

Find The Frequency, F, Of The Voltage In Hertz Given $V_a(t) = 60 \cos(40,000t)$ V.

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Understanding the frequency of a voltage signal is fundamental in electrical engineering, electronics, and related fields. When analyzing sinusoidal voltages such as $V_a(t) = 60 \cos(40,000t)$ V, it is crucial to determine the frequency, which indicates how many cycles the wave completes in one second. This article provides a comprehensive guide to find the frequency, F, of the voltage given the function $V_a(t) = 60 \cos(40,000t)$ V, including the underlying principles, step-by-step procedures, and practical applications.

Understanding the Sinusoidal Voltage Function

General Form of a Sinusoidal Voltage

A sinusoidal voltage can be expressed in the general form:

$$v(t) = V_{\max} \cos(\omega t + \phi)$$

where:

- $V_{\max}$ is the amplitude (peak voltage),
- ω is the angular frequency in radians per second,
- t is time in seconds,
- ϕ is the phase angle in radians.

In our case, the given voltage function is:

$$V_a(t) = 60 \cos(40,000 t), \text{ V}$$

which resembles the general form with:

- Peak voltage, $V_{\max} = 60$ V,
- Argument of cosine, $40,000 t$,
- Phase angle, assumed to be zero since not specified.

From Angular Frequency to Frequency in Hertz

The key to finding the frequency is understanding the relationship between angular frequency (ω) and frequency in Hertz (F):

$$\omega = 2\pi F$$

where:

- ω is the angular frequency (radians per second),
- F is the frequency in Hertz (cycles per second).

Rearranged, the formula to find frequency is:

$$F = \frac{\omega}{2\pi}$$

Step-by-Step Calculation of Frequency F

Step 1: Identify the Angular Frequency ω

In the given function, the argument of the cosine is $40,000 t$.
Comparing to the standard form:

$$\omega t = 40,000 t$$

we see that:

$$\omega = 40,000 \text{ radians/second}$$

Step 2: Apply the Relationship $F = \frac{\omega}{2\pi}$

Using the identified ω :

$$F = \frac{40,000}{2\pi}$$

Calculate the denominator:

$$2\pi \approx 6.2832$$

Therefore,

$$F = \frac{40,000}{6.2832}$$

Perform the division:

$$F \approx 6366.2 \text{ Hz}$$

Step 3: Final Result

The frequency of the voltage $V_a(t)$ is approximately 6366.2 Hz.

Understanding the Significance of the Frequency

What Does a Frequency of 6366.2 Hz Imply?

- The voltage completes approximately 6366.2 cycles every second.
- Such a high frequency suggests applications in high-frequency circuits, RF communication, or specialized electronic systems.
- In typical power systems, the frequency is about 50 Hz or 60 Hz, but in signal processing or RF systems, frequencies are often much higher.

Practical Applications of High-Frequency Voltages

- Radio frequency (RF) transmission
- Microwave communications
- High-frequency inductors and capacitors
- Electronic testing and measurement equipment

Additional Concepts and Related Calculations

Understanding Period and Wavelength

- Period (T): The reciprocal of frequency:

$$T = \frac{1}{F}$$

For our calculated frequency:

$$T = \frac{1}{6366.2} \approx 0.000157 \text{ seconds}$$

- Wavelength (λ): If considering wave propagation in a medium:

$$\lambda = \frac{v}{F}$$

where v is the velocity of wave propagation (speed of light in vacuum is approximately $3 \times 10^8 \text{ m/s}$).

Implications in Circuit Design

- High-frequency signals require special considerations in circuit design to prevent issues like signal attenuation, interference, and parasitic effects.
- Proper shielding, impedance matching, and the use of high-frequency components are essential.

Summary and Key Takeaways

- The frequency (F) of a sinusoidal voltage $(V_a(t) = 60 \cos(40,000 t))$ is approximately 6366.2 Hz.
- The angular frequency (ω) is derived directly from the coefficient of (t) in the cosine argument.
- The relationship between angular frequency and Hertz is fundamental:

$$F = \frac{\omega}{2\pi}$$

- Understanding frequency is vital for applications across power systems, communications, and signal processing.

FAQs about Voltage Frequency Calculation

Q1: Can the frequency be negative?

No, frequency is a scalar quantity representing cycles per second and is always positive.

Q2: Why is the angular frequency expressed in radians per second?

Because the cosine function's argument is in radians, and the angular frequency relates to how many radians the wave cycles through per second.

