Find The Distance Between The Given Parallel Planes. 3x 2y Z = 12, 6x 4y 2z = 3

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Understanding the distance between parallel planes is a fundamental concept in three-dimensional geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or an engineer working on spatial calculations, knowing how to find this distance accurately is essential. In this article, we will explore the methods to determine the shortest distance between two parallel planes, specifically focusing on the planes given by the equations 3x + 2y + z = 12 and 6x + 4y + 2z = 3. We will delve into the mathematical principles, step-by-step procedures, and provide clear examples to enhance your understanding.

Understanding Parallel Planes

What Are Parallel Planes?

Parallel planes are planes in three-dimensional space that never intersect, regardless of how far they extend. They have identical normal vectors but different constant terms in their equations.

Key points about parallel planes:
- They have the same normal vector.
- They do not intersect at any point.
- The distance between them is constant everywhere.

Standard Form of Plane Equations

A plane in three-dimensional space can be represented in the general form:
$$ Ax + By + Cz + D = 0 $$

Where:
- A, B, C are the coefficients that form the normal vector $\vec{n} = (A, B, C)$.
- D is the constant term.

For the given planes:
1. $3x + 2y + z = 12$

2. $6x + 4y + 2z = 3$

We can see both planes have the same normal vector scaled differently:
- Normal vector of plane 1: $\vec{n}_1 = (3, 2, 1)$
- Normal vector of plane 2: $\vec{n}_2 = (6, 4, 2)$

Since $\vec{n}_2 = 2 \times \vec{n}_1$, these planes are parallel.

Steps to Find the Distance Between Parallel Planes

Calculating the distance between two parallel planes involves a systematic approach.

Step 1: Write the equations in standard form

Ensure both planes are expressed as:
$$ Ax + By + Cz + D = 0 $$

Rearranged equations:
- Plane 1: $3x + 2y + z - 12 = 0$
- Plane 2: $6x + 4y + 2z - 3 = 0$

Divide the second plane's equation by 2 to simplify:
$$
\frac{6x + 4y + 2z - 3}{2} \Rightarrow 3x + 2y + z - \frac{3}{2} = 0
$$

Now, both planes share the same normal vector:
- Plane 1: $3x + 2y + z - 12 = 0$
- Plane 2: $3x + 2y + z - \frac{3}{2} = 0$

Step 2: Use the distance formula between two parallel planes

The formula for the distance d between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is:

$$
d = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}
$$

Note: Always verify that the planes are parallel (which they are in this case).

Step 3: Calculate the distance

From the simplified forms:
- \(D_1 = -12\)
- \(D_2 = -\frac{3}{2}\)

Compute the numerator:
\[
|D_2 - D_1| = \left| -\frac{3}{2} - (-12) \right| = \left| -\frac{3}{2} + 12 \right| = \left| \frac{-3 + 24}{2} \right| = \left| \frac{21}{2} \right| = \frac{21}{2}
\]

Calculate the denominator:
\[
\sqrt{A^2 + B^2 + C^2} = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}
\]

Finally, the distance:
\[
d = \frac{\frac{21}{2}}{\sqrt{14}} = \frac{21}{2 \sqrt{14}} = \frac{21}{2 \sqrt{14}}
\]

Rationalize the denominator:
\[
d = \frac{21}{2 \sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{21 \sqrt{14}}{2 \times 14} = \frac{21 \sqrt{14}}{28} = \frac{3 \sqrt{14}}{4}
\]

Result:
\[
\boxed{
\text{Distance} = \frac{3 \sqrt{14}}{4}
}
\]

Additional Insights and Tips for Finding Distance Between Planes

Understanding the Role of Normal Vectors

- The normal vector $\vec{n}$ determines the orientation of the plane.
- Parallel planes share proportional normal vectors.
- When calculating distance, focus on the normal vector's magnitude.

Key Points to Remember

- Always convert plane equations into standard form.
- Verify that the planes are parallel before applying the distance formula.
- Use the absolute value in the numerator to ensure a positive distance.
- Simplify equations where possible to make calculations easier.

Common Mistakes to Avoid

- Forgetting to check if the planes are parallel.
- Mixing up the constants D_1 and D_2.
- Not rationalizing the denominator, leading to less clean results.
- Ignoring the scale factors when the second plane's equation is a multiple of the first.

Practical Applications of Finding Distance Between Parallel Planes

Understanding how to find the distance between parallel planes has numerous applications in various fields:

1. Engineering and Construction
- Ensuring structural components are correctly spaced.
- Calculating the gap between walls or panels.

2. Computer Graphics and 3D Modeling
- Detecting the spacing between different layers or surfaces.

3. Geography and Cartography
- Measuring the distance between parallel geological layers.

4. Physics
- Analyzing fields and potential planes in space.

Summary and Key Takeaways

- Parallel planes have identical normal vectors, scaled versions of each other.
- Convert plane equations into standard form $Ax + By + Cz + D = 0$.
- Use the formula $d = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}$ to find the shortest distance.
- Always verify the planes are parallel before applying the formula.
- The specific example provided yields a distance of $\frac{3 \sqrt{14}}{4}$.

