Find The General Solution Of The Differential Equation Y + 18y'' + 81y = Oy(t) =

Find The General Solution Of The Differential Equation Y + 18y'' + 81y = Oy(t) = is a fundamental problem in differential equations, often encountered in various applications such as physics, engineering, and mathematical modeling. Solving this type of second-order linear homogeneous differential equation involves understanding its characteristic equation, roots, and the resulting general solution. In this comprehensive guide, we will explore the step-by-step process of finding the general solution to the differential equation Y + 18y'' + 81y = 0, providing detailed explanations, techniques, and examples to enhance your understanding.

Understanding the Differential Equation

The given differential equation is:

$$ Y + 18y'' + 81y = 0 $$

It is a linear homogeneous differential equation of second order with constant coefficients. The notation Y can be interpreted as y(t), the dependent variable, and y'' represents the second derivative of y with respect to t.

Rewriting the equation for clarity:

$$ y'' + \frac{Y}{18} + \frac{81}{18} y = 0 $$

But considering the original form, it's clearer to write:

$$ y'' + p y' + q y = 0 $$

In our case, the equation does not contain a y' term, so p = 0, and q = 81/18 = 4.5. However, to clarify, the standard form for second-order linear differential equations with constant coefficients is:

$$ y'' + a y' + b y = 0 $$

Since the original equation does not have a y' term, the form simplifies to:

$$ y'' + c y = 0 $$

where c is a constant.

Note: There appears to be some ambiguity in the original notation, as "Y + 18y'' + 81y = 0y(t) =" seems to have a typographical error or misformatting. Usually, differential equations are written as:

$$ y'' + a y' + b y = 0 $$

Assuming the intended differential equation is:

$$ y'' + 18 y' + 81 y = 0 $$

which is a typical second-order linear differential equation with constant coefficients, then the process to find the general solution proceeds as follows.

Assumption for clarity:
We will assume the differential equation is:

$$ y'' + 18 y' + 81 y = 0 $$

and proceed with solving this equation.

Step 1: Write the Characteristic Equation

To solve the differential equation:

$$ y'' + 18 y' + 81 y = 0 $$

we begin by forming the characteristic equation, which is a quadratic polynomial in terms of the variable r, representing the roots related to the solution of the differential equation:

$$ r^2 + 18 r + 81 = 0 $$

This characteristic equation encapsulates the behavior of the solutions.

Step 2: Solve the Characteristic Equation

The roots of the quadratic are found using the quadratic formula:

$$ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where:

- $a = 1$

- $b = 18$

- $c = 81$

Substituting:

$$ r = \frac{-18 \pm \sqrt{18^2 - 4 \times 1 \times 81}}{2} $$

Calculate discriminant:

$$ \Delta = 18^2 - 4 \times 81 = 324 - 324 = 0 $$

Since the discriminant is zero, the roots are real and equal:

$$ r = \frac{-18}{2} = -9 $$

and both roots are:

$$ r_1 = r_2 = -9 $$

Understanding roots:

- Distinct real roots: lead to solutions involving exponentials.

- Repeated roots: lead to solutions involving exponential functions and polynomial factors.

- Complex conjugate roots: lead to solutions involving sines and cosines.

In our case, with a repeated root, the general solution takes a special form.

Step 3: Write the General Solution

For a second-order homogeneous linear differential equation with constant coefficients, the general solution depends on the roots of the characteristic equation:

- Distinct real roots r_1, r_2:

$$y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

- Repeated real root r:

$$y(t) = (C_1 + C_2 t) e^{r t}$$

- Complex conjugate roots $\alpha \pm i \beta$:

$$y(t) = e^{\alpha t} (C_1 \cos \beta t + C_2 \sin \beta t)$$

Given our roots $r_1 = r_2 = -9$, the general solution is:

$$y(t) = (C_1 + C_2 t) e^{-9 t}$$

where C_1 and C_2 are arbitrary constants determined by initial conditions.

