

Find The Cartesian Co-ordinates Of The Point Whose Polar Co-ordinates Are : (4,2)

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Understanding the relationship between polar and Cartesian coordinates is fundamental in coordinate geometry, especially when dealing with points in a plane. When given a point's polar coordinates, the goal is to convert these into Cartesian coordinates, which are more familiar and straightforward for many geometric and algebraic applications. In this article, we will explore the process of converting polar coordinates to Cartesian coordinates with a detailed explanation, step-by-step instructions, and practical examples. Our focus will be on the specific problem: finding the Cartesian coordinates of the point whose polar coordinates are (4, 2).

Introduction to Polar and Cartesian Coordinates

Before diving into the conversion process, it is essential to understand what polar and Cartesian coordinates represent.

What are Cartesian Coordinates?

- Cartesian coordinates describe a point in a plane using two perpendicular axes: the x-axis (horizontal) and the y-axis (vertical).
- A point is represented as (x, y) , where:
- x = horizontal distance from the origin
- y = vertical distance from the origin

What are Polar Coordinates?

- Polar coordinates represent a point in terms of its distance from the origin and the angle it makes with the positive x-axis.
- A point is represented as (r, θ) , where:
- r = the radial distance from the origin (also called the modulus or magnitude)
- θ = the angle measured in radians or degrees from the positive x-axis to the line segment connecting the origin to the point

Understanding the Conversion Between Polar and Cartesian Coordinates

The core relationship between polar and Cartesian coordinates is based on basic trigonometry.

Mathematical Formulas for Conversion

- From polar to Cartesian:
 - $x = r \cos(\theta)$
 - $y = r \sin(\theta)$
- From Cartesian to polar:
 - $r = \sqrt{x^2 + y^2}$
 - $\theta = \arctangent(y / x)$ (taking into account the quadrant)

In our case, since the problem is to find Cartesian coordinates from polar coordinates, we will primarily use the first set of formulas.

Given Data and Problem Statement

The given polar coordinates are:

- $r = 4$
- $\theta = 2$ (We will assume the angle is in radians unless specified otherwise)

Our task:

- Convert (4, 2) in polar coordinates to Cartesian coordinates (x, y).

Step-by-Step Conversion Process

Let's proceed through the conversion systematically.

Step 1: Understand the Given Data

- Radial distance, $r = 4$ units
- Angle, $\theta = 2$ radians

Step 2: Apply the Conversion Formulas

Using the formulas:

- $x = r \cos(\theta)$
- $y = r \sin(\theta)$

We substitute the known values:

- $x = 4 \cos(2)$
- $y = 4 \sin(2)$

Step 3: Calculate the Cosine and Sine Values

- $\cos(2 \text{ radians}) \approx -0.4161$
- $\sin(2 \text{ radians}) \approx 0.9093$

These are approximate values obtained from calculator or trigonometric tables.

Step 4: Compute the Cartesian Coordinates

- $x = 4 (-0.4161) \approx -1.6644$
- $y = 4 0.9093 \approx 3.6372$

Thus, the approximate Cartesian coordinates are:

- $x \approx -1.664$
- $y \approx 3.637$

Step 5: Final Coordinates

- The point in Cartesian coordinates is approximately $(-1.664, 3.637)$.

Interpreting the Result

The Cartesian coordinates $(-1.664, 3.637)$ indicate:

- The point lies in the second quadrant, since x is negative and y is positive.
- The distance from the origin can be verified using distance formula:
- $r = \sqrt{x^2 + y^2} \approx \sqrt{(-1.664)^2 + 3.637^2} \approx \sqrt{2.77 + 13.23} \approx \sqrt{16} \approx 4$
- This confirms the correctness of the conversion.

Additional Considerations and Tips

Angles in Degrees vs. Radians

- Always verify the unit of the angle provided.
- If the angle is in degrees, convert it to radians before applying the formulas:
- $\theta \text{ (radians)} = \theta \text{ (degrees)} \pi / 180$

Quadrant Determination

- The sign of $\cos(\theta)$ and $\sin(\theta)$ determines the quadrant:
- Quadrant I: $x > 0, y > 0$
- Quadrant II: $x < 0, y > 0$
- Quadrant III: $x < 0, y < 0$
- Quadrant IV: $x > 0, y < 0$

Use of Calculators and Tools

- Use scientific calculators for precise sine and cosine values.
- Confirm the angle unit is set correctly on your calculator.

