Find An Equation Of The Circle That Has Center 5, 6 And Passes Through 1, 1.

Find An Equation Of The Circle That Has Center 5, 6 And Passes Through 1, 1.

Understanding how to formulate the equation of a circle given its center and a point on its circumference is a fundamental aspect of coordinate geometry. In this article, we will explore the step-by-step process to find the equation of a circle with a specific center at the point (5, 6) and passing through the point (1, 1). We will delve into the mathematical concepts involved, including calculating the radius using the distance formula, and then use the standard form of a circle's equation to derive the final expression.

Basics of the Equation of a Circle

Standard Form of a Circle's Equation

The general, or standard, form of the equation of a circle in a coordinate plane is given by:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) is the center of the circle
- r is the radius of the circle

Understanding this form is crucial because, once the center and radius are known, the equation can be directly written.

Key Components

- The center (h, k) defines the position of the circle on the coordinate plane.
- The radius r determines the size of the circle, i.e., the distance from the center to any point on the circle.

Determining the Radius

The Distance Formula

To find the radius r, we need to determine the distance from the center $(5, 6)$ to the point $(1, 1)$ which lies on the circle. This is calculated using the distance formula:

$$ r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

where:

- (x_1, y_1) is the center point $(5, 6)$
- (x_2, y_2) is the point on the circle $(1, 1)$

Calculating the Radius Step-by-Step

Let's substitute the values into the distance formula:

$$ r = \sqrt{(1 - 5)^2 + (1 - 6)^2} $$

Calculating each term:

- $1 - 5 = -4$
- $1 - 6 = -5$

Squaring these:

- $(-4)^2 = 16$
- $(-5)^2 = 25$

Adding these:

$$ r = \sqrt{16 + 25} = \sqrt{41} $$

Hence, the radius of the circle is $\sqrt{41}$.

Formulating the Equation of the Circle

Substituting the Center and Radius

Now that we know:

- Center $(h, k) = (5, 6)$
- Radius $r = \sqrt{41}$

we can substitute these into the standard form:

$$ (x - 5)^2 + (y - 6)^2 = (\sqrt{41})^2 $$

Since $(\sqrt{41})^2 = 41$, the equation simplifies to:

$$ (x - 5)^2 + (y - 6)^2 = 41 $$

This is the required equation of the circle.

Understanding the Equation

Interpreting the Components

- The $x - 5$ term indicates the horizontal shift from the origin to the center at $x=5$.
- The $y - 6$ term indicates the vertical shift from the origin to the center at $y=6$.
- The right side, 41, is the square of the radius, confirming the size of the circle.

Graphical Representation

Plotting the circle involves:

- Marking the center at $(5, 6)$.
- Drawing a circle with radius $\sqrt{41}$, which is approximately 6.4 units.

- Ensuring the circle passes through $(1, 1)$, which lies exactly 6.4 units away from the center.

Alternative Forms and Additional Insights

Expanded Form of the Equation

Expanding the standard form yields:

$$
\begin{aligned}
(x - 5)^2 + (y - 6)^2 &= 41 \\
x^2 - 10x + 25 + y^2 - 12y + 36 &= 41 \\
x^2 + y^2 - 10x - 12y + (25 + 36) &= 41 \\
x^2 + y^2 - 10x - 12y + 61 &= 41 \\
x^2 + y^2 - 10x - 12y &= -20
\end{aligned}
$$

This form is useful for algebraic manipulations and certain types of problem-solving.

Graphing the Circle

To graph:

1. Plot the center at $(5, 6)$.
2. Draw a circle with radius approximately 6.4 units.
3. Confirm it passes through the point $(1, 1)$.

Graphing tools or software can visually verify the correctness of the equation.

Practical Applications and Related Problems

Real-World Applications

- Designing circular tracks or fences.
- Signal coverage areas in telecommunications.
- Mapping zones of influence in spatial analysis.

Related Mathematical Problems

- Finding the equation of a circle given different points.
- Determining the circle passing through three points.
- Tangent lines to the circle at specific points.

