Find The Point On The Line Y = 5x 2 That Is Closest To The Origin. (x, Y) =

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Introduction

In the realm of coordinate geometry, one common problem involves finding the point on a given curve or line that is closest to a specific point, often the origin (0,0). This problem not only deepens understanding of geometric principles but also enhances problem-solving skills through algebraic and calculus-based methods.

Today, we will explore how to find the point on the line $Y = 5x^2$ that is closest to the origin. This involves determining the coordinates (x, y) on the parabola $Y = 5x^2$ that minimizes the distance to (0, 0). The solution combines concepts from algebra, calculus, and geometry, providing a comprehensive approach to the problem.

Understanding the Problem

The goal is to find a point (x, y) on the curve $Y = 5x^2$ such that the distance from (x, y) to the origin (0, 0) is minimized.

Key points:

- The curve is defined by $Y = 5x^2$.

- The point (x, y) must satisfy this equation.
- The distance D from (x, y) to the origin is given by the distance formula:

$$
D = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}
$$

Since the square root function is monotonically increasing, minimizing D is equivalent to minimizing D^2:

$$
D^2 = x^2 + y^2
$$

Step-by-Step Approach to Find the Closest Point

The problem reduces to minimizing $D^2 = x^2 + y^2$, subject to the constraint $y = 5x^2$.

Step 1: Express the Distance Squared in Terms of x

Substitute $y = 5x^2$ into the distance squared formula:

$$
D^2 = x^2 + (5x^2)^2 = x^2 + 25x^4
$$

Now, the problem simplifies to minimizing:

$$
f(x) = x^2 + 25x^4

\]

Using Calculus to Find the Minimum

To find the minimum of f(x), we apply calculus techniques—taking derivatives and analyzing critical points.

Step 2: Find the Derivative of f(x)

Differentiate f(x) with respect to x:

\[
f'(x) = 2x + 100x^3
\]

Factor f'(x):

\[
f'(x) = 2x (1 + 50x^2)
\]

Step 3: Find Critical Points

Set f'(x) = 0:

\[
2x (1 + 50x^2) = 0
\]

This yields:

1. $2x = 0 \Rightarrow x = 0$
2. $1 + 50x^2 = 0 \Rightarrow x^2 = -\frac{1}{50}$, which is invalid since $x^2 \geq 0$.

Thus, the only critical point is at $x = 0$.

Step 4: Analyze the Critical Point

To determine whether $x = 0$ corresponds to a minimum, evaluate the second derivative $f''(x)$:

$$
f''(x) = 2 + 300x^2
$$

At $x = 0$:

$$
f''(0) = 2 + 0 = 2 > 0
$$

Since the second derivative is positive at $x = 0$, the critical point $x = 0$ corresponds to a local minimum.

Step 5: Find the Corresponding y-Coordinate

Recall $y = 5x^2$. At $x = 0$:

$$
y = 5 \times 0^2 = 0
$$

Therefore, the point (0, 0) is on the parabola $Y = 5x^2$ and is the closest point to the origin.

Summary of the Solution

- The point $(x, y) = (0, 0)$ on the parabola $Y = 5x^2$ is the closest to the origin.
- The minimum distance is $D = 0$, which makes intuitive sense because the point (0, 0) is on the curve itself.

Additional Insights and Explanation

Geometric Interpretation

The parabola $Y = 5x^2$ opens upwards and is symmetric about the y-axis. The closest point to the origin is at the vertex of the parabola, which is at (0, 0). Since the vertex is on the curve and coincides with the origin, the shortest possible distance is zero.

Why is the point (0, 0) the closest?

- The parabola passes through the origin.
- No other point on the parabola can be closer because the origin itself is on the curve.
- The distance function $D^2 = x^2 + 25x\square$ is minimized at $x = 0$.

Alternative Methods to Verify the Solution

While calculus is the most straightforward approach, alternative methods include:

- Geometric reasoning: Recognizing the vertex of the parabola as the minimum point.
- Using Lagrange multipliers: For more complex constraints, this method is useful, but in this case, calculus suffices.

Practical Applications and Relevance

Understanding how to find points closest to a given point on a curve has numerous applications:

- Optimization problems: Minimize travel distance, resource allocation.
- Engineering: Design components that must be as close as possible to a reference point.
- Computer graphics: Find the closest point on a shape to a viewer or camera position.
- Robotics: Path planning to avoid obstacles or reach points efficiently.