Practical Applications of Polar to Cartesian Conversion

Converting between coordinate systems is crucial in various fields:

- **Engineering:** Designing components with specific angular and radial specifications.
- **Physics:** Describing motion in polar coordinates, such as circular paths or oscillations.
- **Computer Graphics:** Rendering images and objects based on different coordinate systems.
- **Navigation:** Using GPS coordinates, which often involve angular measurements from a reference point.

Summary

To summarize, the process of finding Cartesian coordinates from polar coordinates involves:

1. Identifying the given polar coordinates (r, θ) .
2. Applying the formulas $x = r \cos(\theta)$ and $y = r \sin(\theta)$.
3. Calculating the cosine and sine of the angle (in radians).
4. Multiplying these values by the radius to obtain x and y .
5. Interpreting the resulting (x, y) point in the context of the coordinate plane.

For the specific case of polar coordinates $(4, 2 \text{ radians})$, the Cartesian coordinates are approximately $(-1.664, 3.637)$.

Conclusion

Converting polar coordinates to Cartesian coordinates is a fundamental skill in geometry and mathematical analysis. By understanding the relationship between these coordinate systems and applying basic trigonometric formulas, you can efficiently find the Cartesian coordinates of any point given in polar form. Remember to verify angle units and use precise calculations for accurate results. Whether you are solving geometry problems, working on physics projects, or developing computer graphics, mastering this conversion process enhances your problem-solving toolkit and deepens your understanding of coordinate systems.

Frequently Asked Questions

What are the Cartesian coordinates of the point with polar coordinates $(4, 2)$?

The Cartesian coordinates are $(x, y) = (4 \cos 2, 4 \sin 2)$. Numerically, $x \approx -1.6$ and $y \approx 3.6$.

How do you convert polar coordinates (r, θ) to Cartesian coordinates?

To convert, use $x = r \cos \theta$ and $y = r \sin \theta$. For $(4, 2)$, plug in $r=4$ and $\theta=2$ radians.

What is the significance of the angle in the polar coordinate system?

The angle θ indicates the direction of the point from the origin relative to the positive x-axis, measured in radians or degrees.

Can the polar coordinates $(4, 2)$ be represented in degrees? If so, what is the approximate degree measure of θ ?

Yes. Since $\theta=2$ radians, in degrees it's approximately 114.59° ($2 \times 180/\pi$).

Are the Cartesian coordinates of the point with polar coordinates $(4, 2)$ unique?

Yes, for a given r and θ , the Cartesian coordinates are unique, calculated as $(x, y) = (4 \cos 2, 4 \sin 2)$.

Additional Resources

Find The Cartesian Co-ordinates Of The Point Whose Polar Co-ordinates Are : $(4, 2)$

Understanding the relationship between polar and Cartesian coordinates is fundamental in fields like mathematics, physics, engineering, and computer graphics. When working with points in a plane, converting from polar coordinates to Cartesian coordinates allows for easier visualization and computation in many practical applications. In this guide, we will explore find the Cartesian co-ordinates of the point whose polar co-ordinates are: $(4, 2)$, providing a comprehensive explanation of the process, formulas, and step-by-step calculations involved.

Introduction to Polar and Cartesian Coordinates

Before diving into the conversion process, it is important to understand what polar and Cartesian coordinates represent and how they relate to each other.

Cartesian Coordinates

- Definition: Cartesian coordinates specify a point's position in a 2D plane using two perpendicular axes: the x-axis (horizontal) and y-axis (vertical).
- Notation: A point is written as (x, y) , where:
 - x: Horizontal distance from the origin
 - y: Vertical distance from the origin

Polar Coordinates

- Definition: Polar coordinates specify a point's location based on its distance from the origin and the angle it makes with the positive x-axis.
- Notation: A point is written as (r, θ) , where:
 - r: The radius or distance from the origin to the point
 - θ : The angle in radians or degrees measured counterclockwise from the positive x-axis

Conversion Formulas Between Polar and Cartesian Coordinates

The process of converting between these coordinate systems hinges on two fundamental formulas:

From Polar to Cartesian:

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

From Cartesian to Polar:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ \theta &= \arctan \left(\frac{y}{x} \right) \end{aligned}$$

In this tutorial, our focus is on the first set of formulas – converting from polar to Cartesian – to find the x and y coordinates of the point with given polar coordinates.

