Find The Area Of The Region Enclosed By The Curves: Y=1/x, Y=1/ X^2 And X=3 .

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Understanding the area enclosed between curves is a fundamental topic in integral calculus. Calculating such an area involves understanding the geometric shape formed when multiple curves intersect and determining the limits of integration accordingly. In this article, we will explore how to find the exact area of the region enclosed by the curves $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, and the vertical line $x = 3$. We will discuss the steps involved in setting up the integral, analyze the points of intersection, and perform the integration to arrive at the precise area.

Understanding the Curves and Their Behavior

Before delving into the calculation, it is essential to understand the behavior of the given curves:

- Curve 1: $y = \frac{1}{x}$
- Hyperbolic curve.
- Decreases monotonically for $x > 0$.
- Approaches zero as $x \to \infty$.
- Has a vertical asymptote at $x = 0$.

- Curve 2: $y = \frac{1}{x^2}$
- Also a hyperbola, steeper than $y=1/x$.
- Decreases monotonically for $x > 0$.
- Approaches zero faster than $y=1/x$ as $x \to \infty$.
- Vertical asymptote at $x=0$.

- Line: $x=3$
- Vertical boundary line at $x=3$.

Since both $y=1/x$ and $y=1/x^2$ are defined for $x > 0$, we focus on $x > 0$. The region of interest is enclosed between these curves and the line $x=3$.

Identifying the Points of Intersection

The first step in calculating the enclosed area is to find the points where the curves intersect. The points of intersection determine the limits of integration.

Step 1: Find where $y=1/x$ and $y=1/x^2$ intersect

Set:

$$
\frac{1}{x} = \frac{1}{x^2}
$$

Multiply both sides by x^2 (assuming $x \neq 0$):

$$
x = 1
$$

Thus, the two curves intersect at:

$$
x=1
$$

At this point, the y-coordinate is:

$$
y = \frac{1}{1} = 1
$$

Step 2: Determine the region between $x=1$ and $x=3$

- For $1 < x < 3$, compare $y=1/x$ and $y=1/x^2$:
- Since $x > 1$, we have $1/x > 1/x^2$, because dividing by a larger number decreases the value.
- Therefore, in this interval, $y=1/x$ is above $y=1/x^2$.

Setting Up the Integral for the Enclosed Area

The region enclosed between the curves from $x=1$ to $x=3$ is bounded by:

- On the top: $y=1/x$
- On the bottom: $y=1/x^2$

The total area A can be computed by integrating the difference between these two curves over the interval $[1,3]$:

$$
A = \int_{x=1}^{3} \left(\frac{1}{x} - \frac{1}{x^2} \right) dx
$$

This integral sums the vertical slices from $x=1$ to $x=3$, measuring the vertical distance between the curves at each point.

Calculating the Integral

Step 1: Write the integral explicitly

$$
A = \int_{1}^{3} \left(\frac{1}{x} - \frac{1}{x^2} \right) dx
$$

Step 2: Simplify the integrand

Recall that $\frac{1}{x^2} = x^{-2}$. So,

$$
A = \int_{1}^{3} \left(x^{-1} - x^{-2} \right) dx
$$

Step 3: Compute the indefinite integral

The indefinite integrals are:

$$
\int x^{-1} dx = \ln|x| + C
$$
$$
\int x^{-2} dx = -x^{-1} + C
$$

Therefore,

$$
A = \left[\ln|x| + x^{-1} \right]_{1}^{3}
$$

Note: As $x>0$ in our interval, absolute value signs can be omitted.

Step 4: Evaluate the definite integral

At $x=3$:

$$
\ln 3 + \frac{1}{3}
$$

At $x=1$:

$$
\ln 1 + 1 = 0 + 1 = 1
$$

Subtract:

$$
A = \left(\ln 3 + \frac{1}{3} \right) - 1
$$

Simplify:

$$
A = \ln 3 + \frac{1}{3} - 1
$$

Final Expression for the Enclosed Area

The exact area enclosed by the curves $y=1/x$, $y=1/x^2$, and the line $x=3$ is:

$$
\boxed{
A = \ln 3 + \frac{1}{3} - 1
}
$$

This is a precise, exact value expressed in terms of natural logarithms.

Numerical Approximation of the Area

To get a numerical estimate of the area:

- Recall that $\ln 3 \approx 1.0986$.

Thus:

$$
A \approx 1.0986 + 0.3333 - 1 = 0.4319
$$

The approximate area is about 0.432 square units.

Summary of the Calculation Process

To summarize, the process involved:

- Recognizing the curves and their behavior.
- Finding points of intersection to determine limits.
- Setting up the integral as the difference between the curves over the interval.
- Computing the integral using basic antiderivatives.
- Simplifying to arrive at the exact area expression.