Step 4: Interpreting the Solution

The solution:

$$y(t) = (C_1 + C_2 t) e^{-9 t}$$

has several important features:

- The exponential term $e^{-9 t}$ indicates the solution decays exponentially as $t \to \infty$, which is typical in systems with damping or stabilization.

- The polynomial factor $(C_1 + C_2 t)$ arises due to the repeated root, reflecting the multiplicity effect in the solution.

- The arbitrary constants C_1 and C_2 allow for the solution to be tailored to specific initial conditions, such as initial position and velocity in mechanical systems.

Step 5: Applying Initial Conditions (Optional)

In many problems, initial conditions are provided, such as:

$$y(0) = y_0$$
$$y'(0) = y_1$$

Using these, you can solve for C_1 and C_2:

1. Substituting $t = 0$:

$$y(0) = (C_1 + C_2 \times 0) e^{0} = C_1$$

so,

$$C_1 = y(0)$$

2. Differentiating the general solution:

$$ y'(t) = \frac{d}{dt} [(C_1 + C_2 t) e^{-9 t}] $$

Using the product rule:

$$ y'(t) = C_2 e^{-9 t} + (C_1 + C_2 t)(-9) e^{-9 t} $$

$$ y'(t) = e^{-9 t} [C_2 - 9 (C_1 + C_2 t)] $$

At $t = 0$:

$$ y'(0) = e^{0} [C_2 - 9 C_1] = C_2 - 9 C_1 $$

Thus,

$$ y'(0) = C_2 - 9 y(0) $$

which allows solving for C_2:

$$ C_2 = y'(0) + 9 y(0) $$

Summary:

- The general solution to the differential equation $y'' + 18 y' + 81 y = 0$ is:

$$ y(t) = (C_1 + C_2 t) e^{-9 t} $$

- Constants C_1 and C_2 are determined by initial conditions.

Additional Notes and Common Variations

1. Different Forms of the Differential Equation

If the original differential equation has different coefficients or additional terms, the solution process will adjust accordingly. The key steps remain:

- Form the characteristic equation.
- Find roots.
- Write the general solution based on the root type.

2. Non-Homogeneous Equations

If the differential equation included a non-zero right-hand side, such as:

$$ y'' + 18 y' + 81 y = f(t) $$

then the solution would involve:

- The complementary (homogeneous) solution (as above).
- A particular solution $y_p(t)$, found via methods like undetermined coefficients or variation of parameters.

3. Significance of Repeated Roots

Repeated roots indicate a multiplicity greater than one in the characteristic equation, leading to solutions involving polynomial factors multiplied by exponentials.

4. Physical Interpretation

In mechanical systems, such differential equations often model damped oscillations. The exponential decay $e^{-9 t}$ signifies damping, while the polynomial term relates to system responses with multiplicity in eigenvalues.

Conclusion

In summary, to find the general solution of the differential equation $y'' + 18 y' + 81 y = 0$, we:

- Derived the characteristic equation: $r^2 + 18 r + 81 = 0$.
- Calculated the roots: a repeated real root $r = -9$.
- Constructed the solution for repeated roots: \(y(t) = (C_

Frequently Asked Questions

What is the general form of the differential equation Y + 18y'' + 81y = 0?

The differential equation is a second-order linear homogeneous differential equation with constant coefficients: Y + 18y'' + 81y = 0.

How do you find the characteristic equation for Y + 18y'' + 81y = 0?

Replace y with e^{rt} to get the characteristic equation: $r^2 + 18r + 81 = 0$.

What are the roots of the characteristic equation r^2 + 18r + 81 = 0?

The roots are $r = -9, -9$, which are repeated roots because the discriminant is zero.

What is the general solution for a differential equation with repeated roots r = -9?

The general solution is y(t) = (C1 + C2 t) e^{-9t}, where C1 and C2 are arbitrary constants.

How do repeated roots affect the form of the solution to the differential equation?

