

Find The Length Of The Third Side. If Necessary, Write In Simplest Radical Form

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Understanding how to determine the length of the third side of a triangle is a fundamental skill in geometry. Whether you're solving for unknowns in a triangle, verifying the validity of a triangle, or preparing for exams, mastering these concepts can greatly enhance your mathematical proficiency. Sometimes, the problem requires expressing the answer in simplest radical form to maintain accuracy and clarity. This comprehensive guide will walk you through various methods to find the third side of a triangle, provide step-by-step instructions, and explain when and how to write your answer in simplest radical form.

Basics of Triangle Side Lengths

Before diving into problem-solving techniques, let's review some essential triangle properties and theorems relevant to finding the third side.

Types of Triangles Based on Sides

- **Equilateral Triangle:** All three sides are equal.
- **Isosceles Triangle:** Two sides are equal.
- **Scalene Triangle:** All sides are different.

Triangle Inequality Theorem

This fundamental rule states that the sum of the lengths of any two sides must be greater than the length of the remaining side. Formally, for sides a , b , and c :

- $a + b > c$
- $a + c > b$
- $b + c > a$

This theorem is crucial for verifying whether a set of three side lengths can form a triangle.

Law of Cosines

A key formula for finding an unknown side when two sides and the included angle are known, or when all sides are known:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

where:

- a and b are known sides,
- C is the included angle,
- c is the side opposite angle C.

Methods to Find the Third Side

Depending on the information provided, different methods can be used to determine the third side's length.

1. Using the Law of Cosines

This method is particularly useful when you know:

- Two sides and the included angle, or
- All three sides (to verify or find a missing side).

Steps:

1. Identify the known values: sides a, b, and the included angle C.
2. Apply the Law of Cosines formula: $c^2 = a^2 + b^2 - 2ab \cos C$.
3. Calculate the value of c^2 .
4. Take the square root of c^2 to find c. If necessary, write in simplest radical form.

Example:

Suppose two sides of a triangle are 7 units and 10 units, and the included angle is 60° . Find the third side.

Solution:

$$\begin{aligned} c^2 &= 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ \\ c^2 &= 49 + 100 - 140 \times 0.5 \\ c^2 &= 149 - 70 = 79 \\ c &= \sqrt{79} \end{aligned}$$

Since 79 is prime, the simplest radical form is $\boxed{\sqrt{79}}$.

2. Using the Pythagorean Theorem (Special Cases)

Applicable when the triangle is right-angled.

Steps:

1. Identify the hypotenuse or legs.
2. Use the Pythagorean theorem: $c^2 = a^2 + b^2$ (for right triangles).
3. Calculate the missing side, simplifying radicals if needed.

Example:

Find the hypotenuse of a right triangle with legs of 3 and 4 units.

Solution:

```
\[
c^2 = 3^2 + 4^2 = 9 + 16 = 25
\]
\[
c = \sqrt{25} = 5
\]
```

Note: If the radical cannot be simplified further, leave it as is.

3. Using the Law of Sines

Useful when:

- Two angles and a side are known, or
- Two sides and a non-included angle are known.

Steps:

1. Set up the Law of Sines ratio: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
2. Use known values to find the unknown side or angle.
3. Solve algebraically and simplify radicals if necessary.

Example:

Given a triangle with angles $(A=30^\circ)$, $(B=45^\circ)$, and side $(a=10)$, find side c .

Solution:

First, find angle C :

```
\[
```

$$C = 180^\circ - 30^\circ - 45^\circ = 105^\circ$$

\]

Apply Law of Sines:

\[

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

\]

\[

$$\frac{10}{\sin 30^\circ} = \frac{c}{\sin 105^\circ}$$

\]

\[

$$\frac{10}{0.5} = \frac{c}{\sin 105^\circ}$$

\]

\[

$$20 = \frac{c}{\sin 105^\circ}$$

\]

\[

$$c = 20 \times \sin 105^\circ$$

\]

\[

$$c \approx 20 \times 0.9659 \approx 19.318$$

\]

Expressed exactly:

\[

$$c = 20 \times \sin 105^\circ$$

\]

$$\text{Since } (\sin 105^\circ = \sin (180^\circ - 75^\circ) = \sin 75^\circ =$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}),$$

\[

$$c = 20 \times \frac{\sqrt{6} + \sqrt{2}}{4} = 5 (\sqrt{6} + \sqrt{2})$$

\]

Thus, in simplest radical form:

\[

$$\boxed{c = 5 (\sqrt{6} + \sqrt{2})}$$

\]

Expressing the Third Side in Simplest Radical Form

Sometimes, after calculations, the side length involves radicals that can be simplified further. Here's how to write radicals in simplest form:

Steps to Simplify Radicals

1. Factor the radical's radicand into prime factors.
2. Identify perfect squares within the factors.
3. Rewrite the radical as a product of a simplified radical and an integer.