Additional Insights and Generalizations

- The approach demonstrated here can be generalized for other regions bounded by different curves.
- When dealing with more complicated curves, numerical methods or advanced integration techniques may be necessary.
- Always verify the points of intersection and the relative positions of the curves before setting up the integral.
- For regions extending beyond $x=1$ and $x=3$, similar analysis can be performed, considering the domain restrictions of the functions.

Conclusion

Calculating the area enclosed between curves such as $y=1/x$ and $y=1/x^2$, bounded by the vertical line $x=3$, is a classic application of integral calculus. By carefully analyzing the intersection points, setting up the correct limits, and integrating the difference of the functions, we obtain an exact expression for the area:

\[
\boxed{
A = \ln 3 + \frac{1}{3} - 1
}
\]

This problem exemplifies the importance of understanding the behavior of functions, their intersections, and the power of definite integrals in computing areas between curves. Whether for academic purposes or practical applications, mastering such techniques is fundamental for any calculus student or enthusiast.

Frequently Asked Questions

How do I determine the points of intersection between the curves y = 1/x, y = 1/x^2, and the line x = 3?

To find the intersection points, set y = 1/x equal to y = 1/x^2, which gives 1/x = 1/x^2. Solving this yields x = 1 (since $x \neq 0$), and substituting back, y = 1. The line x = 3 intersects with the curves at x = 3, y = 1/3 (from y = 1/x) and y = 1/9 (from y = 1/x^2).

What is the correct order of integration when finding the area enclosed by these curves?

The area can be found by integrating with respect to y or x, but typically, setting up the integral with respect to x is straightforward here. First, identify the x-intervals where the curves enclose the region, then integrate the difference between the top and bottom curves accordingly.

How do I set up the integral to find the area of the region enclosed by y = 1/x, y = 1/x^2, and x = 3?

The region is bounded between x = 1 and x = 3, with y varying between y = 1/x^2 (bottom) and y = 1/x (top). The area A is given by A = ∫ from x=1 to x=3 of [(1/x) - (1/x^2)] dx.

How do I evaluate the integral for the area enclosed by these curves?

Evaluate the integral A = $\int_1^3$ (1/x - 1/x^2) dx. This simplifies to $\int_1^3$ 1/x dx - $\int_1^3$ 1/x^2 dx. Integrate each term separately: ∫ 1/x dx = ln|x|, and ∫ 1/x^2 dx = -1/x. Substitute the limits and compute the difference to find the area.

What is the final value of the area enclosed by the curves $y = 1/x$, $y = 1/x^2$, and $x = 3$?

Calculating the integrals, we have: $A = [\ln|x|]_1^3 - [-1/x]_1^3 = (\ln 3 - \ln 1) - (-1/3 + 1/1) = (\ln 3 - 0) - (-1/3 + 1) = \ln 3 - (-1/3 + 1) = \ln 3 - (2/3)$. Therefore, the area is $\ln 3 - 2/3$.

Are there any special considerations when calculating the area between these curves?

Yes, ensure that the curves are correctly identified as the upper and lower boundaries over the interval, especially since $y = 1/x$ is above $y = 1/x^2$ for $x > 1$. Also, confirm the limits of integration correspond to the intersection points, and verify the region is correctly enclosed before integrating.

Additional Resources

Find The Area Of The Region Enclosed By The Curves: $y=1/x$, $y=1/x^2$, and $x=3$

Introduction

In the realm of calculus and analytical geometry, the determination of the area enclosed by curves is both a fundamental and intricate problem. It involves understanding the behavior of functions, identifying intersection points, and applying integral calculus techniques to compute the exact area. This investigation focuses on a specific configuration: the region enclosed by the curves $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, and the vertical line $x=3$.

This problem exemplifies classic methods in calculus, including setting up integrals for bounded regions, analyzing the intersections of curves, and considering the implications of the functions' behavior over specified domains. The goal is to precisely determine the total area of the region enclosed by these curves and the line, providing a detailed methodology that can serve as a guide for similar problems.

Understanding the Curves and Their Behavior

Before delving into the calculations, it is crucial to understand the properties of the curves involved:

The Curve $y = \frac{1}{x}$

- Defined for $x \neq 0$.

- Hyperbolic in nature, decreasing monotonically for $x > 0$.
- Approaches $y \to 0$ as $x \to \infty$, and $y \to \infty$ as $x \to 0^+$.

The Curve $y = \frac{1}{x^2}$

- Also defined for $x \neq 0$.
- Decreasing for $x > 0$, with a steeper decline compared to $y = 1/x$.
- Approaches $y \to 0$ as $x \to \infty$, and $y \to \infty$ as $x \to 0^+$.

The Vertical Line $x = 3$

- Acts as a boundary on the right side of the region.
- The region of interest is confined between $x=0$ (or some other lower boundary) and $x=3$.

Identifying the Region of Interest

To find the enclosed area, the initial step involves determining the intersection points of the curves $y=1/x$ and $y=1/x^2$.