Summary and Conclusion

In summary, to find the equation of a circle with a given center and passing through a specific point, the key steps involve:

1. Calculating the radius using the distance formula.
2. Substituting the center coordinates and radius into the standard form.
3. Simplifying the equation for clarity.

Applying these steps to the problem at hand, with center $(5, 6)$ and passing through $(1, 1)$, the equation of the circle is:

$$ (x - 5)^2 + (y - 6)^2 = 41 $$

This equation precisely describes the circle's position and size on the coordinate plane, providing a complete mathematical characterization.

Understanding such concepts enhances problem-solving skills in geometry and analytical mathematics, and serves as a foundation for more complex geometrical analyses and applications.

Frequently Asked Questions

What is the general form of the equation of a circle with center (h, k) and radius r?

The general form is $(x - h)^2 + (y - k)^2 = r^2$.

How do you find the radius of the circle given the center and a point it passes through?

Use the distance formula: $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) is the center and (x_2, y_2) is the point on the circle.

What are the steps to find the equation of a circle with center at (5, 6) passing through (1, 1)?

First, calculate the radius using the distance formula between (5, 6) and (1, 1), then substitute the center and radius into the circle equation $(x - 5)^2 + (y - 6)^2 = r^2$.

What is the radius of the circle with center (5, 6) passing through (1, 1)?

The radius is $\sqrt{(1 - 5)^2 + (1 - 6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41}$.

What is the equation of the circle with center (5, 6) passing through (1, 1)?

The equation is $(x - 5)^2 + (y - 6)^2 = 41$.

How can you verify that a point lies on the circle's equation?

Substitute the point's coordinates into the circle's equation; if the equation holds true, the point lies on the circle.

Additional Resources

Finding an Equation of a Circle Given Its Center and a Point on the Circle

Understanding how to determine the equation of a circle when given its center and a point on the circle is a fundamental concept in coordinate geometry. This process involves applying the standard form of a circle's equation and calculating the radius based on the distance between the center and the point. This detailed guide will walk you through the entire method, providing clarity, step-by-step calculations, and insights into the underlying principles.

Introduction to the Problem

Suppose you are given:

- The center of the circle at the coordinates (5, 6)
- A point on the circle at (1, 1)

Your goal is to find the equation of the circle that satisfies these conditions.

Such problems are common in algebra and coordinate geometry and are foundational for understanding more complex geometric figures and their properties.

Understanding the Basic Equation of a Circle

Before delving into calculations, it’s crucial to recall the standard form of a circle's equation:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) is the center of the circle
- r is the radius of the circle

Given this form, the primary task becomes determining the radius r. Once r is known, plugging values into the standard form yields the complete equation.

Step-by-Step Approach to Find the Equation

Step 1: Identify the Center Coordinates

From the problem statement:

- Center $C = (h, k) = (5, 6)$

Step 2: Use the Given Point on the Circle

- Point $P = (x, y) = (1, 1)$

Since P lies on the circle, it must satisfy the circle's equation once r is known.

Step 3: Calculate the Radius r

The radius is the distance between the center C and the point P. Use the distance formula:

$$
r = \sqrt{(x - h)^2 + (y - k)^2}
$$

Substitute the known values:

$$
r = \sqrt{(1 - 5)^2 + (1 - 6)^2}
$$

Calculate each term:

- $(1 - 5)^2 = (-4)^2 = 16$
- $(1 - 6)^2 = (-5)^2 = 25$

Sum:

$$
r = \sqrt{16 + 25} = \sqrt{41}
$$

Step 4: Write the Equation of the Circle

Using the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

\]

Plugging in the known values:

\[
(x - 5)^2 + (y - 6)^2 = (\sqrt{41})^2
\]

Simplify:

\[
(x - 5)^2 + (y - 6)^2 = 41
\]

This is the equation of the circle.

