Find The Average Value Of F(x,y)=xy Over The Region Bounded By Y=x2 And Y=73x.

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Understanding how to find the average value of a function over a specific region is a fundamental concept in multivariable calculus. In this article, we will explore the process of determining the average value of the function $f(x, y) = xy$ over the region bounded by the curves $y = x^2$ and $y = 73x$. This problem involves setting up and evaluating a double integral, transforming it into a suitable coordinate system, and interpreting the results. Whether you're a student preparing for calculus exams or a professional applying mathematical concepts, this comprehensive guide will help you grasp the methodology step-by-step.

Understanding the Problem and the Region of Integration

Function to Find the Average Value Of

The function in question is:

$$f(x, y) = xy$$

The goal is to compute the average value of this function over a specific region R, which is bounded by two curves:

- $y = x^2$ (a parabola opening upwards)
- $y = 73x$ (a straight line)

The average value f_{avg} of a function $f(x, y)$ over a region R in the xy-plane is given by:

$$
f_{avg} = \frac{1}{\text{Area}(R)} \iint_R f(x, y) \, dA
$$

where dA is the differential area element, typically $dx\,dy$ in Cartesian coordinates.

Visualizing the Region

To better understand the region, consider plotting the two curves:

- The parabola $y = x^2$
- The line $y = 73x$

The intersection points of these curves define the limits of integration.

Finding the Points of Intersection

Set the Curves Equal to Find Intersections

To find where the curves intersect:

$$
x^2 = 73x
$$

Rearranged as:

$$
x^2 - 73x = 0
$$
$$
x(x - 73) = 0
$$

Thus, the solutions are:

$$
x = 0 \quad \text{or} \quad x = 73
$$

Corresponding y-values are:

- When $x = 0$, $y = 0^2 = 0$ and $y = 73 \times 0 = 0$
- When $x = 73$, $y = 73^2 = 5329$ and $y = 73 \times 73 = 5329$

Therefore, the curves intersect at the points:

$$
(0, 0) \quad \text{and} \quad (73, 5329)
$$

This means the region R is bounded between $x=0$ and $x=73$.

Setting Up the Double Integral

Choosing the Order of Integration

To evaluate the double integral, we need to decide whether to integrate with respect to y first or x first.

Given the boundaries:

- y varies between the parabola $y = x^2$ and the line $y = 73x$
- x varies from 0 to 73

It is often easier to integrate with respect to y first, then x.

Expressing the Region in Terms of x and y

For each x in $[0, 73]$, the y-values range from:

$$
y = x^2 \quad \text{(lower boundary)}
$$
to
$$
y = 73x \quad \text{(upper boundary)}
$$

Thus, the double integral becomes:

$$
\iint_R xy \, dA = \int_{x=0}^{73} \int_{y=x^2}^{73x} xy \, dy \, dx
$$

Calculating the Area of the Region

The area A of the region R is:

$$
A = \int_{x=0}^{73} \int_{y=x^2}^{73x} dy \, dx
$$

which simplifies to:

$$
A = \int_{0}^{73} \left((73x) - (x^2) \right) dx
$$

Calculating:

$$
A = \int_{0}^{73} (73x - x^2) dx
$$

Let's evaluate this integral.

Computing the Area

$$
A = \int_{0}^{73} 73x \, dx - \int_{0}^{73} x^2 \, dx
$$

Calculate each separately:

- $\int 73x \, dx = \frac{73}{2} x^2$
- $\int x^2 \, dx = \frac{1}{3} x^3$

Plugging in the limits:

$$
A = \left[\frac{73}{2} x^2 \right]_0^{73} - \left[\frac{1}{3} x^3 \right]_0^{73}
$$

Compute each term:

$$
A = \frac{73}{2} \times (73)^2 - \frac{1}{3} \times (73)^3
$$

Calculate 73^2:

$$
73^2 = 5329
$$

Calculate 73^3:

$$
73^3 = 73 \times 5329 = 73 \times 5329

\]

Compute:

$$
73 \times 5329 = (70 + 3) \times 5329 = 70 \times 5329 + 3 \times 5329 = 373,030 + 15,987 = 389,017
$$

Therefore:

$$
A = \frac{73}{2} \times 5329 - \frac{1}{3} \times 389,017
$$

Calculate each:

$$
\frac{73}{2} \times 5329 = \frac{73 \times 5329}{2}
$$
$$
73 \times 5329 = 389,017 \quad \text{(as above)}
$$

