 2, -6) satisfies the plane equation:

$$
x - y + 1 = 0
$$

$$
1 - 2 + 1 = 0
$$

Yes, it satisfies the equation.

- Check if the line's point $R_0 = (4, 5, 1)$:

$$
4 - 5 + 1 = 0
$$

Yes, it satisfies the plane equation.

- Check if the line's direction vector is perpendicular to the normal vector:

$$
\mathbf{n} \cdot \mathbf{d} = (-2, 2, 0) \cdot (7, 7, 5) = -2 \times 7 + 2 \times 7 + 0 \times 5 = -14 + 14 + 0 = 0
$$

Since the dot product is zero, the direction vector is perpendicular to the normal vector, confirming the line lies within the plane.

Additional Considerations and General Methods

When solving similar problems, consider the following:

- Always identify a point on the line and the direction vector.
- Use vector subtraction to find vectors in the plane.
- Cross product calculations are crucial to find the normal vector.
- Confirm the plane passes through given points by substituting into the equation.
- For more complex scenarios, parametric equations or vector algebra methods can be combined.

Applications of Finding Planes in Space

Understanding how to find a plane given specific points and lines has numerous practical applications:

- Computer Graphics: Rendering 3D objects requires defining planes for surfaces.
- Engineering: Designing structural components often involves plane equations.
- Navigation and Robotics: Calculating planes helps in spatial orientation and path planning.
- Physics: Planes are used to model surfaces and boundary conditions.

Conclusion

In summary, to find a plane containing a specific point and a line, follow these key steps:

- Identify a point on the line.
- Find the direction vector of the line.
- Determine a vector from

Frequently Asked Questions

How do I find a plane that contains the point (1, 2, -6) and the line R(t) = (4, 5, 1) + t(7, 7, 5)?

To find such a plane, identify a point on the line (say, R(0) = (4, 5, 1)) and the direction vector of the line (v = (7, 7, 5)). Then, find a vector from the point (1, 2, -6) to a point on the line, and use the cross product of this vector and the direction vector to determine the plane's normal vector. Finally, use the point-normal form of the plane equation.

What is the first step in determining the plane containing point (1, 2, -6) and line R(t)?

The first step is to select a specific point on the line, such as R(0) = (4, 5, 1), and find the vector connecting this point to the given point (1, 2, -6).

How do I find the normal vector of the plane in this problem?

The normal vector can be found by taking the cross product of two vectors: one from the point on the line to (1, 2, -6), and the direction vector of the line. The resulting vector is perpendicular to both, thus normal to the plane.

What is the role of the cross product in this problem?

The cross product of the two vectors gives the normal vector to the plane, which is essential for writing the plane's equation.

How do I write the equation of the plane after finding the normal vector?

Use the point-normal form: $n \cdot (r - r_0) = 0$, where n is the normal vector, r is the general point (x, y, z), and r_0 is a known point on the plane, such as (4, 5, 1).

Can you provide the explicit equation of the plane containing (1, 2, -6) and the line R(t)?

Yes. First, find the vectors: from (4, 5, 1) to (1, 2, -6) is (-3, -3, -7). The line's direction vector is (7, 7, 5). The cross product of these two vectors gives the normal vector, which leads to the plane equation.

What is the importance of verifying that both the point and line lie on the plane?

Verifying ensures that the calculated plane correctly contains the given point and the entire line, confirming the accuracy of your solution.

Is the line R(t) contained entirely within the plane once the plane is found?

Yes, since the plane contains the line R(t) by construction, the entire line lies within the plane.

Additional Resources

Find A Plan Containing The Point (1,2,-6) And The Line R(t) = (4, 5, 1) + t(7, 7, 5)

Introduction

In the realm of vector calculus and analytical geometry, problems involving the determination of planes through specific points and lines are fundamental. Such problems are not mere exercises but crucial for understanding more complex spatial relationships and for applications spanning physics, engineering, and computer graphics.

This detailed exploration focuses on a particular problem:

> Find a plane that contains a given point (1, 2, -6) and a specified line R(t) = (4, 5, 1) + t(7, 7, 5).

This task involves understanding the geometric structures involved, extracting necessary vector information, and applying the mathematical principles to derive the plane's equation. We will dissect each step carefully to provide clarity and depth.

Understanding the Problem

The Geometric Elements

To approach this problem systematically, we first interpret the given elements:

- Point $P_0 = (1, 2, -6)$: This is a specific point through which the plane must pass.
- Line $R(t) = (4, 5, 1) + t(7, 7, 5)$: This parametric line is given, and the plane must contain this entire line.

Objective

- Find the equation of the plane that contains the point P_0 and the line $R(t)$.

