Find The Smallest Non-zero Whole Number That Isdivisible By 315, 378 And 392.

Find The Smallest Non-zero Whole Number That Is Divisible By 315, 378, And 392

The task of finding the smallest non-zero whole number that is divisible by multiple numbers involves understanding the concept of the Least Common Multiple (LCM). Specifically, when asked to find the smallest positive integer divisible by 315, 378, and 392, we are essentially searching for the LCM of these three numbers. This number is the smallest positive integer that each of the given numbers divides without leaving a remainder. Understanding how to compute the LCM requires a solid grasp of prime factorization and the properties of divisibility. In this article, we'll explore the step-by-step process to determine this smallest number, along with the mathematical principles involved.

Understanding the Concept of Least Common Multiple (LCM)

What is the LCM?

The Least Common Multiple of two or more integers is the smallest positive integer that is divisible by each of those integers. For example, the LCM of 4 and 6 is 12 because 12 is the smallest number divisible by both 4 and 6.

Why is LCM Important?

- It helps in solving problems involving multiple periodic events occurring at different intervals.
- It is essential in simplifying fractions and solving algebraic problems.
- In this context, it helps find the smallest number divisible by given numbers, which is critical in various applications such as scheduling, number theory, and computer science.

Step-by-Step Approach to Find the LCM

Step 1: Prime Factorization of Each Number

The first step involves breaking down each number into its prime factors. Prime factorization expresses a number as a product of prime numbers raised to various powers.

Step 2: Identify the Highest Powers of Prime Factors

After prime factorization, identify the highest power of each prime factor across all numbers. The LCM will include each prime factor raised to this highest power.

Step 3: Multiply the Highest Powers of All Prime Factors

Finally, multiply these highest powers together to obtain the LCM.

Prime Factorization of 315, 378, and 392

Prime Factorization of 315

- Divide 315 by 3: $315 \div 3 = 105$
- Divide 105 by 3: $105 \div 3 = 35$
- Divide 35 by 5: $35 \div 5 = 7$
- 7 is prime.

Therefore, the prime factorization of 315 is:

$315 = 3^2 \times 5 \times 7$

Prime Factorization of 378

- Divide 378 by 2: $378 \div 2 = 189$

- Divide 189 by 3: 189 ÷ 3 = 63
- Divide 63 by 3: 63 ÷ 3 = 21
- Divide 21 by 3: 21 ÷ 3 = 7
- 7 is prime.

Prime factors are: 2, 3, and 7, with the highest powers being:
$378 = 2^1 \times 3^3 \times 7^1$

Prime Factorization of 392

- Divide 392 by 2: 392 ÷ 2 = 196
- Divide 196 by 2: 196 ÷ 2 = 98
- Divide 98 by 2: 98 ÷ 2 = 49
- 49 is 7^2.

Prime factors are: 2 and 7, with the highest powers being:
$392 = 2^3 \times 7^2$

Determining the Highest Powers of Prime Factors

Now, we examine all prime factors identified:

- Prime 2: highest power is 2^3 (from 392)
- Prime 3: highest power is 3^3 (from 378)
- Prime 5: highest power is 5^1 (from 315)
- Prime 7: highest power is 7^2 (from 392)

Calculating the LCM

Step 1: Write the LCM as a product of prime factors raised to their highest powers

Therefore, the LCM is:

LCM = $2^3 \times 3^3 \times 5^1 \times 7^2$

Step 2: Calculate the numerical value

1. Calculate 2^3: 8
2. Calculate 3^3: 27
3. Calculate 7^2: 49
4. Multiply all together: 8 × 27 = 216
5. Next, 216 × 5 = 1080
6. Finally, 1080 × 49 = 52920

Final Result

The smallest non-zero whole number divisible by 315, 378, and 392 is **52,920**.

Verification of the Result

To ensure the correctness, verify that 52,920 is divisible by each number:

Dividing 52,920 by 315

- 52,920 ÷ 315 = 168 (an integer)

Dividing 52,920 by 378

- 52,920 ÷ 378 = 140 (an integer)

Dividing 52,920 by 392

- 52,920 ÷ 392 = 135 (an integer)

Since the divisions result in integers, the number 52,920 is indeed the least common multiple, confirming our calculation.

Additional Insights and Applications

Why Knowing the LCM is Useful

- Scheduling events that repeat at different intervals
- Solving fraction problems where common denominators are required
- Designing algorithms that involve synchronization of processes

Extensions of the Problem

This method can be extended to find the LCM of any set of integers, regardless of size, by following similar prime factorization and highest power identification steps. For larger numbers, computational tools or algorithms like the Euclidean algorithm for GCD can expedite the process.