Summary Table

Step	Action	Result
1	Write the distance squared function	$f(x) = x^2 + 25x^4$
2	Compute derivative	$f'(x) = 2x + 100x^3$
3	Find critical points	$x=0$ only
4	Second derivative test	$f''(0) = 2 > 0$ □ minimum at $x=0$
5	Find y-coordinate	$y=5(0)^2=0$
Conclusion	Closest point	(0, 0) with distance 0

Final Thoughts

Finding the closest point on a curve to a given point combines algebra, calculus, and geometric insights. In the specific problem of $Y = 5x^2$, the solution is straightforward once the problem is translated into minimizing the squared distance function. Recognizing the vertex of the parabola plays a key role in understanding why the minimum occurs at the origin.

By mastering these techniques, you can handle more complex problems involving shortest distances, optimization in geometry, and applications across various scientific and engineering disciplines.

Keywords for SEO Optimization

- Closest point on a curve
- Distance minimization in coordinate geometry
- Find the point on $Y=5x^2$ closest to the origin
- Calculus optimization problems
- Distance formula in geometry
- Parabola vertex and minimum distance
- Algebraic methods for geometric problems
- Coordinate geometry problem-solving
- Optimization techniques in math
- Calculus-based distance minimization

Conclusion

In conclusion, the point on the line $Y = 5x^2$ that is closest to the origin is (0, 0). This point not only minimizes the distance but also coincides with the vertex of the parabola, making the problem elegant and straightforward. Understanding how to approach such problems enhances your mathematical toolkit for tackling a wide range of geometric and optimization challenges.

Frequently Asked Questions

How do I find the point on the line $y = 5x^2$ that is closest to the origin?

You can minimize the distance from the origin to a point (x, y) on the line, which involves minimizing the distance function $d(x) = \sqrt{x^2 + (5x^2)^2}$. Alternatively, minimize the squared distance to simplify calculations.

What is the formula for the distance from the origin to a point (x, y) on the line $y = 5x^2$?

The distance is given by $d(x) = \sqrt{x^2 + (5x^2)^2} = \sqrt{x^2 + 25x^4}$.

How do I find the x-value that minimizes the distance to the origin on the line $y = 5x^2$?

Minimize the squared distance function $D(x) = x^2 + 25x^4$ by taking its derivative, setting it to zero, and solving for x.

What is the derivative of the squared distance function $D(x) = x^2 + 25x^4$?

The derivative is $D'(x) = 2x + 100x^3$.

How do I solve for x when finding the closest point to the origin on $y = 5x^2$?

Set $D'(x) = 0$: $2x + 100x^3 = 0$, which simplifies to $x(1 + 50x^2) = 0$. The solutions are $x = 0$ or $x^2 = -1/50$ (discarding the negative root since x^2 □ 0). So, the only real solution is $x = 0$.

What is the closest point on the line $y = 5x^2$ to the origin?

The closest point is at (0, 0), since $x = 0$ minimizes the distance, and $y = 5(0)^2 = 0$, making the point (0, 0).

Additional Resources

Find The Point On The Line $Y = 5x^2$ That Is Closest To The Origin. (x, Y) =

In the realm of coordinate geometry, one of the fundamental problems involves finding the point on a given curve that is nearest to a specific location, often the origin. This problem is not only central to theoretical mathematics but also bears practical implications in fields such as physics, engineering, and data analysis, where minimizing distances is crucial. Today, we explore how to determine the point on the curve defined by the equation $Y = 5x^2$ that is closest to the origin (0,0). We'll delve into the mathematical techniques involved, frame the problem precisely, and walk through the solution step by step, providing clarity for readers with varying levels of mathematical background.

Understanding the Problem: The Geometric Context

Before jumping into calculations, it's essential to understand what the problem entails geometrically. The curve given by:

$Y = 5x^2$

is a parabola opening upward, symmetric about the y-axis. Our goal is to find the point on this parabola—say, (x, y)—that minimizes the distance to the origin.

Key question: Which point on the parabola yields the shortest straight-line distance to the point (0,0)?

The shortest distance between a point and a curve is a classic optimization problem. It involves:

- Expressing the distance from a general point on the curve to the origin.
- Finding the value(s) of x that minimize this distance.
- Confirming that the found point indeed gives the minimum distance.

Formulating the Mathematical Problem

Let us formalize the problem:

- Any point on the parabola can be represented as (x, y), with $y = 5x^2$.
- The distance D from this point to the origin is given by the distance formula:

$$D = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

Since $y = 5x^2$, substitute to express D solely in terms of x:

$$D(x) = \sqrt{x^2 + (5x^2)^2} = \sqrt{x^2 + 25x^4}$$

To simplify calculations, note that minimizing D is equivalent to minimizing D^2, since the square root function is monotonically increasing. Minimizing D^2 avoids dealing with the square root and makes differentiation straightforward.