Repeated roots lead to solutions involving t multiplied by the exponential term, resulting in terms like t e^{rt}.

What are the arbitrary constants in the general solution, and what do they represent?

C1 and C2 are arbitrary constants representing the family of solutions satisfying initial conditions or boundary conditions.

Can the solution be written in a simplified form when roots are repeated?

Yes, the solution simplifies to y(t) = (C1 + C2 t) e^{-9t}.

What is the significance of the exponential term e^{-9t} in the solution?

The exponential term e^{-9t} indicates the rate of decay of the solution over time due to the negative root.

How do I verify that the obtained general solution satisfies the

differential equation?

Differentiate y(t) twice, substitute y, y', y'' into the original equation, and check if the equation simplifies to zero for all t.

Additional Resources

Find The General Solution Of The Differential Equation Y + 18y'' + 81y = 0

Understanding and solving differential equations is fundamental in mathematics, physics, engineering, and various applied sciences. They describe how quantities change over time or space and are crucial for modeling real-world phenomena such as mechanical vibrations, electrical circuits, population dynamics, and heat transfer. Among the many types of differential equations, linear homogeneous equations with constant coefficients occupy a prominent position due to their relatively straightforward solution methods and broad applicability. In this article, we delve into the comprehensive process of finding the general solution of the differential equation:

$$ y + 18 y'' + 81 y = 0 $$

This equation exemplifies a second-order linear homogeneous differential equation with constant coefficients, and understanding its solution structure provides insights into more complex differential systems.

Understanding the Differential Equation

The Form and Significance

The given differential equation,

$$ y + 18 y'' + 81 y = 0, $$

can be recognized as a second-order linear homogeneous differential equation with constant coefficients. Its structure suggests that the solution involves exponential functions, which are typical for such equations.

The general form of a second-order linear homogeneous differential equation with constant coefficients is:

$$ a y'' + b y' + c y = 0, $$

where a, b, c are constants, and y is a function of the independent variable, often denoted as t.

In our case, the equation simplifies to:

$$ 18 y'' + y + 81 y = 0, $$

which, after rearrangement, is:

$$ 18 y'' + (1 + 81) y = 0, $$

though it's more precise to keep it as is for characteristic equation derivation.

Methodology for Solving the Differential Equation

1. Recognizing the Type of Equation

The key to solving this differential equation is to identify its structure and apply the appropriate method—namely, the auxiliary or characteristic equation method. Since the equation is linear, homogeneous, and with constant coefficients, the standard approach involves assuming solutions of a specific exponential form.

2. Establishing the Characteristic Equation

Assuming a solution of the form:

$$ y(t) = e^{r t}, $$

where r is a constant to be determined, we substitute into the differential equation to find the characteristic roots.

Deriving and Solving the Characteristic Equation

Step 1: Substitute $y(t) = e^{r t}$ into the differential equation

Differentiating the assumed solution:

- First derivative:

$$ y' = r e^{r t} $$

- Second derivative:

$$ y'' = r^2 e^{r t} $$

Plugging these into the original differential equation:

$$ y + 18 y'' + 81 y = 0 $$

becomes:

$$ e^{r t} + 18 r^2 e^{r t} + 81 e^{r t} = 0. $$

Step 2: Factor out the exponential term

Since $e^{r t} \neq 0$, divide through:

$$ 1 + 18 r^2 + 81 = 0. $$

However, notice that in the original differential equation, the structure is:

$$ y + 18 y'' + 81 y = 0. $$

Substituting the derivatives:

$$ e^{r t} (1 + 18 r^2 + 81) = 0. $$

Therefore, the characteristic equation is:

$$ 18 r^2 + 1 + 81 = 0. $$

But this appears inconsistent because the original differential equation is:

$$ y + 18 y'' + 81 y = 0. $$

Rearranged:

$$ 18 y'' + y + 81 y = 0, $$

which simplifies to:

$$ 18 y'' + 82 y = 0. $$

So, the precise characteristic equation is obtained by replacing y, y', y'' with their exponential derivatives:

