Find The Area Under Y = 2x On [0, 3] In The First Quadrant. Explain Your Method.

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Understanding how to compute the area under a curve is a fundamental concept in calculus, particularly in integral calculus. When asked to find the area under the graph of a function over a specific interval, the standard approach involves setting up and evaluating an integral. In this article, we will explore the process of determining the area under the curve $y = 2x$ on the interval [0, 3], focusing on the first quadrant, where all coordinates are positive. We will walk through the method step-by-step, including the formulation of the integral, the calculation process, and interpreting the results.

Understanding the Problem

Before jumping into calculations, it is crucial to understand what the problem asks for:

- The function in question: $y = 2x$
- The interval: from $x = 0$ to $x = 3$
- The region of interest: the area in the first quadrant, where $y \geq 0$ and $x \geq 0$

Our goal is to find the exact numerical value of the area enclosed between the curve $y = 2x$, the x-axis, and the vertical lines $x = 0$ and $x = 3$.

Methodology for Finding the Area

The standard method for finding the area under a curve between two points involves the use of definite integrals. Here's an overview of the steps:

1. Set up the definite integral of the function $y = 2x$ over the interval [0, 3].
2. Calculate the antiderivative (also known as the indefinite integral) of $2x$.
3. Evaluate the definite integral by applying the Fundamental Theorem of Calculus.
4. Interpret the result as the area under the curve, which should be positive given the curve lies above the x-axis in this interval.

Let's examine each step in detail.

Step 1: Setting Up the Integral

The area A under the curve $y = 2x$ from $x = 0$ to $x = 3$ is expressed as:

$$A = \int_0^3 2x \, dx$$

This integral represents the sum of infinitely many infinitesimal rectangles under the curve, whose heights are determined by the function value at each x, and widths are infinitesimally small.

Step 2: Finding the Antiderivative

To evaluate the integral, we need the antiderivative of $2x$. Recall:

- The power rule for integration: $\int x^n \, dx = (x^{n+1}) / (n+1) + C$, for $n \neq -1$
- Applying this to $2x$:

$$\int 2x \, dx = 2 \int x \, dx = 2 (x^2 / 2) + C = x^2 + C$$

Thus, the indefinite integral of 2x is x^2 + C.

Step 3: Evaluating the Definite Integral

Using the antiderivative, evaluate at the bounds:

A = [x^2] from x = 0 to x = 3

= (3)^2 - (0)^2 = 9 - 0 = 9

Therefore, the area under the curve y = 2x from 0 to 3 is 9 square units.

Additional Considerations

While the calculation seems straightforward, it's worth noting:

- Since y = 2x is a linear, increasing function in [0, 3], the area under the curve forms a right triangle with vertices at (0, 0), (3, 0), and (3, 6).
- The area of this triangle can also be computed as (1/2) base height = (1/2) 3 6 = 9, confirming our integral result.

Visualizing the Area

Visual representations can help in understanding the problem:

- Plot the function y = 2x over [0, 3].
- Highlight the region between the curve and the x-axis.

- Observe the triangular shape formed, with base along the x-axis from 0 to 3 and height reaching $y = 6$ at $x = 3$.

This visual confirms that the area is indeed a triangle with area 9.

Summary of the Method

To find the area under $y = 2x$ on [0, 3]:

1. Write the integral: $A = \int_0^3 2x \, dx$
2. Find the antiderivative: $x^2 + C$
3. Evaluate at bounds: $9 - 0 = 9$
4. Conclude that the area is 9 square units

Additional Practice and Applications

Understanding this process is vital for tackling more complex functions and intervals. Here are some related exercises:

- Find the area under $y = x^2$ on [1, 4]
- Calculate the area between $y = 3x + 1$ and the x-axis on [0, 2]
- Determine the area enclosed by $y = \sqrt{x}$ and $y = x$ on [0, 1]

Each of these involves setting up appropriate integrals, finding antiderivatives, and evaluating bounds.