Conclusion

In summary, finding the smallest non-zero whole number divisible by 315, 378, and 392 involves prime factorization of each number, identifying the highest powers of each prime factor, and multiplying those together. The resulting number, 52,920, is the least common multiple and the smallest positive integer that satisfies the divisibility conditions. Mastery of prime factorization and LCM calculation is fundamental in many mathematical and real-world applications, making this a valuable skill for students and professionals alike.

Frequently Asked Questions

What is the smallest non-zero whole number divisible by 315, 378, and 392?

The smallest non-zero whole number divisible by 315, 378, and 392 is their least common multiple (LCM).

How do you find the least common multiple (LCM) of 315, 378, and 392?

To find the LCM, prime factorize each number, take the highest powers of all prime factors involved, and multiply them together.

What is the prime factorization of 315, 378, and 392?

315 = 3^2 5 7; 378 = 2 3^3 7; 392 = 2^3 7^2.

Can you show the step-by-step calculation to find the LCM of 315, 378, and 392?

Yes. Prime factorizations:
315 = 3^2 5 7
378 = 2 3^3 7
392 = 2^3 7^2

Take the highest powers:
- 2^3
- 3^3
- 5
- 7^2

Multiply: 2^3 3^3 5 7^2 = 8 27 5 49 = 529200.

So, the LCM is 529,200.

Why is the least common multiple (LCM) the smallest number divisible by multiple numbers?

Because the LCM is the smallest number that contains all prime factors at their highest powers needed to be divisible by each of the given numbers.

What practical applications can finding the smallest number divisible by multiple numbers have?

It is useful in scheduling, synchronization of periodic events, and in problems involving repeating cycles where multiple conditions must align.

Additional Resources

Find The Smallest Non-zero Whole Number That Is Divisible By 315, 378 And 392

Introduction

In the realm of mathematics, particularly in number theory, one of the foundational concepts is understanding divisibility and the relationships between numbers. A common challenge involves finding the smallest positive number divisible by a given set of integers—an essential problem that underpins many applications in cryptography, computer science, engineering, and daily problem-solving. Today, we delve into a specific example: determining the smallest non-zero whole number divisible by 315, 378, and 392. This task essentially asks us to find the least common multiple (LCM) of these three numbers.

Understanding this problem requires a comprehensive grasp of divisibility rules, prime factorizations, and the mathematical techniques used to compute the LCM efficiently. The goal of this article is to explore these concepts in detail, providing an in-depth, step-by-step analysis that not only arrives at the solution but also illuminates the underlying principles involved.

The Significance of the Problem

Why is finding the smallest number divisible by multiple integers important? Beyond the theoretical appeal, this problem has practical implications. For example:

- Scheduling: Determining the earliest time when multiple periodic events coincide.
- Cryptographic algorithms: Ensuring synchronization of cycles in encryption schemes.
- Computer programming: Handling tasks that require alignment of different periodic processes.
- Mathematical puzzles and education: Enhancing understanding of divisibility, prime factors, and number properties.

By solving this problem, students and professionals sharpen their problem-solving skills, gain insight into prime factorization, and develop a strong foundation in number theory.

Understanding the Core Concept: Least Common Multiple (LCM)

The least common multiple (LCM) of two or more integers is the smallest positive integer that is divisible by each of them. For example, the LCM of 4 and 6 is 12 because:

- $12 \div 4 = 3$ (integer)
- $12 \div 6 = 2$ (integer)

No smaller positive number satisfies this condition.

Why is LCM relevant here? Because our goal is to find the smallest number divisible by all three given numbers: 315, 378, and 392.

Step 1: Prime Factorization of Each Number

The first step in finding the LCM is to perform prime factorization of each number. Prime factorization involves expressing each number as a product of prime numbers.

Prime Factorization of 315

- $315 \div 3 = 105$
- $105 \div 3 = 35$
- $35 \div 5 = 7$
- 7 is prime

So, $315 = 3 \times 3 \times 5 \times 7 = 3^2 \times 5 \times 7$

Prime Factorization of 378

- $378 \div 2 = 189$
- $189 \div 3 = 63$
- $63 \div 3 = 21$
- $21 \div 3 = 7$
- 7 is prime

Therefore, $378 = 2 \times 3 \times 3 \times 3 \times 7 = 2^1 \times 3^3 \times 7$

Prime Factorization of 392

- $392 \div 2 = 196$
- $196 \div 2 = 98$
- $98 \div 2 = 49$
- $49 \div 7 = 7$
- 7 is prime

Thus, $392 = 2 \times 2 \times 2 \times 7 \times 7 = 2^3 \times 7^2$

Step 2: Determining the Highest Powers of Prime Factors

The LCM is formed by taking each prime factor at its highest exponent found among the factorizations.