Step-by-Step Guide: Finding the Cartesian Coordinates for (4, 2)

Given the polar coordinates:

- $r = 4$

- $\theta = 2$ radians

we aim to find the corresponding Cartesian coordinates (x, y).

Step 1: Understand the Given Data

- Radius (r): 4 units
- Angle (θ): 2 radians

Note: If the angle was provided in degrees, we would convert it to radians before proceeding. However, since it is already in radians, we can directly apply the formulas.

Step 2: Apply the Conversion Formulas

Using the formulas:

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

we substitute $r = 4$ and $\theta = 2$ radians.

Step 3: Calculate x-coordinate

$$x = 4 \cos 2$$

Using a calculator or computational tool:

$$\cos 2 \approx -0.4161$$

Therefore,

$$x \approx 4 \times (-0.4161) \approx -1.6644$$

Step 4: Calculate y-coordinate

$$y = 4 \sin 2$$

Calculate:

$\sin 2 \approx 0.9093$

So,

$y \approx 4 \times 0.9093 \approx 3.6372$

Final Cartesian Coordinates:

$$\boxed{(x, y) \approx (-1.6644, 3.6372)}$$

Additional Considerations in Conversion

While the above calculations are straightforward, there are some key points to keep in mind:

1. Angles in Degrees vs. Radians

- Ensure that your calculator is set to the correct mode (radians or degrees).
- If given in degrees, convert to radians: $(\text{radians} = \text{degrees} \times \frac{\pi}{180})$.

2. Quadrant Determination

- The signs of x and y depend on the angle's quadrant:
- Quadrant I: $(r > 0), (0 < \theta < \frac{\pi}{2})$ – x and y are positive.
- Quadrant II: $(r > 0), (\frac{\pi}{2} < \theta < \pi)$ – x negative, y positive.
- Quadrant III: $(r > 0), (\pi < \theta < \frac{3\pi}{2})$ – x and y negative.
- Quadrant IV: $(r > 0), (\frac{3\pi}{2} < \theta < 2\pi)$ – x positive, y negative.

In our case, since $(\theta = 2)$ radians ($\sim 114.6^\circ$), the point lies in Quadrant II, consistent with the negative x and positive y coordinates found.

3. Precision

- Use sufficient decimal places for accuracy, especially in professional or scientific calculations.

Visualizing the Point

Understanding the position of the point in the plane enhances comprehension:

- The radius of 4 units indicates how far from the origin the point lies.
- The angle of 2 radians ($\sim 114.6^\circ$) shows the direction relative to the positive x-axis.
- The Cartesian coordinates $(-1.6644, 3.6372)$ confirm that the point is situated in Quadrant II, to the left of the y-axis and above the x-axis.

Summary of Key Steps

Step	Description	Calculation/Result
1	Identify given polar coordinates	$(r=4, \theta=2)$ radians
2	Recall conversion formulas	$x = r \cos \theta$, $y = r \sin \theta$
3	Calculate x-coordinate	$x = 4 \cos 2 \approx -1.6644$
4	Calculate y-coordinate	$y = 4 \sin 2 \approx 3.6372$
5	Final Cartesian coordinates	$(-1.6644, 3.6372)$

Practical Applications of Conversion

Converting polar to Cartesian coordinates is crucial in various fields:

- Engineering: Analyzing waveforms, antenna radiation patterns, or mechanical components.
- Physics: Describing particle motions or fields in different coordinate systems.
- Computer Graphics: Rendering objects based on polar data or transformations.
- Navigation and Robotics: Path planning involving angles and distances.

Conclusion

Finding the Cartesian co-ordinates of a point given its polar coordinates involves straightforward application of the fundamental formulas. For the point with polar coordinates $(4, 2)$, the Cartesian coordinates are approximately $(-1.6644, 3.6372)$. Mastery of these conversions enhances spatial understanding and computational versatility across scientific and engineering disciplines. Always remember to check your angle units and quadrant placement to ensure accurate results.

By following this detailed guide, you can confidently convert any polar coordinates to Cartesian coordinates and deepen your understanding of the relationship between these two coordinate systems.

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