Alternative Forms and Additional Insights

General Equation of the Circle

The standard form is most useful for geometric interpretation, but sometimes the equation is expressed in expanded form:

\[
x^2 + y^2 + Dx + Ey + F = 0
\]

To convert, expand the standard form:

\[
(x - 5)^2 + (y - 6)^2 = 41
\]

which becomes:

\[
x^2 - 10x + 25 + y^2 - 12y + 36 = 41
\]

Combine like terms:

$$
x^2 + y^2 - 10x - 12y + (25 + 36) = 41
$$

$$
x^2 + y^2 - 10x - 12y + 61 = 41
$$

Subtract 41 from both sides:

$$
x^2 + y^2 - 10x - 12y + 20 = 0
$$

This is the expanded form of the circle's equation.

Center-Radius Form vs. General Form

- Center-radius form: $(x - h)^2 + (y - k)^2 = r^2$
- General form: $x^2 + y^2 + Dx + Ey + F = 0$

Both describe the same circle but serve different purposes. The center-radius form emphasizes the geometric features, while the general form is often used for algebraic manipulations and solving systems involving circles.

Key Concepts and Important Points

- Distance Formula: The calculation of the radius relies on the distance formula, which is derived from the Pythagorean theorem:

$$
\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

- Radius: The radius is always a positive value and represents the distance from the center to any point on the circle.

- Unique Circle: Given a center and a point on the circle, the circle's equation is uniquely determined because the radius is fixed.

- Symmetry: Circles are symmetric about their center. This symmetry can be useful for graphical representation and understanding geometric properties.

Applications and Extended Topics

1. Finding Equations of Circles with Different Conditions

- Given the center and radius: Directly substitute into the standard form.
- Given two points on the circle: Calculate the radius as the distance from the center to either point.
- Given the general form: Complete the square to find the center and radius.

2. Graphical Representation

Plotting the circle involves:

- Marking the center at (5, 6)
- Drawing a circle with radius $\sqrt{41}$ around it

This visual helps in understanding the location and size of the circle relative to coordinate axes.

3. Equation of a Circle in Real-World Problems

- Engineering: Design of circular components
- Physics: Motion paths
- Computer Graphics: Rendering circles

Understanding how to derive the equation from given data is vital in these applications.

Common Mistakes and Tips

- Mixing up the center and point coordinates: Carefully identify the given data.
- Incorrect calculation of the radius: Always double-check the distance formula and calculations.
- Forgetting to square the radius when substituting into the equation.

- Misplacing signs in the standard form: Remember that the equation uses $(x - h)^2$ and $(y - k)^2$.

Summary and Final Thoughts

The process of finding the equation of a circle given its center and a point on the circle is straightforward once you understand the relationship between the center, the point, and the radius. The key steps involve:

- Recognizing the standard form of the circle's equation
- Calculating the radius using the distance formula
- Substituting the center coordinates and radius into the standard form

In this specific problem:

- Center: (5, 6)
- Point on circle: (1, 1)
- Radius: $\sqrt{41}$

The final equation:

$$
\boxed{
(x - 5)^2 + (y - 6)^2 = 41
}
$$

or, in expanded form:

$$
x^2 + y^2 - 10x - 12y + 20 = 0
$$

Mastering this procedure enhances your ability to solve a variety of geometric problems involving circles, and it forms a foundation for understanding more complex concepts such as circle intersections, tangent lines, and circle equations in higher dimensions.

In conclusion, whether for academic purposes or practical applications, knowing how to derive a circle's equation from its center and a point on it is a vital skill in geometry. With practice, this process becomes intuitive, enabling you to analyze and interpret geometric figures with confidence and precision.

Find An Equation Of The Circle That Has Center 5 6 And Passes Through 1 1

Find An Equation Of The Circle That Has Center 5 6 And Passes Through 1 1

Related Articles

- If I Failed Three Quarters Can I Still Pass If I Do Well In The Last Quarter?
- True Or False: On A Scatter Diagram, Costs Are Plotted On The Horizontal Axis.
- Is The Statement True Or False? If F(x, Y) Is A Constant Function, Then Df = 0.

Back to Home