Conclusion

Calculating the distance between parallel planes is a straightforward process once you understand the underlying principles. By converting plane equations into standard form, verifying parallelism, and applying the appropriate formula, you can accurately measure the shortest gap between any two parallel planes. The example of $3x + 2y + z = 12$ and $6x + 4y + 2z = 3$ demonstrates this process clearly, providing a template for similar problems in geometry, engineering, and physics.

Understanding these concepts enhances spatial reasoning and problem-solving skills, essential for advanced mathematics and practical applications alike. Whether you're tackling homework problems or real-world projects, mastering the method to find the distance between parallel planes will serve as a valuable tool in your mathematical toolkit.

Keywords for SEO Optimization:
- Distance between parallel planes
- Find the shortest distance between planes
- Parallel plane equations
- Distance formula for planes
- Geometry problems on planes
- Calculating distance between planes in 3D
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Frequently Asked Questions

What is the general formula to find the distance between two parallel planes?

The distance between two parallel planes $Ax + By + Cz + D1 = 0$ and $Ax + By + Cz + D2 = 0$ is given by $|D2 - D1| / \sqrt{A^2 + B^2 + C^2}$.

Are the planes 3x + 2y + z = 12 and 6x + 4y + 2z = 3 parallel?

Yes, because the second plane's coefficients are multiples of the first plane's coefficients (6 = 23, 4=22, 2=21), indicating they are parallel.

How do you rewrite the second plane 6x + 4y + 2z = 3 in the standard form?

Divide the entire equation by 2 to get $3x + 2y + z = 1.5$, which is in the same form as the first plane.

What is the value of D for the plane 3x + 2y + z = 12?

Rearranged into standard form: $3x + 2y + z - 12 = 0$, so $D = -12$.

What is the value of D for the plane 3x + 2y + z = 1.5?

Rearranged: $3x + 2y + z - 1.5 = 0$, so $D = -1.5$.

How do you calculate the distance between these two parallel planes?

Use the formula $|D2 - D1| / \sqrt{A^2 + B^2 + C^2}$. Here, $D1 = -12$, $D2 = -1.5$, and $A=3$, $B=2$, $C=1$.

What is the numerical value of the distance between the planes?

Distance = $|(-1.5) - (-12)| / \sqrt{3^2 + 2^2 + 1^2} = 10.5 / \sqrt{14} \approx 10.5 / 3.7417 \approx 2.80$.

Why do we divide the difference of D-values by the square root of the sum of squares of A, B, and C?

Because this accounts for the normal vector's magnitude, providing the perpendicular distance between the planes.

Can this method be used for any pair of parallel planes?

Yes, as long as the planes are parallel and in the form Ax + By + Cz + D = 0, the formula applies directly.

What is the importance of identifying that the planes are parallel before calculating the distance?

Because the distance formula applies only to parallel planes; non-parallel planes do not have a constant distance between them.

Additional Resources

Find The Distance Between The Given Parallel Planes is a fundamental problem in coordinate geometry that often appears in algebra and calculus coursework. Understanding how to determine the shortest distance between two parallel planes not only enhances spatial reasoning but also deepens comprehension of the geometric relationships in three-dimensional space. In this article, we will explore the methods to find the distance between the given planes: 3x + 2y + z = 12 and 6x + 4y + 2z = 3. Through detailed explanations, step-by-step calculations, and insights into the features of these planes, you'll gain a comprehensive understanding of this important geometric concept.

Understanding the Equations of the Planes

Before diving into the calculation of the distance, it’s essential to understand the nature of the given planes, their equations, and their orientations.

Standard Form of Plane Equations

Typically, a plane in three-dimensional space is represented by the general equation:

$$ Ax + By + Cz + D = 0 $$

where A, B, C are the coefficients that define the normal vector to the plane, and D shifts the plane relative to the origin.

In our case, the two planes are:

1. $3x + 2y + z = 12$
2. $6x + 4y + 2z = 3$

Notice that the second plane can be rewritten as:

$$ 6x + 4y + 2z = 3 \Rightarrow \text{Divide entire equation by 2:} $$
$$ 3x + 2y + z = \frac{3}{2} $$

This reveals that both planes share the same normal vector:

$$ \mathbf{n} = (3, 2, 1) $$

and are therefore parallel because their normal vectors are scalar multiples of each other.

Features:

- Parallel planes: Since their normal vectors are proportional, the planes are parallel.
- Different constants: The right-hand sides differ (12 and 1.5), implying they are distinct planes with some distance between them.