Geometric Foundations

Planes in 3D Space

A plane in three-dimensional space can be represented by a point and a normal vector:

$$
\text{Plane Equation:} \quad \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0
$$

where:

- $\mathbf{n}$ is the normal vector to the plane.
- $\mathbf{r} = (x, y, z)$ is a generic point on the plane.
- $\mathbf{r}_0$ is a specific point on the plane.

Necessary Components

To find the plane, we need:

1. A point on the plane: Given as $P_0 = (1, 2, -6)$.
2. The normal vector $\mathbf{n}$: To determine $\mathbf{n}$, we need to find two independent vectors lying on the plane and then take their cross product.

Step-by-Step Solution Approach

Step 1: Identify a Point on the Line

Since the line $R(t)$ is given by:

$$
R(t) = (4, 5, 1) + t(7, 7, 5)
$$

- For any real t, the point $R(t)$ lies on the line.
- Let's select a specific value of t, for simplicity $t=0$:

$$
\mathbf{R}_0 = (4, 5, 1)
$$

- This point lies on the line and, consequently, on the plane we seek (since the plane contains the entire line).

Step 2: Find Direction Vectors

To determine the plane's normal vector, we need two vectors lying on the plane.

- Vector along the line $R(t)$:

$$
\mathbf{v}_1 = \text{direction vector of } R(t) = (7, 7, 5)
$$

- Vector from the given point P_0 to a point on the line $R(t)$:

$$
\mathbf{v}_2 = \mathbf{R}_0 - P_0 = (4 - 1, 5 - 2, 1 - (-6)) = (3, 3, 7)
$$

Note: Alternatively, you could consider the vector from P_0 to any point on the line, such as \(R(0) \).

Step 3: Find the Normal Vector to the Plane

- The normal vector \(\mathbf{n} \) is perpendicular to both \(\mathbf{v}_1 \) and \(\mathbf{v}_2 \).
- Therefore, compute:

\[
\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2
\]

Computing the Cross Product

Cross Product Formula

Given vectors \(\mathbf{a} = (a_1, a_2, a_3) \) and \(\mathbf{b} = (b_1, b_2, b_3) \):

\[
\mathbf{a} \times \mathbf{b} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3
\end{vmatrix}
= (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}
\]

Applying to Our Vectors

- \(\mathbf{v}_1 = (7, 7, 5) \)
- \(\mathbf{v}_2 = (3, 3, 7) \)

Compute:

\[
\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
7 & 7 & 5 \\
3 & 3 & 7
\end{vmatrix}
\]

Calculate component-wise:

- \(n_x = (7)(7) - (5)(3) = 49 - 15 = 34 \)
- \(n_y = - [(7)(7) - (5)(3)] = - (49 - 15) = -34 \)

Wait, that indicates a calculation issue: the formula for the y-component is:

$$
n_y = - [a_1b_3 - a_3b_1] = - [7 \times 7 - 5 \times 3] = - (49 - 15) = -34
$$

Similarly, for n_z:

$$
n_z = (7)(3) - (7)(3) = 21 - 21 = 0
$$

Hold on, this suggests a potential inconsistency, so let's double-check the calculations:

Recalculating step-by-step:

- $n_x = (a_2b_3 - a_3b_2) = (7)(7) - (5)(3) = 49 - 15 = 34$
- $n_y = - [a_1b_3 - a_3b_1] = - [7 \times 7 - 5 \times 3] = - (49 - 15) = -34$
- $n_z = a_1b_2 - a_2b_1 = 7 \times 3 - 7 \times 3 = 21 - 21 = 0$

Therefore, the normal vector is:

$$
\boxed{
\mathbf{n} = (34, -34, 0)
}
$$

For simplicity, divide by 34:

$$
\mathbf{n} = (1, -1, 0)
$$

Final Equation of the Plane

Step 4: Write the Plane Equation

Using the point-normal form:

$$
\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0
$$

where:

- $\mathbf{n} = (1, -1, 0)$
- $\mathbf{r}_0 = (1, 2, -6)$

Let $\mathbf{r} = (x, y, z)$, then:

$$
(1, -1, 0) \cdot (x - 1, y - 2, z + 6) = 0
$$

Calculate the dot product:

$$
1 \cdot (x - 1) + (-1) \cdot (y - 2) + 0 \cdot (z + 6) = 0
$$

Simplify:

$$
x - 1 - y + 2 = 0
$$

$$
x - y + 1 = 0
$$

Final Equation of the Plane:

$$
\boxed{
x - y + 1 = 0
}
$$

Verification and Consistency Checks

1. Does the plane contain point $P_0 = (1, 2, -6)$?

Substitute $x=1$, $y=2$:

$$
1 - 2 + 1 = 0 \quad \Rightarrow \quad 0=0
$$

Yes, the point satisfies the plane equation.

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