Example:

Simplify $\sqrt{72}$.

Solution:

Factor 72:

```
\[
72 = 36 \times 2
\]
```

Since 36 is a perfect square:

```
\[
\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6 \sqrt{2}
\]
```

Result:

```
\[
\boxed{6 \sqrt{2}}
\]
```

Practical Tips for Solving and Simplifying

- Always check if the radical can be simplified further by prime factorization.
- When using the Law of Cosines or Sines, double-check your calculations for accuracy.
- Use a calculator for approximate values but express exact answers in radical form whenever possible.
- Remember to verify if the given sides satisfy the triangle inequality theorem to confirm the possibility of forming a triangle.
- Be mindful of units and angles, especially when working with degrees versus radians.

Sample Problems and Solutions

Problem 1:

In triangle ABC, side $a = 8$ units, side $b = 15$ units, and the included angle $C = 60^\circ$. Find side c in simplest radical form.

Solution:

Apply Law of Cosines:

```
\[
c^2 = 8^2 + 15^2 - 2 \times 8 \times 15 \times \cos 60^\circ
\]
```

```
\[
c^2 = 64 + 225 - 240 \times 0.5
\]
```

```
\[
c^2 = 289 - 120 = 169
\]
```

```
\[
c = \sqrt{169} = 13
\]
Answer: \(\boxed{13}\)
```

Problem 2:

A right triangle has legs of 9 and 12 units. Find the length of the hypotenuse in simplest radical form.

Solution:

```
\[
c^2 = 9^2 + 12^2 = 81 + 144 = 225
```

Frequently Asked Questions

How do I find the length of the third side in a triangle when two sides and the included angle are known?

Use the Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos(C)$. Solve for c to find the third side's length.

What is the process to write the answer in simplest radical form?

Simplify any square roots by factoring out perfect squares, and reduce fractions to their lowest terms to express the length in simplest radical form.

Can the Law of Cosines be used when all three sides are known to find the third side?

No, if all three sides are known, you already have the lengths. The Law of Cosines is typically used to find a side when two sides and the included angle are known or to find angles with given sides.

What are common mistakes to avoid when calculating the third side in radical form?

Avoid incorrect simplification of radicals, mixing units, or using the wrong formula. Always check if the radical can be simplified further and ensure calculations follow proper order of operations.

How do I determine if I need to use the Law of Cosines or the Pythagorean Theorem?

Use the Pythagorean Theorem only for right triangles. For oblique triangles, apply the Law of Cosines. If you know two sides and the included angle, Law

of Cosines is appropriate.

What is an example of finding the third side in simplest radical form?

Given sides $a=5$ and $b=7$ with included angle $C=60^\circ$, $c^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cos(60^\circ) = 25 + 49 - 70(0.5) = 74 - 35 = 39$. So, $c = \sqrt{39}$, which simplifies to $\sqrt{39}$ (already in simplest radical form).

How can I check if my radical expression for the third side is in simplest form?

Ensure the radical has no perfect square factors other than 1, and the radical is fully simplified. For example, $\sqrt{50}$ can be simplified to $5\sqrt{2}$, which is in simplest radical form.

Is it necessary to convert decimal answers to radical form?

Not necessarily. Use radical form when exact values are required or when the radical can be simplified. Decimals are acceptable for approximate measures, but radicals provide exact lengths.

Additional Resources

Find The Length Of The Third Side. If Necessary, Write In Simplest Radical Form

Understanding the properties and calculations related to triangles is fundamental in geometry, whether you're a student, educator, or an enthusiast. One common challenge involves determining the length of the third side of a triangle when the other two sides and certain angles are known. This process often requires applying key theorems such as the Law of Cosines or the Law of Sines, and sometimes involves simplifying radical expressions to their simplest form. In this article, we explore the techniques used to find the third side of a triangle and how to express the answer in simplest radical form when necessary.

The Foundations: Understanding Triangle Types and Key Theorems

Before diving into calculations, it is essential to recognize the type of triangle involved and the information provided. Triangles can be classified based on side lengths (equilateral, isosceles, scalene) or angles (acute, right, obtuse). Depending on the known elements, different methods are employed:

- Side-Side-Side (SSS): All three sides are known; the third side can be found using the Law of Cosines if an angle is known or the Law of Cosines directly.
- Side-Angle-Side (SAS): Two sides and the included angle are known; the Law of Cosines is used.
- Angle-Side-Angle (ASA): Two angles and a side are known; the Law of Sines is applicable.

- Angle-Angle-Side (AAS): Two angles and a non-included side are known; again, the Law of Sines is useful.
- Right Triangle: Pythagoras' theorem simplifies calculations when one angle is 90° .