Finding Intersection Points

Set $\frac{1}{x} = \frac{1}{x^2}$:

$$
\frac{1}{x} = \frac{1}{x^2}
$$

Multiply both sides by x^2:

$$
x^2 \times \frac{1}{x} = x^2 \times \frac{1}{x^2}
$$
$$
x = 1
$$

Thus, the two curves intersect at $x=1$. To verify the corresponding y-coordinate:

$$
y = \frac{1}{1} = 1
$$

The intersection point is at $(1,1)$.

Analyzing the Behavior in the Domain $x \in (0,3]$

- For $0 < x < 1$, note that $y = 1/x$ is greater than $y = 1/x^2$, since:

$$
\frac{1}{x} > \frac{1}{x^2}
$$

because $x < 1 \Rightarrow 1/x > 1/x^2$.

- For $x > 1$, the inequality reverses:

$$
\frac{1}{x} < \frac{1}{x^2}
$$

which indicates that above $x=1$, $y=1/x^2$ is the larger of the two functions.

Visualizing the Enclosed Area

The region enclosed by these curves and the line $x=3$ can be visualized as follows:

- From $x=0$ (or an appropriate lower limit approaching zero) to $x=1$, the area between $y=1/x$ (upper curve) and $y=1/x^2$ (lower curve).
- From $x=1$ to $x=3$, the area between $y=1/x^2$ (upper curve) and $y=1/x$ (lower curve).
- The line $x=3$ acts as the right boundary of the region.

Since the functions tend toward infinity as $x \to 0^+$, and the area from 0 to some small positive value is unbounded, the region's area is finite only if the region is considered starting at some positive value $a > 0$. Typically, in such problems, the lower limit is taken as approaching zero but not including it explicitly, or the problem is constrained to $x \in [a, 3]$ for some $a > 0$.

For the purpose of this analysis, we assume the region is bounded between $x = \varepsilon > 0$ and $x=3$. As $\varepsilon \to 0^+$, the area approaches a finite limit, which will be examined.

Setting Up the Integrals

The total enclosed area A can be expressed as the sum of two integrals, representing the regions where each curve is on top:

$$
A = \underbrace{\int_{a}^{1} \left(\frac{1}{x} - \frac{1}{x^2} \right) dx}_{\text{from } x=a \text{ to } x=1} + \underbrace{\int_{1}^{3} \left(\frac{1}{x^2} - \frac{1}{x} \right) dx}_{\text{from } x=1 \text{ to } x=3}
$$

The choice of $a \to 0^+$ recognizes the potential divergence at zero, but for practical finite calculations, we analyze from a small positive a.

Calculating the Integrals

Integral from a to 1:

$$
I_1 = \int_{a}^{1} \left(\frac{1}{x} - \frac{1}{x^2} \right) dx
$$

Compute separately:

$$
I_1 = \int_{a}^{1} \frac{1}{x} dx - \int_{a}^{1} \frac{1}{x^2} dx
$$

$$
I_1 = \left[\ln|x| \right]_{a}^{1} - \left[-\frac{1}{x} \right]_{a}^{1}
$$

$$
I_1 = \left(\ln 1 - \ln a \right) - \left(-\frac{1}{1} + \frac{1}{a} \right)
$$

$$
I_1 = (0 - \ln a) - (-1 + \frac{1}{a}) = - \ln a + 1 - \frac{1}{a}
$$

As $a \to 0^+$:

- $\ln a \to -\infty$,
- $1/a \to +\infty$.

Thus, the integral diverges to infinity, indicating the area is unbounded near zero unless the region is truncated at some positive a.

Integral from 1 to 3:

$$
I_2 = \int_{1}^{3} \left(\frac{1}{x^2} - \frac{1}{x} \right) dx
$$

\]

Compute separately:

\[
I_2 = \int_{1}^{3} \frac{1}{x^2} dx - \int_{1}^{3} \frac{1}{x} dx
\]

\[
I_2 = \left[-\frac{1}{x} \right]_1^{3} - \left[\ln|x| \right]_1^{3}
\]

\[
I_2 = \left(-\frac{1}{3} + \frac{1}{1} \right) - (\ln 3 - \ln 1) = \left(-\frac{1}{3} + 1 \right) - (\ln 3 - 0)
\]

\[
I_2 = \left(\frac{2}{3} \right) - \ln 3
\]

Final Expression for the Enclosed Area

Combining the integrals, the total area \(A \) (from \(a \) to 3) is:

\[
A(a) = I_1 + I_2 = \left(- \ln a + 1 - \frac{1}{a} \right) + \left(\frac{2}{3} - \ln 3 \right)
\]

As \(a \to 0^+ \), \(A(a) \to \infty \), which implies the area is infinite in the vicinity of \(x=0 \). To have a finite, well-defined area, the region must be restricted to \(x \in [a, 3] \) with \(a >

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