Define:

$$D^2(x) = x^2 + 25x^4$$

Our goal is to find the value(s) of x that minimize $D^2(x)$.

Optimization Technique: Calculus Approach

To find the minimum, we use calculus. Specifically, we:

1. Compute the derivative of $D^2(x)$ with respect to x.
2. Find critical points where this derivative is zero.
3. Confirm that these points correspond to minima.

Step 1: Derivative of $D^2(x)$

$$\frac{d}{dx} D^2(x) = \frac{d}{dx} (x^2 + 25x^4)$$

Using basic differentiation rules:

$$\frac{d}{dx} D^2(x) = 2x + 100x^3$$

Step 2: Find critical points

Set the derivative equal to zero:

$$2x + 100x^3 = 0$$

Factor out common terms:

$$2x (1 + 50x^2) = 0$$

This product equals zero when either:

- $2x = 0 \Rightarrow x = 0$
- $1 + 50x^2 = 0 \Rightarrow x^2 = -\frac{1}{50}$

Since x^2 cannot be negative for real x, the second solution is extraneous (complex solutions). Therefore, the only critical point in the real domain is:

$$x = 0$$

Step 3: Confirm the nature of the critical point

To verify whether x=0 gives a minimum, examine the second derivative:

$$\frac{d^2}{dx^2} D^2(x) = \frac{d}{dx} (2x + 100x^3) = 2 + 300x^2$$

At $x=0$:

$$\frac{d^2}{dx^2} D^2(0) = 2 + 0 = 2 > 0$$

Since the second derivative is positive, the critical point at $x=0$ corresponds to a minimum.

Calculating the Closest Point

Now that we know the minimizing x-value is zero, find the corresponding y-value:

$$y = 5x^2 = 5 \times 0^2 = 0$$

Thus, the point on the parabola closest to the origin is:

$$(x, y) = (0, 0)$$

In other words: The origin itself lies on the parabola $y = 5x^2$ at $x=0$. Therefore, the closest point to the origin on this parabola is the origin itself.

Interpreting the Result: The Geometric and Practical Perspective

This solution indicates that the parabola $y=5x^2$ passes through the origin. Consequently, the shortest distance from the origin to the parabola is zero; the origin is a point on the curve. The point closest to itself is trivially the origin, which confirms the intuitive understanding that a point is always at zero distance from itself.

However, in more complicated scenarios—such as when the curve does not pass through the origin—the same calculus approach can be employed to find the closest point. The key steps involve expressing the distance squared, differentiating, and solving for critical points.

Broader Applications and Considerations

While this particular problem yields a straightforward result, the methodology extends to more complex situations. Here are some key takeaways and applications:

- Optimization in Path Planning: Engineers designing minimal-distance routes often need to find points on curves or surfaces closest to specific locations.
- Data Fitting and Regression: Minimizing distances between data points and model curves is fundamental in statistics.
- Collision Detection: In computer graphics and robotics, determining the nearest points on objects can prevent collisions.

- Physics and Engineering: Calculating minimal potential energy configurations often involves similar minimization problems.

Furthermore, when the curve does not pass through the point of interest, the calculus-based approach remains robust, involving derivatives and critical point analysis, often supplemented by second derivative tests or boundary considerations.

Final Thoughts: The Significance of Mathematical Rigor

This exploration underscores how calculus provides a powerful toolkit for solving geometric optimization problems. Starting from basic geometric principles, translating into algebraic expressions, and employing derivatives to find minima, this approach exemplifies mathematical rigor with practical relevance.

In the case of the parabola $y = 5x^2$, the simplicity of the curve ensures that the closest point is at the origin itself. Yet, the systematic process outlined here is universally applicable, equipping students, engineers, and scientists with a reliable method for tackling similar problems across various domains.

Summary

- The problem involves finding the point on the parabola $y=5x^2$ closest to the origin.
- Expressed the distance squared $D^2 = x^2 + (5x^2)^2$.
- Simplified to $D^2 = x^2 + 25x^4$.
- Used calculus to find critical points by differentiating and setting to zero.
- Confirmed the critical point at $x=0$ is a minimum via second derivative test.
- The closest point is at the origin $(0,0)$, which lies on the parabola.
- The minimal distance is zero, confirming the parabola passes through the origin.

This systematic approach demonstrates how mathematical tools can elegantly solve what might seem like complex geometric problems, providing insights that extend well beyond the specific example.

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