Thus,

$$
\frac{389,017}{2} = 194,508.5
$$

And

$$
\frac{1}{3} \times 389,017 \approx 129,672.33
$$

Finally, the area:

$$
A = 194,508.5 - 129,672.33 \approx 64,836.17
$$

Calculating the Double Integral of $f(x,y) = xy$

Recall the integral:

$$
\iint_R xy \, dA = \int_{x=0}^{73} \int_{y=x^2}^{73x} xy \, dy \, dx
$$

First, evaluate the inner integral:

$$
I(x) = \int_{y=x^2}^{73x} xy \, dy
$$

Since x is treated as a constant during integration with respect to y:

$$
I(x) = x \int_{y=x^2}^{73x} y \, dy
$$

Calculate:

$$
I(x) = x \left[\frac{1}{2} y^2 \right]_{y=x^2}^{73x} = x \left(\frac{1}{2} (73x)^2 - \frac{1}{2} (x^2)^2 \right)
$$

Simplify:

$$
I(x) = x \times \frac{1}{2} \left(73^2 x^2 - x^4 \right) = \frac{x}{2} \left(5329 x^2 - x^4 \right)
$$

Distribute:

$$
I(x) = \frac{1}{2} \left(5329 x^3 - x^5 \right)
$$

Now, compute the outer integral:

$$
\iint_R xy \, dA = \int_{0}^{73} I(x) \, dx = \frac{1}{2} \int_0^{73} \left(5329 x^3 - x^5 \right) dx
$$

Evaluate separately:

$$
\frac{1}{2} \left[\int_0^{73} 5329 x^3 \, dx - \int_0^{73} x^5 \, dx \right]
$$

Calculate each integral:

- \(\int x^3 \, dx = \frac{1}{4} x^4\)
- \(\int x^5 \, dx = \frac{1}{6} x^6\)

Plug in limits:

\[
\frac{1}{2} \left[5329 \

Frequently Asked Questions

How do I set up the double integral to find the average value of f(x, y) = xy over the region bounded by y = x^2 and y = 73x?

To find the average value, you set up the double integral of f(x, y) over the region B, then divide by the area of B. The bounds are determined by solving y = x^2 and y = 73x; typically, x ranges between their intersection points, and y varies between the lower and upper curves for each x.

What are the intersection points of the curves y = x^2 and y = 73x, and why are they important?

The intersection points are found by solving x^2 = 73x, which gives x = 0 and x = 73. These points define the limits of integration for x when calculating the double integral, crucial for accurately setting up the region.

How do I compute the area of the region bounded by y = x^2 and y = 73x?

Calculate the area by integrating with respect to x from 0 to 73: Area = $\int_0^{73} (73x - x^2)$ dx. This integral computes the total area between the curves, which is needed for the average value calculation.

What is the step-by-step process to find the average value of f(x, y) = xy over the given region?

First, set up the double integral of xy over the region, with y from x^2 to 73x and x from 0 to 73. Then, compute the integral, divide by the area of the region, and simplify to find the average value.

Can you provide the final formula for the average value of f(x, y) = xy over the bounded region?

Yes. The average value is given by:

$$\text{Average} = (1 / \text{Area}) \int_0^{73} \int_{x^2}^{73x} xy \, dy \, dx,$$

where $\text{Area} = \int_0^{73} (73x - x^2) \, dx$. Evaluating these integrals yields the numerical value for the average.

Additional Resources

Find The Average Value Of F(x,y) = xy Over The Region Bounded By $y = x^2$ And $y = 73x$

Introduction: The Significance of Analyzing Function Averages Over Regions

In the realm of mathematical analysis, especially in calculus and multivariable calculus, understanding the average value of a function over a specified region holds profound importance. It provides insights into the function's behavior across that area, offering a measure of its typical magnitude. This concept is vital across numerous applied disciplines such as physics, engineering, economics, and environmental science, where it assists in modeling real-world phenomena.

Specifically, when dealing with functions of two variables, like $f(x,y) = xy$, and regions bounded by curves, calculating the average value involves integrating the function over the region and then normalizing by the area. This process requires a methodical approach, involving the determination of the region's boundaries, setting up the appropriate double integral, and performing the integration accurately.

In this article, we delve into the task of finding the average value of the function $f(x, y) = xy$ over a region bounded by the curves $y = x^2$ and $y = 73x$. This problem exemplifies the synergy between geometric intuition and analytical rigor, illustrating the process of translating a geometric boundary into an algebraic form suitable for integration and analysis.

Understanding the Region: Geometric Boundaries and Their Significance

The Curves Defining the Region

The region B of interest is bounded by two curves:

- The parabola: $y = x^2$
- The straight line: $y = 73x$

Graphing these curves provides valuable insight into the shape and extent of the region:

- The parabola $y = x^2$ opens upward, with its vertex at the origin $(0,0)$.
- The line $y = 73x$ passes through the origin with a steep positive slope.

The intersection points of these two curves determine the region's limits:

$$
x^2 = 73x
$$

Rearranged as:

$$
x^2 - 73x = 0
$$
$$
x(x - 73) = 0
$$

Thus, the solutions are:

$$
x = 0 \quad \text{and} \quad x = 73
$$

Corresponding y-values are:

- At $x=0$, $y=0$
- At $x=73$, $y=73 \times 73 = 5329$

These points, $(0,0)$ and $(73, 5329)$, mark the intersection points where the parabola and the line meet.

