Find The First Partial Derivatives Of The Function. F(x,y)=intyx Cos(e^t)dt

Find The First Partial Derivatives Of The Function. F(x,y)=int_{y}^{x} Cos(e^t) dt

Calculus plays a crucial role in understanding how functions behave, especially when dealing with multivariable functions. In particular, partial derivatives provide insight into how a function changes as one variable varies while holding others constant. In this article, we focus on a specific function defined by a definite integral:

$F(x,y) = \int_y^x Cos(e^t)\, dt$

Our goal is to find the first partial derivatives of this function with respect to both variables, x and y. This process involves applying the Fundamental Theorem of Calculus, along with the Leibniz rule for differentiating integrals with variable limits. Understanding these derivatives is essential in fields ranging from differential equations to applied physics, where such integrals often represent accumulated quantities over a range.

Understanding the Function F(x,y)

Definition and Context

The function F(x,y) is defined as a definite integral with variable limits:

$$F(x,y) = \int_y^x Cos(e^t)\, dt$$

Here, the integrand is Cos(e^t), a composite function involving the exponential and cosine functions. The limits of integration are y (lower limit) and x (upper limit), both of which are variables. This structure makes F(x,y) a function of two variables, x and y, and understanding its partial derivatives involves differentiating with respect to each variable separately.

Why Are Partial Derivatives Important?

Partial derivatives measure how a multivariable function changes as one variable changes, keeping the other variables fixed. For F(x,y),:

- **∂F/∂x** describes how F changes as x varies, with y held constant.
- **∂F/∂y** describes how F changes as y varies, with x held constant.

These derivatives are foundational in optimization, sensitivity analysis, and solving differential equations involving multiple variables.

Applying the Fundamental Theorem of Calculus

Fundamental Theorem of Calculus (Part 1)

The Fundamental Theorem of Calculus states that if a function is continuous on an interval, then its definite integral from a fixed point to a variable endpoint is differentiable with respect to that endpoint. Specifically:

- The derivative of the integral with respect to its upper limit x is the integrand evaluated at x.
- The derivative with respect to the lower limit y is the negative of the integrand evaluated at y.

This principle simplifies the process of finding partial derivatives of functions defined as integrals with variable limits.

Leibniz Rule for Differentiation Under the Integral Sign

When both limits of the integral depend on the variables, the Leibniz rule states:

$$d/dx \left[\int_{\alpha(x,y)}^{\beta(x,y)} f(t)\, dt\right] = f(\beta(x,y))\ d\beta/dx - f(\alpha(x,y))\ d\alpha/dx$$

Similarly, for differentiation with respect to y:

$$d/dy \left[\int_{\alpha(x,y)}^{\beta(x,y)} f(t)\, dt\right] = f(\beta(x,y))\ d\beta/dy - f(\alpha(x,y))\ d\alpha/dy$$

In our case, the limits are functions of x and y, so applying these rules allows us to compute the partial derivatives methodically.

Calculating the First Partial Derivative with Respect to x

Step 1: Identify the limits and the integrand

- Upper limit: x
- Lower limit: y
- Integrand: $f(t) = \text{Cos}(e^t)$

Step 2: Apply the Fundamental Theorem of Calculus

Since the upper limit is x, which depends on x, and the lower limit y, which does not depend on x, the derivative of F with respect to x is:

$$\partial F/\partial x = f(x) \quad d(x)/dx - f(y) \quad d(y)/dx$$

But $d(y)/dx = 0$ because y is independent of x, and $d(x)/dx = 1$. Therefore,

$$\partial F/\partial x = \text{Cos}(e^x) \quad 1 - 0 = \text{Cos}(e^x)$$

Final expression for ∂F/∂x

$\partial F/\partial x = \text{Cos}(e^x)$

Calculating the First Partial Derivative with Respect to y

Step 1: Identify the limits and the integrand

- Upper limit: x (constant with respect to y)
- Lower limit: y
- Integrand: $f(t) = Cos(e^t)$

Step 2: Apply Leibniz rule for differentiation with respect to y

Since y appears as the lower limit, and x is independent of y:

$$\partial F/\partial y = -f(y) \; d(y)/dy + 0$$

Note the negative sign because y is the lower limit.

- $d(y)/dy = 1$

Therefore,

$$\partial F/\partial y = -Cos(e^y) \; 1 = -Cos(e^y)$$

Final expression for ∂F/∂y

$\partial F/\partial y = -Cos(e^y)$

Summary of the First Partial Derivatives

- Partial derivative with respect to x: **$\partial F/\partial x = Cos(e^x)$**
- Partial derivative with respect to y: **$\partial F/\partial y = -Cos(e^y)$**

Interpreting the Results

The derivatives obtained reveal how the integral-based function $F(x,y)$ responds to changes in its variables:

- **$\partial F/\partial x = \cos(e^x)$** indicates that as x increases, the rate of change of the integral depends on the cosine of e^x. Since e^x is always positive, the cosine oscillates depending on the value of x, leading to periodic behavior in the derivative.