Understanding which theorem applies based on the given data is the first step in accurately determining the unknown side.

Applying the Law of Cosines to Find the Third Side

The Law of Cosines is a powerful tool for calculating an unknown side when two sides and the included angle are known, or when all three sides are unknown but some elements are given in a way that allows its application.

The Law of Cosines formula:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

where:

- a and b are known sides,
- C is the angle opposite side c ,
- c is the side to be calculated.

Steps to find the third side:

1. Identify known values: Determine which sides and angles are given in your problem.
2. Select the correct formula: Use the Law of Cosines if you know two sides and the included angle, or all three sides.
3. Plug in the known values: Substitute the known lengths and cosine of the known angle.
4. Simplify the expression: Carry out algebraic operations carefully.
5. Solve for the unknown side: Take the square root to find the length, being cautious of potential radical simplifications.

Example:

Suppose in a triangle, sides $a = 7$ units, $b = 10$ units, and included angle $C = 60^\circ$. Find side c .

Applying the Law of Cosines:

$$c^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ$$

$$c^2 = 49 + 100 - 2 \times 7 \times 10 \times \frac{1}{2}$$

$$c^2 = 149 - 70$$

$$c^2 = 79$$

Therefore,

$$\begin{aligned} & \backslash[\\ c &= \sqrt{79} \\ & \backslash] \end{aligned}$$

Since 79 is a prime number and cannot be simplified further, the length of the third side in simplest radical form is $\sqrt{79}$.

Simplifying Radicals to Their Simplest Form

When calculating the length of the third side, the square root might sometimes be reducible. Simplifying radicals involves factoring out perfect squares from under the radical sign.

Procedure for radical simplification:

1. Factor the radicand: Express the number inside the radical as a product of its prime factors.
2. Identify perfect squares: Look for perfect square factors.
3. Extract perfect squares: For each perfect square factor, take its square root and multiply it outside the radical.
4. Rewrite the radical: Express the simplified radical with the remaining radical part.

Example:

Suppose after calculations, you get:

$$\begin{aligned} & \backslash[\\ c &= \sqrt{180} \\ & \backslash] \end{aligned}$$

Factoring 180:

$$\begin{aligned} & \backslash[\\ 180 &= 36 \times 5 \\ & \backslash] \end{aligned}$$

Since 36 is a perfect square:

$$\begin{aligned} & \backslash[\\ c &= \sqrt{36 \times 5} = \sqrt{36} \times \sqrt{5} = 6 \sqrt{5} \\ & \backslash] \end{aligned}$$

Thus, the simplest radical form of c is $6\sqrt{5}$.

Practical Application: When and Why to Simplify

Simplifying radicals is more than a matter of aesthetic clarity; it provides a more precise and manageable form for further calculations or interpretations. For instance:

- Precision: Radical forms often retain more exact values than decimal approximations.

- Ease of comparison: Simplified radicals make it easier to compare lengths or angles.
- Further calculations: Simplified radicals facilitate algebraic manipulations in complex problems.

In real-world scenarios like engineering or architecture, expressing lengths in simplest radical form can lead to more accurate measurements and clear communication of results.

Common Challenges and How to Overcome Them

While applying these methods, several challenges can arise:

- Complex radicals: Radicals involving large numbers or multiple factors can be difficult to simplify.
- Misapplication of formulas: Using the wrong law or formula based on the given data can lead to errors.
- Rounding errors: Approximations may cause inaccuracies, especially when dealing with radical expressions.

Tips for overcoming these challenges:

- Carefully analyze the problem: Confirm which sides and angles are known.
- Practice radical factoring: Familiarize yourself with prime factorization techniques.
- Use exact values: When possible, leave answers in radical form rather than decimals.
- Double-check your work: Verify calculations and ensure the correct application of formulas.

Real-World Examples and Applications

Understanding how to find the third side of a triangle extends beyond classroom exercises:

- Navigation: Calculating the shortest path between two points when certain distances and angles are known.
- Construction: Determining lengths of materials when constructing triangles with known angles.
- Physics: Resolving vectors into components when forces and directions are specified.
- Computer Graphics: Computing distances and angles for rendering and modeling.

In each case, expressing results in simplest radical form ensures precision and clarity, especially when precise measurements are crucial.

Conclusion

Finding the length of the third side of a triangle when given certain elements is a fundamental skill in geometry, blending algebraic manipulation with geometric reasoning. Whether using the Law of Cosines, Law of Sines, or Pythagoras' theorem, the key lies in correctly identifying the given data,

applying the appropriate formula, and simplifying the radical expressions to their simplest form. This not only yields exact solutions but also enhances understanding and accuracy in various mathematical and real-world contexts. Mastery of these techniques empowers learners and professionals to handle complex geometric problems with confidence and precision.

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