Envisioning the Region

Between these intersection points:

- For \(x \in [0, 73] \), the line \(y=73x \) is above the parabola \(y = x^2 \). To verify, check at an intermediate point, say \(x=10 \):

\[
y_{\text{line}} = 730, \quad y_{\text{parabola}}=100
\]
Since \(730 > 100 \), the line is above the parabola in this interval.

- The region \(B \) is thus bounded below by the parabola \(y = x^2 \) and above by the line \(y = 73x \), for \(x \in [0, 73] \).

This understanding is crucial for setting up the integral correctly, as the limits of integration will correspond to this \(x\)-interval, and the bounds for \(y \) are between the lower curve \(y = x^2 \) and the upper curve \(y = 73x \).

Mathematical Framework for Finding the Average Value

Definition of the Average Value of a Function over a Region

The average value \(f_{\text{avg}}\) of a function \(f(x,y)\) over a region \(D\) in the plane is given by:

\[
f_{\text{avg}} = \frac{1}{\text{Area of } D} \iint_D f(x,y) \, dA
\]

where:

- \(\iint_D f(x,y) \, dA\) denotes the double integral of \(f\) over the region \(D\),
- \(dA\) is the differential area element.

To evaluate this, the steps generally involve:

1. Determining the region \(D\)'s boundaries.
2. Computing the area of \(D\).
3. Computing the double integral of \(f\) over \(D\).
4. Dividing the integral by the area.

Choosing the Appropriate Coordinates and Limits

Given the boundaries, it is most straightforward to set up the integral in terms of x and y:

- For $x \in [0, 73]$,
- For each fixed x, y varies from $y = x^2$ (lower boundary) to $y=73x$ (upper boundary).

Thus, the region D can be characterized as:

$$
D = \{ (x,y) \mid 0 \leq x \leq 73, \quad x^2 \leq y \leq 73x \}
$$

Calculating the Area of the Region

Before computing the average value, the area A of the region D must be determined:

$$
A = \iint_D dA
$$

Expressed in the chosen coordinate system:

$$
A = \int_{x=0}^{73} \int_{y=x^2}^{73x} dy \, dx
$$

Performing the inner integral:

$$
A = \int_0^{73} \left[y \right]_{y=x^2}^{y=73x} dx = \int_0^{73} (73x - x^2) dx
$$

Now, compute:

$$
A = \int_0^{73} 73x\, dx - \int_0^{73} x^2\, dx
$$

Calculate each separately:

- $\int 73x\, dx = \frac{73}{2} x^2$

- $\int x^2\, dx = \frac{x^3}{3}$

Evaluate at the bounds:

$$
A = \left[\frac{73}{2} x^2 \right]_0^{73} - \left[\frac{x^3}{3} \right]_0^{73}
$$

Compute:

$$
A = \frac{73}{2} \times (73)^2 - \frac{(73)^3}{3}
$$

Calculate each term:

- $73^2 = 5329$
- $73^3 = 73 \times 5329 = 73 \times 5329$

Let's compute 73×5329:

$$
73 \times 5329 = (70 + 3) \times 5329 = 70 \times 5329 + 3 \times 5329
$$
$$
= 70 \times 5329 + 3 \times 5329
$$

Calculate each:

- $70 \times 5329 = 70 \times 5329 = 373,030$
- $3 \times 5329 = 15,987$

Sum:

$$
373,030 + 15,987 = 389,017
$$

Therefore:

$$
A = \frac{73}{2} \times 5329 - \frac{389,017}{3}
$$

Compute:

$$
\frac{73}{2} \times 5329 = 36.5 \times 5329
$$

Calculate 36.5×5329:

- $36 \times 5329 = 192,044$
- $0.5 \times 5329 = 2,664.5$

Sum:

$$
192,044 + 2,664.5 = 194,708.5
$$

Now, the area:

$$
A = 194,708.5 - \frac{389,017}{3}
$$

Expressed as a decimal:

$$
\frac{389,017}{3} \approx 129,672.33
$$

Therefore:

$$
A \approx 194,708.5 - 129,672.33 = 65,036.17
$$

For exactness, write:

$$
A = 194,708.5 - 129,672.\overline{3}
$$

Alternatively, as fractions:

$$
A = \frac{36.5 \times 5329 \times 2}{2} - \frac{389,017}{3}
$$

But for practical purposes, the approximate area is about 65,036.17 square units.

Computing the Double Integral of \(f(x,y) =

xy\)

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Having established the region and area, next, we compute:

\[
I = \iint_D xy \,
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Find The Average Value Of F X Y Xy Over The Region Bounded By Y X2 And Y 73x

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