Conclusion

Calculating the area under a curve using definite integrals is a fundamental skill in calculus. For the specific function y = 2x over [0, 3], the process involves straightforward integration and application of the Fundamental Theorem of Calculus. The resulting area of 9 square units aligns with geometric intuition, as it corresponds to the area of a right triangle formed by the line, the x-axis, and the vertical boundary at x = 3. Mastery of this method provides a foundation for tackling more complex integration problems in mathematics and related fields.

Keywords: area under curve, definite integral, calculus, y = 2x, integral calculation, first quadrant, geometric interpretation, antiderivative, integral bounds

Frequently Asked Questions

What is the goal when finding the area under the curve y = 2x on [0, 3]?

The goal is to calculate the total region enclosed between the curve y = 2x, the x-axis, and the vertical lines x = 0 and x = 3, which represents the definite integral of the function over that interval.

Which mathematical method is used to find the area under the curve y = 2x from 0 to 3?

The method used is definite integration, specifically computing the integral of 2x with respect to x from 0 to 3.

How do you set up the integral for the area under y = 2x over [0, 3]?

You set up the integral as $\int_0^3 2x \, dx$, where 2x is the function to be integrated over the interval from x=0 to x=3.

What is the antiderivative of y = 2x?

The antiderivative of 2x is $x^2 + C$, where C is the constant of integration (not needed for definite integral).

How do you evaluate the definite integral to find the area?

Evaluate the antiderivative x^2 at the upper limit (x=3) and lower limit (x=0), then subtract: $(3)^2 - (0)^2 = 9 - 0 = 9$.

What is the final area under y = 2x between x=0 and x=3?

The area is 9 square units, calculated as the definite integral of 2x from 0 to 3.

Why is it important to specify the interval [0, 3] when finding the area under the curve?

Because the definite integral computes the area specifically between x=0 and x=3; changing the interval would change the area calculated.

Additional Resources

Finding the Area Under the Curve y = 2x on [0, 3]: An In-Depth Approach

When exploring the fascinating world of calculus and integral geometry, one of the fundamental tasks is to determine the area under a curve within a specified interval. In this case, we focus on the function y = 2x over the interval [0, 3], situated firmly within the first quadrant where both x and y are non-negative. Understanding how to accurately compute this area not only reinforces foundational calculus concepts but also enhances problem-solving skills applicable in diverse fields such as physics, engineering, and economics.

This comprehensive guide will delve into the nuances of this problem, explaining the methodology step-by-step, elucidating the underlying principles, and demonstrating the calculation process with clarity and depth.

Understanding the Problem

Before diving into the solution, it is essential to clearly interpret what the problem asks:

- Function: $y = 2x$
- Interval: from $x = 0$ to $x = 3$
- Objective: Find the total area bounded between the curve $y = 2x$, the x-axis, and the vertical lines $x = 0$ and $x = 3$, all situated in the first quadrant.

The first quadrant condition implies that $x \geq 0$ and $y \geq 0$, which aligns with the chosen interval and the function, ensuring the area we compute is positive and meaningful in the geometric sense.

Graphical Representation and Intuition

Visualizing the problem enhances understanding:

- Plot the line $y = 2x$ within the interval [0, 3].
- The line passes through the origin (0, 0) and the point (3, 6).
- The area under the curve forms a right-angled triangle with vertices at:
- The origin (0, 0),
- The point (3, 0) on the x-axis,
- The point (3, 6) on the curve.

Graphical insight confirms that the area under the curve over [0, 3] corresponds to the area of this triangle, which provides an initial intuition about the expected magnitude of the area.

Mathematical Approach: The Integral Method

The most rigorous and systematic method for determining the area under a curve is through definite integrals in calculus. The process involves summing infinitesimally small vertical strips under the curve between $x = 0$ and $x = 3$.

Why Use Integration?

- Integration generalizes the process of summing areas of infinitesimal slices.
- It accounts for the changing height of the function $y = 2x$ across the interval.
- The definite integral calculates the accumulated area precisely.

Prerequisites for the Method

- Understanding of the concept of limits and Riemann sums.

- Familiarity with basic integral calculus, including antiderivatives.