Verifying Parallelism of the Planes

To confirm the planes are parallel, observe their normal vectors:

- Plane 1: $\mathbf{n}_1 = (3, 2, 1)$
- Plane 2: $\mathbf{n}_2 = (6, 4, 2)$

Since $\mathbf{n}_2 = 2 \times \mathbf{n}_1$, the normal vectors are scalar multiples, confirming the planes are parallel.

Implication:

The planes do not intersect (they are parallel) and are separated by some finite distance, which we need to compute.

Method to Find the Distance Between Parallel Planes

The distance d between two parallel planes given by:

$$ A x + B y + C z + D_1 = 0 $$
$$ A x + B y + C z + D_2 = 0 $$

is calculated using the formula:

$$ d = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}} $$

This formula is derived from the point-to-plane distance formula and applies because the planes are parallel.

Key points:

- The numerator is the absolute difference of the constant terms (after rewriting in standard form).
- The denominator is the magnitude of the normal vector.

Rewriting the Given Planes in Standard Form

Let's convert our equations to the standard form $A x + B y + C z + D = 0$.

Plane 1:

$$ 3x + 2y + z = 12 \Rightarrow 3x + 2y + z - 12 = 0 $$
Here, $D_1 = -12$.

Plane 2:

$$ 6x + 4y + 2z = 3 $$

Divide through by 2:

$$ 3x + 2y + z = \frac{3}{2} \Rightarrow 3x + 2y + z - \frac{3}{2} = 0 $$

Here, $D_2 = -\frac{3}{2}$.

Calculating the Distance Between the Planes

Using the formula:

$$ d = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}} $$

Substitute the values:

$$ d = \frac{\left| -\frac{3}{2} - (-12) \right|}{\sqrt{3^2 + 2^2 + 1^2}} $$

Simplify numerator:

$$ -\frac{3}{2} + 12 = -\frac{3}{2} + \frac{24}{2} = \frac{21}{2} $$

Absolute value:

$$ \left| \frac{21}{2} \right| = \frac{21}{2} $$

Calculate denominator:

$$ \sqrt{9 + 4 + 1} = \sqrt{14} $$

Therefore:

$$ d = \frac{\frac{21}{2}}{\sqrt{14}} $$

Simplify numerator:

$$ \frac{21}{2} = \frac{21 \sqrt{14}}{2 \sqrt{14}} $$

Alternatively, write as:

$$ d = \frac{21}{2 \sqrt{14}} $$

To rationalize the denominator:

$$ d = \frac{21}{2 \sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{21 \sqrt{14}}{2 \times 14} = \frac{21 \sqrt{14}}{28} $$

Simplify numerator and denominator:

$$ d = \frac{3 \sqrt{14}}{4} $$

Final answer:

$$ \boxed{d = \frac{3 \sqrt{14}}{4}} $$

This is the shortest distance between the two parallel planes.

Features and Insights

Advantages of this method:

- Simplicity: The formula provides a quick way to compute the distance once the planes are in standard form.

- Applicability: Works for any pair of parallel planes, regardless of their orientation.
- Clarity: The calculation explicitly shows the relationship between the constants and the normal vector.

Limitations:

- Requires standard form: The equations must be converted to standard form.
- Assumes parallelism: The formula is valid only if the planes are parallel; otherwise, the shortest distance would be zero or the planes would intersect.

Alternative Approaches and Considerations

While the direct formula is most straightforward, other methods include:

- Point-to-Plane Distance Method: Choose a point on one plane and compute its perpendicular distance to the other plane.
- Vector Projection Method: Use vector projections to verify and find the shortest distance, especially in more complex scenarios where the planes are not parallel.

In our context, since the planes are parallel, the direct formula is optimal.

Summary and Conclusion

Finding the distance between two parallel planes involves understanding their normal vectors and rewriting the equations in standard form. Recognizing that the planes are scalar multiples of each other's normal vectors confirms their parallelism. Applying the distance formula yields:

\[\boxed{d = \frac{3 \sqrt{14}}{4}} \]

which simplifies the process and provides an exact measure of the separation between these two planes.

Key takeaways:

- Confirm parallelism by comparing normal vectors.
- Convert equations to standard form for easy application of the formula.
- Use the distance formula for a quick and accurate result.
- Understand the geometric significance: the shortest segment connecting the planes is perpendicular to both, aligned with the normal vector.

This problem exemplifies the power of algebraic manipulation combined with geometric insight, making it a valuable example in the study of three-dimensional geometry.

Pros & Cons Summary:

Pros:

- Straightforward calculation using a simple formula.
- Clear geometric interpretation.
- Applicable to all parallel planes with known equations.

Cons:

- Requires careful algebraic manipulation.
- Not applicable if planes are not parallel.
- Misidentification of normal vectors can lead to errors.

By mastering this approach, students and enthusiasts can confidently solve similar problems involving distances between parallel planes, enhancing their spatial reasoning and algebraic skills.

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