Find The Equation Of The Line Tangent To The Graph Of F(x)=-3x+4x+3 At X = 2.

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Introduction

Understanding how to find the tangent line to a function at a specific point is a fundamental concept in calculus. The tangent line provides the best linear approximation to the function at that point and is essential in analyzing the behavior of functions, especially in physics, engineering, and mathematics. This article will walk through the step-by-step process of determining the equation of the tangent line to the graph of the function $f(x) = -3x + 4x + 3$ at $x = 2$. We will explore the concepts of derivatives, slope calculation, and point-slope form of a line to arrive at the final answer.

Understanding the Function $f(x) = -3x + 4x + 3$

Simplifying the Function

The given function is:

$$
f(x) = -3x + 4x + 3
$$

Combine like terms:

$$
f(x) = (-3x + 4x) + 3 = x + 3
$$

Thus, the simplified form of the function is:

$$
f(x) = x + 3
$$

This is a linear function with a slope of 1 and a y-intercept of 3.

Implication of the Simplified Function

Since the function simplifies to $f(x) = x + 3$, its graph is a straight line with:

- Slope $m = 1$
- Y-intercept at $(0, 3)$

Because the graph is a straight line, the tangent line at any point on this line is the line itself. Therefore, the tangent line at $x=2$ is simply the line $f(x) = x + 3$.

Calculating the Slope of the Tangent Line

Derivative of the Function $f(x)$

In calculus, the slope of the tangent line at any point $x = a$ is given by the derivative of the function evaluated at a:

$$
f'(x) = \frac{d}{dx} [x + 3]
$$

Since the derivative of x is 1, and the derivative of a constant is 0:

$$
f'(x) = 1
$$

This derivative indicates that the slope of the tangent line is constant everywhere, consistent with the function being linear.

Evaluating the Slope at $x = 2$

Given that $f'(x) = 1$, the slope at $x=2$ is:

$$
m = f'(2) = 1
$$

Note: Since the function is linear, the slope is the same at all points, including $x=2$.

Finding the Point of Tangency

Calculating $f(2)$

To find the specific point of tangency, evaluate the function at $x=2$:

$$
f(2) = 2 + 3 = 5
$$

Thus, the point of tangency is $(2, 5)$.

Formulating the Equation of the Tangent Line

Using the Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m (x - x_1)
$$

Where:

- (x_1, y_1) is a point on the line
- m is the slope

Substituting the known point $(2, 5)$ and slope $m=1$:

$$
y - 5 = 1 (x - 2)
$$

Simplify:

$$
y - 5 = x - 2
$$

Add 5 to both sides:

\[
y = x + 3
\]

Result: The equation of the tangent line is \(y = x + 3\), which aligns with our earlier understanding since the function itself is a straight line.

Final Equation of the Tangent Line

The equation of the line tangent to the graph of \(f(x) = -3x + 4x + 3\) at \(x=2\) is:

\[
\boxed{y = x + 3}
\]

This line passes through the point \((2, 5)\) and has a slope of 1. Because the original function simplifies to a linear form, the tangent line coincides with the function itself at every point.

Summary and Key Takeaways

- The original function simplifies to a linear function: \(f(x) = x + 3\).
- The derivative \(f'(x)\) is constant and equals 1, indicating a constant slope.
- The point of tangency at \(x=2\) is \((2, 5)\).
- The equation of the tangent line at \(x=2\) is \(y = x + 3\).
- For linear functions, the tangent line at any point is the line itself.

Conclusion

Finding the tangent line to a function at a specific point involves understanding the derivative — the slope of the tangent — and the point of contact. In this case, since the function is linear, the tangent line is identical to the function graph. This example underscores the importance of derivatives in calculus and how they facilitate understanding the behavior of functions at particular points. Whether dealing with nonlinear or linear functions, the process remains fundamentally the same: compute the derivative, evaluate at the point of interest, and use the point-slope form to derive the tangent line's equation.

Frequently Asked Questions

What is the first step to find the equation of the tangent line to the graph of f(x) at x = 2?

The first step is to find the derivative of f(x) to determine the slope of the tangent line at x = 2.

How do you find the derivative of the function f(x) = -3x + 4x + 3?

Combine like terms to simplify f(x) to (1x) + 3, then differentiate to get f'(x) = 1.

What is the value of f'(x) at x = 2?

Since f'(x) = 1, the slope of the tangent line at x = 2 is 1.

How do you find the point on the graph at x = 2?

Plug x = 2 into the original function: f(2) = -3(2) + 4(2) + 3, which simplifies to -6 + 8 + 3 = 5.

What is the equation of the tangent line at x = 2?

Using point-slope form: y - 5 = 1(x - 2). Simplified, the tangent line is y = x + 3.

What is the final answer for the equation of the tangent line to the graph at x = 2?