Q3: What happens if the voltage function has a phase shift?

The phase shift (ϕ) does not affect the frequency; it only shifts the waveform along the time axis.

Q4: How does increasing the value inside the cosine affect the frequency?

A larger coefficient (like 40,000 in this case) increases the angular frequency, thus increasing the frequency in Hertz.

Q5: Are there practical limits to high-frequency voltages?

Yes, at very high frequencies, issues such as signal loss, interference, and component parasitics become significant.

Conclusion

Accurately determining the frequency of a sinusoidal voltage is essential for designing, analyzing, and troubleshooting electrical and electronic systems. By recognizing the form of the voltage function and understanding the relationship between angular frequency and Hertz, engineers and students can efficiently find the frequency of any sinusoidal waveform. In this case, given $V_a(t) = 60 \cos(40,000 t)$ V, the frequency is approximately 6366.2 Hz, highlighting its application in high-frequency electronic systems and communications.

Keywords: sinusoidal voltage, frequency calculation, angular frequency, Hertz, voltage waveform analysis, high-frequency signals, electrical engineering

Frequently Asked Questions

What is the general form of the voltage signal given as $V_a(t) = 60 \cos(40,000t)$ V?

The voltage is expressed as a cosine function with an amplitude of 60 volts and an angular frequency of 40,000 radians per second.

How do you find the frequency F in Hertz from the given voltage function $V_a(t) = 60 \cos(40,000t)$ V?

First, identify the angular frequency $\omega = 40,000$ rad/s. Then, use the relation $F = \omega / (2\pi)$ to find the frequency in Hertz.

What is the value of the frequency F for $V_a(t) = 60 \cos(40,000t)$ V?

The frequency F is approximately 6,366 Hz, calculated as $40,000 / (2\pi) \approx 6,366$ Hz.

Why is it important to convert angular frequency to frequency in Hertz?

Converting to Hertz makes it easier to understand the number of cycles per second, which is a standard way to express frequency in electrical and signal analysis.

Can you show the calculation for the frequency F given $\omega = 40,000$ rad/sec?

Yes. $F = \omega / (2\pi) = 40,000 / (2 \times 3.1416) \approx 40,000 / 6.2832 \approx 6,366$ Hz.

What units should be used for frequency F when calculating from angular frequency?

Frequency F should be expressed in Hertz (Hz), which is cycles per second.

If the angular frequency ω in the voltage equation is doubled, how does that affect the frequency F ?

Doubling ω doubles the frequency F , since $F = \omega / (2\pi)$.

Is the given voltage function $V_a(t) = 60 \cos(40,000t)$ V typical for AC power systems?

No, typical AC power systems operate at frequencies like 50 or 60 Hz. A frequency of approximately 6,366 Hz is much higher and more typical of high-frequency applications.

What practical applications might involve a voltage oscillating at approximately 6,366 Hz?

Such high-frequency voltages are used in radio frequency transmissions, RF circuits, and communication systems rather than standard power distribution.

How can understanding the frequency of a voltage signal help in electrical engineering?

Knowing the frequency helps in designing filters, transformers, and communication systems, and in analyzing how signals propagate and interact in circuits.

Additional Resources

Find The Frequency, F , Of The Voltage In Hertz Given $V_a(t) = 60 \cos(40,000t)$ V is a fundamental problem in electrical engineering and physics, particularly in the analysis of AC (alternating current) circuits and signal processing. Understanding how to extract the frequency from a time-dependent voltage expression is crucial for designing and troubleshooting electrical systems, communication devices, and electronic circuits. This article aims to provide a comprehensive, detailed explanation of how to determine the frequency, F , from the given voltage function, $V_a(t) = 60 \cos(40,000t)$ V, including the theoretical background, step-by-step solution, and real-world

applications.

Understanding the Voltage Function: $V_a(t) = 60 \cos(40,000t)$ V

Before delving into the process of finding the frequency, it's essential to understand the structure of the given voltage function.

Components of the Voltage Expression

- Amplitude (A): The coefficient of the cosine function, which in this case is 60 V. It indicates the maximum voltage value, or peak voltage, of the AC signal.
- Angular Frequency (ω): The coefficient inside the cosine function's argument, here 40,000. It relates to how rapidly the voltage oscillates over time.
- Time variable (t): The independent variable representing time, typically in seconds.