- **$\partial F/\partial y = -\cos(e^y)$** shows that increasing y decreases the value of F, with the rate of decrease modulated by the cosine of e^y. The negative sign indicates the inverse relationship between y and the integral's value—raising y reduces the integral's value because it lowers the lower limit.

Applications of Partial Derivatives in Real-World Contexts

1. Physics and Engineering

- In thermodynamics, integrals similar to $F(x,y)$ describe accumulated quantities like heat or work, and their derivatives help analyze system sensitivity.
- Structural analysis often involves integrals of stress or strain over a span, requiring derivatives to understand how modifications affect the system.

2. Economics and Finance

- Cost and revenue functions often involve integrals over time or other variables, with derivatives indicating marginal costs or revenues.

3. Mathematical Modeling and Optimization

- Partial derivatives enable optimization of functions involving integrals, such as maximizing efficiency or minimizing costs in engineering designs.

Conclusion

Calculating the first partial derivatives of the function $F(x,y) = \int_y^x \cos(e^t)\,dt$ involves applying fundamental calculus principles, notably the Fundamental Theorem of Calculus and Leibniz rule. The derivatives are straightforward to compute once the limits and integrand are identified:

- $\partial F/\partial x = \cos(e^x)$
- $\partial F/\partial y = -\cos(e^y)$

These results not only deepen our understanding of how the integral function varies with its variables but also demonstrate the utility of calculus tools in analyzing complex functions defined via integrals. Mastery of these concepts is essential for tackling advanced problems in mathematics, physics, economics, and engineering, where such functions frequently appear.

Frequently Asked Questions

How do I find the first partial derivative of F(x,y) with respect to x for F(x,y) = ∫_{y}^{x} cos(e^t) dt?

Using the Fundamental Theorem of Calculus, the partial derivative with respect to x is the integrand evaluated at $t = x$, so $\partial F/\partial x = \cos(e^x)$.

What is the first partial derivative of F(x,y) with respect to y for F(x,y) = ∫_{y}^{x} cos(e^t) dt?

Applying Leibniz's rule, the partial derivative with respect to y is the negative of the integrand evaluated at $t = y$, so $\partial F/\partial y = -\cos(e^y)$.

Can I use Leibniz's rule to compute the partial derivatives of an integral with variable limits like F(x,y) = ∫_{y}^{x} cos(e^t) dt?

Yes, Leibniz's rule allows you to compute derivatives of integrals with variable limits. For the upper limit x, $\partial/\partial x$ adds the integrand at x; for the lower limit y, $\partial/\partial y$ subtracts the integrand at y.

Are the partial derivatives ∂F/∂x and ∂F/∂y continuous functions for F(x,y) = ∫_{y}^{x} cos(e^t) dt?

Yes, since $\cos(e^t)$ is continuous for all real t, and the partial derivatives are expressed as evaluations of

$\cos(e^t)$, they are continuous functions of x and y.

What is the significance of finding the first partial derivatives of F(x,y) in understanding the behavior of the function?

Calculating the first partial derivatives helps determine how F(x,y) changes with x and y, revealing slopes, increasing/decreasing behavior, and potential critical points of the function.

Additional Resources

Partial Derivatives of the Function $F(x, y) = \int_x^y \cos(e^t)\, dt$: A Comprehensive Analysis

In the realm of multivariable calculus, understanding how functions change with respect to their variables is fundamental. One of the most powerful tools for this purpose is the concept of partial derivatives, which measure the rate of change of a function with respect to one variable while holding others constant. Today, we delve into a particularly interesting function:

$$F(x, y) = \int_{x}^{y} \cos(e^t) \, dt$$

This integral-based function is a prime candidate to explore the application of the Leibniz rule for differentiation under the integral sign, an elegant and versatile technique in calculus.

Understanding the Function $F(x, y)$

Before diving into derivatives, let's analyze the structure of $F(x, y)$:

- Type of Function: It is a definite integral with variable limits, where both the lower limit (x) and the upper limit (y) depend on the variables we are differentiating with respect to.
- Integrand: The integrand, $\cos(e^t)$, is a continuous function of t. Its properties are crucial because the Leibniz rule applies smoothly when the integrand is continuous and differentiable.
- Domain: The function is defined for all real numbers x and y where the integral makes sense, especially considering the exponential inside the cosine.

Mathematical Foundations: Differentiation of Integrals with Variable Limits

The core principle used to find the partial derivatives of $F(x, y)$ is the Leibniz rule for differentiation under the integral sign. It states that:

$$
\frac{\partial}{\partial x} \int_{a(x)}^{b(x)} f(t) \, dt = f(b(x)) \cdot b'(x) - f(a(x)) \cdot a'(x)
$$

Similarly, when differentiating with respect to y:

$$
\frac{\partial}{\partial y} \int_{a(x)}^{b(y)} f(t) \, dt = f(b(y)) \cdot b'(y)
$$

In our specific case, the limits are directly the variables x and y, which simplifies the derivatives since the derivatives of the limits are straightforward.