Step-by-Step Solution

Let's walk through the detailed process to find the area:

1. Set Up the Integral

The area A under the curve $y = 2x$ from $x = 0$ to $x = 3$ is expressed as:

$$
A = \int_{0}^{3} y\, dx = \int_{0}^{3} 2x\, dx
$$

This integral sums the areas of vertical slices of width dx, with height $y = 2x$.

2. Find the Antiderivative of the Function

The integrand function is $2x$, which is straightforward to integrate:

$$
\int 2x\, dx = x^2 + C
$$

where C is the constant of integration (not relevant for definite integrals).

3. Evaluate the Definite Integral

Apply the Fundamental Theorem of Calculus:

$$
A = \left[x^2 \right]_0^3 = (3)^2 - (0)^2 = 9 - 0 = 9
$$

Thus, the area under the curve from $x = 0$ to $x = 3$ is 9 square units.

Interpreting the Result

The calculation yields a numerical area of 9 square units. Several points are worth noting:

- The area corresponds to the region enclosed by the curve, the x-axis, and the vertical boundaries at $x = 0$ and $x = 3$.
- The shape, as visualized earlier, is a right-angled triangle. Its area can also be computed geometrically as:

$$
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}
$$

Where:

- Base: 3 units (along the x-axis from 0 to 3),
- Height: 6 units (the y-value at $x = 3$, which is $y = 23 = 6$).

Calculating:

$$
\text{Area} = \frac{1}{2} \times 3 \times 6 = \frac{1}{2} \times 18 = 9
$$

This confirms our integral calculation, illustrating the consistency and reliability of calculus methods with geometric intuition.

Deeper Insights and Variations

While the problem at hand is straightforward, understanding its broader context enhances appreciation:

1. Connection to Geometric Shapes

- Recognizing that the area under $y = 2x$ over $[0, 3]$ forms a triangle simplifies understanding.
- For more complex functions, the integral approach becomes indispensable.

2. Calculus in Action

- Demonstrates how integration calculates areas bounded by curves.
- Highlights the importance of antiderivatives and the Fundamental Theorem of Calculus.

3. Generalization

- For a function $y = f(x)$, the area under the curve over $[a, b]$ is:

$$
A = \int_a^b f(x) \, dx
$$

- The method applies universally, whether the function is linear, quadratic, or more complex.

Additional Considerations and Extensions

Beyond the basic problem, several related topics can deepen understanding:

1. Area Between Two Curves

- If asked to find the area between $y = 2x$ and another curve, say $y = x^2$, over $[0, 3]$, the process involves subtracting the integrals:

$$
A = \int_0^3 [2x - x^2] \, dx
$$

- This extends the methodology to more complex regions.

2. Using Numerical Methods

- For functions without elementary antiderivatives, techniques like Simpson's rule or trapezoidal rule approximate the area.

3. Visualizing the Integral

- Software tools (e.g., Desmos, GeoGebra) can plot the function and shaded region, providing visual reinforcement.

Conclusion: The Significance of the Method

Determining the area under $y = 2x$ from 0 to 3 exemplifies the power of calculus to translate geometric problems into algebraic computations. The integral approach offers precision, flexibility, and a deep understanding of the relationship between functions and the areas they enclose.

By following a structured methodology—setting up the integral, finding the antiderivative, and evaluating—it becomes possible to handle a wide array of similar problems with confidence and accuracy. This process underscores the elegance of calculus: transforming the intuitive idea of "area under a curve" into a rigorous mathematical procedure.

In summary:

- The area under $y = 2x$ on [0, 3] is 9 square units.
- The integral method involves calculating $\int_0^3 2x\, dx$.
- The geometric interpretation aligns with the integral result, showcasing calculus's harmony with

geometry.

- Understanding this foundational problem equips learners for tackling more complex integral-based area calculations in advanced mathematics and applied sciences.

Final thoughts: Mastering the process of integrating simple functions like y = 2x not only builds confidence in calculus techniques but also lays the groundwork for understanding more sophisticated topics such as areas under curves, volumes of revolution, and multivariable calculus. Whether approached visually or algebraically, the calculation of such areas remains a cornerstone of mathematical analysis, enriching our comprehension of the continuous world.

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