The general form of a sinusoidal voltage function can be written as:

$$v(t) = A \cos(\omega t + \phi)$$

where ϕ is the phase angle. In our case, $\phi = 0$.

Connecting Angular Frequency to Frequency in Hertz

To find the frequency in Hertz (Hz), we need to understand the relationship between angular frequency (ω) and frequency (F).

Theoretical Background

- Angular frequency (ω) is measured in radians per second (rad/sec).
- Frequency (F) in Hertz (Hz) tells us how many oscillations occur in one second.

The relationship between ω and F is:

$$\omega = 2\pi F$$

This formula indicates that the angular frequency is simply (2π) times the frequency in Hz.

Key Point:

Given (ω) , you can directly calculate (F) as:

$$F = \frac{\omega}{2\pi}$$

Calculating the Frequency From the Given Function

In the function $(V_a(t) = 60 \cos(40,000 t) \text{ V})$, the argument of the cosine function is $(40,000 t)$. Comparing this to the general form:

$$\omega t = 40,000 t$$

It follows that:

$$\omega = 40,000 \text{ rad/sec}$$

Applying the relationship:

$$F = \frac{\omega}{2\pi}$$

we substitute $(\omega = 40,000)$:

$$F = \frac{40,000}{2\pi}$$

Calculating:

$$F \approx \frac{40,000}{6.2832} \approx 6,366.2, \text{ Hz}$$

Result:

The frequency of the voltage waveform is approximately 6366.2 Hertz.

Step-by-Step Summary of the Calculation

1. Identify the argument of the cosine function: $(40,000 t)$.
2. Recognize that this argument equals (ωt) , where (ω) is angular frequency.
3. Extract $(\omega = 40,000)$ rad/sec.
4. Use the relation $(F = \frac{\omega}{2\pi})$.
5. Compute $(F \approx 6366.2)$ Hz.

This systematic approach ensures clarity and accuracy in deriving the frequency from the sinusoidal voltage expression.

Practical Significance of the Calculated Frequency

Understanding the frequency of an AC voltage is vital for multiple reasons:

- Design of Electrical Components: Many components, such as transformers and filters, are designed to operate optimally at specific frequencies.
- Power Systems: Standard mains power frequency varies by country (e.g., 50 Hz in most of the world, 60 Hz in the US). The calculated frequency here (~6366 Hz) is much higher, indicating a signal used in specialized applications such as communication or RF systems.
- Signal Analysis: Accurate frequency determination helps in modulation, demodulation, and spectral analysis.

Features and Characteristics of Sinusoidal Voltage Signals

Features:

- Pure Sinusoid: The voltage $(V_a(t))$ varies smoothly and periodically, which simplifies analysis.
- Peak Voltage: 60 V, indicating the maximum instantaneous voltage.
- Frequency: As calculated, approximately 6366 Hz, which influences how the signal propagates and interacts with other circuit elements.

Pros:

- Sinusoidal signals are easy to analyze mathematically.

- They are fundamental in AC power systems and RF communications.
- The relationship between angular frequency and frequency is straightforward and well-understood.

Cons:

- High-frequency signals like 6366 Hz may require specialized equipment for accurate measurement.
- In real-world circuits, signals may not be pure sinusoids due to noise, distortion, or non-linearities.

Real-World Applications and Implications

Communication Systems:

High-frequency signals, such as those around 6.3 kHz, are common in radio, audio transmission, and other wireless communication systems.

Signal Processing:

Knowing the frequency helps in filtering, amplification, and processing of signals for clarity and efficiency.

Electrical Engineering Education:

Understanding how to extract frequency from time-domain signals is foundational for students learning about AC circuits, Fourier analysis, and signal theory.

Conclusion

Determining the frequency of a sinusoidal voltage waveform from its mathematical expression is an essential skill in electrical engineering. Given the voltage function $V_a(t) = 60 \cos(40,000 t) \text{ V}$, the process involves recognizing the angular frequency ω , applying the fundamental relationship $F = \frac{\omega}{2\pi}$, and performing straightforward arithmetic to find that the frequency is approximately 6366.2 Hz. This insight not only aids in understanding the nature of the signal but also informs design, analysis, and application in various technological fields. Mastery of this concept is critical for anyone working with AC signals, communications, or electronic systems, highlighting the importance of bridging mathematical understanding with practical engineering applications.

Find The Frequency F Of The Voltage In Hertz Given $V_a = 60 \cos 40000t$ V

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