Calculating the First Partial Derivatives

Let's now proceed to compute $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

Partial Derivative with Respect to x: $\frac{\partial F}{\partial x}$

Given:

$$
F(x, y) = \int_x^y \cos(e^t) \, dt
$$

Applying Leibniz rule for the lower limit x:

$$
\frac{\partial F}{\partial x} = - \cos(e^x) \cdot \frac{\partial x}{\partial x} = - \cos(e^x)
$$

Explanation:

- The negative sign appears because the lower limit is x.
- The integrand evaluated at the lower limit $t = x$ is $\cos(e^x)$.
- The derivative of the lower limit $a(x) = x$ with respect to x is 1.
- Therefore:

$$
\boxed{
\frac{\partial F}{\partial x} = - \cos(e^x)
}
$$

This result intuitively makes sense: increasing x decreases the integral's value at the current point, hence the negative sign.

Partial Derivative with Respect to y: $\frac{\partial F}{\partial y}$

Similarly, for the upper limit:

$$
\frac{\partial F}{\partial y} = \cos(e^y) \cdot \frac{\partial y}{\partial y} = \cos(e^y)
$$

Explanation:

- The upper limit is y, whose derivative with respect to itself is 1.
- The integrand evaluated at $t=y$ is $\cos(e^y)$.
- Since increasing y adds more area under the curve from x to y, the derivative is positive.

$$
\boxed{
\frac{\partial F}{\partial y} = \cos(e^y)
}
$$

Intuitive Insights and Practical Applications

Understanding these derivatives offers more than mere mathematical curiosity; it provides insights into how the integral's value shifts as the limits vary:

- Sensitivity Analysis: The derivatives quantify the sensitivity of F to small changes in x and y. For example, a small increase in x decreases the value of F proportionally to $\cos(e^x)$.
- Numerical Approximations: In computational mathematics, knowing these derivatives facilitates numerical methods like finite difference approximations or gradient-based algorithms.
- Physical Interpretations: If F models a physical quantity (say, accumulated quantity over a variable interval), these derivatives tell us how the quantity changes as the bounds shift.

Extending the Analysis: Higher-Order and Mixed Partial Derivatives

While the focus here is on the first partial derivatives, the methodology extends naturally to higher-order derivatives:

- Second partial derivatives involve differentiating $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ again with respect to x or y.
- Mixed partial derivatives such as $\frac{\partial^2 F}{\partial y \partial x}$ or $\frac{\partial^2 F}{\partial x \partial y}$ can reveal the symmetry properties of the function, provided the necessary smoothness conditions are satisfied.

For instance, differentiating $\frac{\partial F}{\partial x} = - \cos(e^x)$ with respect to x:

$$
\frac{\partial^2 F}{\partial x^2} = - \frac{d}{dx} \left[\cos(e^x) \right] = - \left[- \sin(e^x) \cdot e^x \right] = \sin(e^x) \cdot e^x
$$

Similarly, differentiating $\frac{\partial F}{\partial y}$:

$$
\frac{\partial^2 F}{\partial y^2} = \frac{d}{dy} \left[\cos(e^y) \right] = - \sin(e^y) \cdot e^y
$$

Summary of Key Results

Variable	First Partial Derivative	Explanation
x	$\frac{\partial F}{\partial x} = - \cos(e^x)$	Negative sign due to lower limit x
y	$\frac{\partial F}{\partial y} = \cos(e^y)$	Positive, as the upper limit adds to the integral

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Final Remarks and Practical Considerations

The clear application of Leibniz's rule in this context underscores its power in handling integrals with variable limits. This approach simplifies potentially complex differentiation tasks and provides exact results that are crucial in both theoretical and applied mathematics.

Practical tips for students and practitioners:

- Always verify the conditions for Leibniz rule application: continuity and differentiability of the integrand.
- Remember the signs associated with differentiating with respect to upper vs. lower limits.
- Use chain rule carefully when the integrand involves composite functions like e^t.

This detailed exploration not only clarifies the process of finding the first partial derivatives of $F(x, y)$ but also enriches your understanding of the interplay between integration and differentiation in multivariable calculus.

In conclusion, the first partial derivatives of $F(x, y) = \int_x^y \cos(e^t) \, dt$ are elegantly expressed as:

$$
\boxed{
\frac{\partial F}{\partial x} = - \cos(e^x), \quad \frac{\partial F}{\partial y} = \cos(e^y)
}
$$

This foundational knowledge paves the way for more advanced analyses, including optimization, differential equations, and modeling scenarios where integral bounds are dynamic.

Find The First Partial Derivatives Of The Function F X Y Intyx Cos E T Dt

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