The equation of the tangent line is y = x + 3.

Additional Resources

Find The Equation Of The Line Tangent To The Graph Of F(x)=-3x+4x+3 At X = 2

Understanding how to determine the tangent line to a function at a specific point is a fundamental skill in calculus, with applications spanning physics, engineering, and economics. This article offers a clear, step-by-step guide to finding the tangent line to the function $f(x) = -3x + 4x + 3$ at $x=2$. Whether you're a student tackling calculus for the first time or a professional brushing up on derivatives, this explanation will provide both clarity and depth.

Clarifying the Function: Simplify and Understand

Before diving into the derivative and tangent line calculations, it’s essential to scrutinize the given function:

$$
f(x) = -3x + 4x + 3
$$

Simplification of the Function

Combine like terms:

$$
f(x) = (-3x + 4x) + 3 = (1x) + 3 = x + 3
$$

This simplifies the problem significantly. The original expression looks complicated but reduces to a simple linear function:

$$
f(x) = x + 3
$$

Significance of the Simplification

Since $f(x)$ is linear, the derivative is constant, and the tangent line at any point on the graph is the line itself. However, understanding the process is still valuable, especially when dealing with more complex functions. For educational completeness, we’ll explore the derivative and tangent line step by step.

Deriving the Slope: Calculating the Derivative

What is a Derivative?

The derivative $f'(x)$ of a function $f(x)$ provides the slope of the tangent line to the graph at any point x. For linear functions, the derivative is straightforward, but the process illustrates the fundamental principles.

Computing the Derivative

Given:

$$
f(x) = x + 3
$$

The derivative with respect to x:

$$
f'(x) = \frac{d}{dx}(x + 3) = 1 + 0 = 1
$$

This indicates that the slope of the tangent line at any point x on the graph is 1.

Interpretation

- The slope of the tangent line at $x=2$ is 1.
- Since the derivative is constant, the tangent line is the same as the original line across all points.

Finding the Coordinates at $x=2$

Before writing the equation of the tangent line, determine the specific point on the graph where $x=2$.

Calculating $f(2)$

Using the simplified function:

$$
f(2) = 2 + 3 = 5
$$

Thus, the point of tangency is $(2, 5)$.

Significance

Knowing this point enables us to write the tangent line equation precisely. Even though the function is linear, this process applies to more complex functions where the tangent’s point must be computed explicitly.

Formulating the Equation of the Tangent Line

The Point-Slope Form

The general equation of a line given a point (x_0, y_0) and slope m:

$$
y - y_0 = m(x - x_0)

\]

For our problem:

- $x_0 = 2$
- $y_0 = 5$
- $m = 1$

Substituting Values

$$
y - 5 = 1(x - 2)
$$

Simplify:

$$
y - 5 = x - 2
$$

$$
y = x - 2 + 5
$$

$$
y = x + 3
$$

Final Equation

The tangent line at $x=2$ is:

$$
\boxed{y = x + 3}
$$

This matches the original function, confirming that the tangent line coincides with the graph itself because the function is linear.

Broader Implications and Applications

While this particular case is straightforward, the process illustrates core calculus principles applicable to more complex functions:

For Nonlinear Functions

- Derivative Calculation: Find the derivative $f'(x)$ to determine the slope at the point of interest.
- Coordinate Identification: Compute $f(x_0)$ to find the point on the graph.

- Equation Formation: Use the point-slope form to write the tangent line.

Real-World Applications

- Physics: Calculating the velocity of an object at a specific moment.
- Economics: Determining the marginal cost or revenue at a certain production level.
- Engineering: Analyzing the slope of stress-strain curves for material analysis.

Challenges with More Complex Functions

When dealing with nonlinear functions such as quadratics, cubics, or transcendental functions, the process involves:

- Applying derivative rules (product rule, quotient rule, chain rule)
- Carefully evaluating the derivative at the point of interest
- Computing the corresponding function value for that point

Summary and Key Takeaways

- The original function $f(x) = -3x + 4x + 3$ simplifies to $f(x) = x + 3$.
- The derivative $f'(x) = 1$ indicates a constant slope.
- At $x=2$, the point on the graph is $(2, 5)$.
- The tangent line at this point is $y = x + 3$, which coincides with the original line due to its linearity.

In essence, the tangent line to a linear function at any point is the function itself. For more complex functions, the derivative provides the slope necessary to construct the tangent line, offering insights into the local behavior of the graph.

Final Thoughts

Understanding the process of finding tangent lines deepens comprehension of calculus concepts and enhances problem-solving skills across various disciplines. Although in this particular case, the function's linearity makes the tangent line identical to the graph, the methodology outlined here equips you to tackle more intricate functions with confidence. Remember, mastering derivatives and tangent line equations is a crucial step toward understanding the dynamic nature of functions and their real-world applications.

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