$$ 18 r^2 e^{r t} + e^{r t} + 81 e^{r t} = 0. $$

Dividing through by $e^{r t}$:

$$ 18 r^2 + 1 + 81 = 0. $$

which simplifies to:

$$ 18 r^2 + 82 = 0. $$

Thus, the characteristic equation is:

$$ 18 r^2 + 82 = 0, $$

or

$$ 18 r^2 = -82, $$

leading to:

$$ r^2 = -\frac{82}{18} = -\frac{41}{9}. $$

Step 3: Solve for r

Since the roots involve a negative number, the solutions are complex conjugates:

$$ r = \pm i \sqrt{\frac{41}{9}} = \pm i \frac{\sqrt{41}}{3}. $$

Constructing the General Solution

1. Roots of the Characteristic Equation

The roots found are:

$$ r = \pm i \frac{\sqrt{41}}{3}. $$

Because these roots are purely imaginary, the general solution involves sinusoidal functions, expressed via Euler's formula:

$$ y(t) = C_1 \cos \left(\frac{\sqrt{41}}{3} t \right) + C_2 \sin \left(\frac{\sqrt{41}}{3} t \right), $$

where C_1 and C_2 are arbitrary constants determined by initial conditions.

2. Final Expression of the General Solution

The complete general solution to the differential equation:

$$ y + 18 y'' + 81 y = 0 $$

is:

$$ \boxed{
y(t) = C_1 \cos \left(\frac{\sqrt{41}}{3} t \right) + C_2 \sin \left(\frac{\sqrt{41}}{3} t \right),
} $$

where $C_1, C_2 \in \mathbb{R}$.

Interpretation and Significance of the Solution

Nature of the Solution

The solution manifests as a combination of sinusoidal functions, indicating oscillatory behavior. This is characteristic of systems with purely imaginary roots in their characteristic equations, often representing undamped harmonic oscillators or systems with neutral stability.

The angular frequency:

$$ \omega = \frac{\sqrt{41}}{3} $$

dictates how rapidly the oscillations occur. The amplitude and phase are determined by initial conditions, which specify the system's initial displacement and velocity.

Physical and Engineering Contexts

Such solutions are common in mechanical systems like pendulums, mass-spring systems, electrical circuits with inductance and capacitance, and other harmonic systems. The absence of damping terms (like damping coefficients) indicates sustained oscillations without decay.

Extensions and Related Concepts

1. Particular Solutions and Non-Homogeneous Equations

If the differential equation had non-zero forcing functions (non-homogeneous terms), the solution would involve particular solutions in addition to the complementary (homogeneous) solution described here.

2. Initial Conditions and Specific Solutions

To determine the constants C_1 and C_2, initial conditions such as initial displacement $y(0)$ and initial velocity $y'(0)$ are necessary. These allow for the complete description of the specific solution fitting a particular physical scenario.

3. Stability and Oscillatory Behavior

The purely imaginary roots suggest neutral stability; the system oscillates indefinitely without damping or growth. If damping or other effects are introduced, roots would acquire real parts, leading to decay or amplification of oscillations.

Conclusion

Solving the differential equation $y + 18 y'' + 81 y = 0$ exemplifies the classical approach to linear differential equations with constant coefficients. By assuming exponential solutions, deriving the characteristic equation, and analyzing its roots, we find that the general solution comprises sinusoidal functions reflecting oscillatory dynamics. This methodology underscores the power of mathematical techniques in deciphering the behavior of complex systems modeled by differential equations.

The insights gained extend beyond the specific equation, illustrating foundational principles applicable across physics, engineering, and applied mathematics. Whether modeling mechanical vibrations, electrical oscillations, or other periodic phenomena, understanding the structure of solutions informs both theoretical analysis and practical design.

References and Further Reading:

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In summary, the comprehensive solution to the differential equation reveals a harmonic oscillatory pattern, characteristic of systems governed by

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