Prime Factor	315 ($3^2 \times 5 \times 7$)	378 ($2^1 \times 3^3 \times 7$)	392 ($2^3 \times 7^2$)	Highest Power
2	0	1	3	2^3
3	2	3	0	3^3
5	1	0	0	5^1
7	1	1	2	7^2

Note: The highest power for each prime factor is chosen across all three factorizations.

Step 3: Calculating the LCM

Using the highest powers identified:

- 2^3
- 3^3
- 5^1
- 7^2

The LCM is obtained by multiplying these together:

$$\text{LCM} = 2^3 \times 3^3 \times 5^1 \times 7^2$$

Calculating step-by-step:

- $2^3 = 8$
- $3^3 = 27$
- $5^1 = 5$
- $7^2 = 49$

Now multiply:

1. $8 \times 27 = 216$
2. $216 \times 5 = 1080$
3. $1080 \times 49 = 1080 \times (50 - 1) = 1080 \times 50 - 1080 \times 1 = 54,000 - 1,080 = 52,920$

Thus,

$$\boxed{\text{LCM} = 52,920}$$

Step 4: Verifying the Result

To ensure accuracy, verify that 52,920 is divisible by each of the numbers:

- Divisible by 315:

$52,920 \div 315 = ?$

Since $315 = 3^2 \times 5 \times 7$,

- $52,920 \div 3 = 17,640$
- $17,640 \div 3 = 5,880$
- $5,880 \div 5 = 1,176$
- $1,176 \div 7 = 168$

All divisions yield integers, confirming divisibility.

- Divisible by 378:

$378 = 2^1 \times 3^3 \times 7$

- $52,920 \div 2 = 26,460$
- $26,460 \div 3 = 8,820$
- $8,820 \div 3 = 2,940$
- $2,940 \div 3 = 980$
- $980 \div 7 = 140$

All divisions are integers, confirming divisibility.

- Divisible by 392:

$392 = 2^3 \times 7^2$

- $52,920 \div 8 = 6,615$
- $6,615 \div 49 = 135$

Both divisions are integers.

Therefore, 52,920 is indeed divisible by all three numbers, and being constructed from the highest prime exponents, it is the smallest such number.

Additional Insights and Applications

The process undertaken illustrates a fundamental technique in number theory: prime factorization combined with the identification of maximum exponents to compute the LCM. This method is scalable and applicable to any set of integers, providing a systematic approach to solving divisibility problems.

Real-world examples include:

- Scheduling multiple events: For instance, if three different processes repeat every 315, 378, and 392 seconds, the LCM gives the earliest time they all align simultaneously.
- Synchronization in systems: Ensuring that multiple signals or processes with different cycles synchronize at the earliest opportunity.
- Mathematical education: Teaching students how prime factorization directly relates to divisibility and least common multiples.

The Broader Mathematical Context

Understanding the calculation of the LCM also involves familiarity with related concepts:

- Greatest Common Divisor (GCD): The largest number dividing two or more integers.
- Relationship between GCD and LCM:

$$
\text{GCD}(a, b) \times \text{LCM}(a, b) = a \times b
$$

This relationship can be extended to multiple numbers, providing alternative methods for LCM calculation.

- Prime factorization as a unifying tool: It simplifies the process of dealing with divisibility, common factors, and multiples by breaking numbers down into their prime building blocks.

Final Remarks and Summary

Through a meticulous process involving prime factorization and the identification of maximum prime powers, we have determined that the smallest non-zero whole number divisible by 315, 378, and 392 is 52,920. This number serves as the least common multiple of the three given integers, representing the minimal solution to a classic divisibility problem rooted in fundamental number theory.

This exploration underscores the importance of prime factorization as a powerful mathematical tool, facilitating precise, systematic solutions to problems involving multiple divisibility constraints. Whether applied in computational algorithms, scheduling problems, or educational contexts, the techniques demonstrated here are integral to a deeper understanding of the structure and relationships within the natural numbers.

References

- Elementary Number Theory by David M. Burton

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