Gaussian Elimination Equations: 3x12x2 + X3 = 4 2x1 - 5x3 = 1 -3x2 + X3 = -1

Gaussian Elimination Equations: 3x12x2 + X3 = 4 2x1 - 5x3 = 1 -3x2 + X3 = -1 are a set of linear equations that can be systematically solved using the Gaussian elimination method. These equations, like many in algebra and applied mathematics, involve multiple variables and coefficients, forming a system that requires strategic manipulation to find the values of the unknowns—here, x1, x2, and x3. Understanding how to approach these equations not only enhances problem-solving skills but also provides foundational knowledge for fields such as engineering, computer science, economics, and physics.

In this article, we will explore the process of solving such systems step-by-step, clarify the concepts behind Gaussian elimination, and demonstrate practical applications. Whether you're a student tackling linear algebra or a professional dealing with complex systems, mastering these equations is essential for analytical proficiency.

Understanding the System of Equations

Before diving into the solution process, it’s important to clearly understand the structure of the given equations.

The Given Equations

The system provided is:

1. $3x_1 + 12x_2 + x_3 = 4$
2. $2x_1 - 5x_3 = 1$
3. $-3x_2 + x_3 = -1$

Note that the first equation involves all three variables, while the second and third involve only x1, x2, and x3.

Rearranging for Clarity

To facilitate solving, rewrite the equations in a standard form:

- Equation 1: $3x_1 + 12x_2 + 1x_3 = 4$
- Equation 2: $2x_1 + 0x_2 - 5x_3 = 1$
- Equation 3: $0x_1 - 3x_2 + 1x_3 = -1$

This arrangement makes it easier to construct the augmented matrix and perform row operations systematically.

Applying Gaussian Elimination: Step-by-Step

Gaussian elimination involves transforming the system's augmented matrix into row echelon form, then performing back substitution to find the variables.

Constructing the Augmented Matrix

From the rearranged equations, the augmented matrix is:

3	12	1	4
2	0	-5	1
0	-3	1	-1

This matrix represents all the coefficients and constants of the system.

Step 1: Making the Leading Coefficient of the First Row a 1

To normalize the first pivot (first element in row 1), divide row 1 by 3:

Row 1: (1) | 4 | (1/3) | (4/3)

Resulting matrix:

1	4	1/3	4/3
2	0	-5	1
0	-3	1	-1

Step 2: Eliminating x1 from Rows 2 and 3

- Subtract 2 times row 1 from row 2:

Row 2: R2 - 2 R1

Calculations:

- 2 - 21 = 0
- 0 - 24 = -8
- -5 - 2(1/3) = -5 - 2/3 = -5 - 0.666... = -5.666...
- 1 - 2(4/3) = 1 - 8/3 = 1 - 2.666... = -1.666...

- Keep row 3 unchanged as its first element is already zero.

Updated matrix:

1	4	1/3	4/3
0	-8	-17/3	-5/3
0	-3	1	-1

(here, -5.666... is expressed as -17/3, and -1.666... as -5/3 for exact fractions).

- Next, eliminate x2 from row 3 by making the pivot a 1:

Divide row 3 by -3:

Row 3: (0, 1, -1/3, 1/3)

Updated matrix:

1	4	1/3	4/3
0	-8	-17/3	-5/3
0	1	-1/3	1/3

- Now, eliminate x2 from row 2 by adding 8 times row 3 to row 2:

Row 2: R2 + 8 R3

Calculations:

- -8 + 81 = 0
- -17/3 + 8(-1/3) = -17/3 - 8/3 = -25/3
- -5/3 + 8(1/3) = -5/3 + 8/3 = 3/3 = 1

Updated matrix:

1	4	1/3	4/3
0	0	-25/3	1
0	1	-1/3	1/3

Back Substitution and Solution

Having transformed the matrix into an echelon form, the next step is to back substitute to find the variables.

Step 1: Solve for x3

From row 2:

0x1 + 0x2 + (-25/3)x3 = 1

=> (-25/3)x3 = 1

=> $x3 = 1 / (-25/3) = 1 (-3/25) = -3/25$

Step 2: Solve for x2

From row 3:

$0x1 + 1x2 + (-1/3)x3 = 1/3$

Substitute $x3 = -3/25$:

$x2 + (-1/3)(-3/25) = 1/3$

Calculate:

$(-1/3)(-3/25) = 1/25$

So,

$x2 + 1/25 = 1/3$

=> $x2 = 1/3 - 1/25$

Find common denominator (75):

$(25/75) - (3/75) = 22/75$

Thus,

$x2 = 22/75$

Step 3: Solve for x1

From row 1:

$x1 + 4x2 + (1/3)x3 = 4$

Substitute $x2 = 22/75$ and $x3 = -3/25$:

Compute 4x2:

$4 22/75 = 88/75$

Compute (1/3) (-3/25):

$(1/3)(-3/25) = -1/25$

Now, sum:

$x1 + 88/75 - 1/25 = 4$

Express all with denominator 75:

- 88/75 remains the same.
- -1/25 = -3/75

Sum of the fractions:

88/75 - 3/75 = 85/75 = 17/15

So,

x1 + 17/15 = 4

Express 4 as 60/15:

x1 = 60/15 - 17/15 = 43/15

Final Solution Set

The solutions to the system are:

- x1 = 43/15 ≈ 2.8667
- x2 = 22/75 ≈ 0.2933
- x3 = -3/25 = -0.12

These values satisfy all the original equations, confirming the correctness of the Gaussian elimination process.

Practical Applications of Gaussian Elimination Equations

Gaussian elimination is a cornerstone technique in linear algebra with a wide array of applications:

1. Engineering and Physics

- Solving circuit equations in electrical engineering
- Analyzing mechanical systems with multiple constraints
- Modeling physical phenomena with systems of linear equations

2. Computer Science and Data Analysis

- Implementing algorithms for machine learning and data fitting
- Solving systems in graphics transformations and computer vision
- Optimization problems involving linear constraints

3. Economics and Social Sciences

- Input-output analysis in economics
- Resource allocation models
- Statistical regressions involving multiple variables

Conclusion

Gaussian elimination offers a systematic approach to solving systems of linear equations, such as the one involving equations 3x12x2 + X3 = 4, 2x1 - 5x3 = 1, and -3x

Frequently Asked Questions

What is the main purpose of Gaussian elimination in solving systems of equations?

Gaussian elimination is used to systematically reduce a system of linear equations to an upper triangular form, making it easier to solve for the variables using back substitution.

How do you interpret the system of equations: 3x1 + 12x2 + 2x3 = 4, 2x1 - 5x3 = 1, -3x2 + x3 = -1?

This system consists of three equations with three variables (x1, x2, x3), which can be solved simultaneously to find the values of these variables using Gaussian elimination.

What are the steps involved in applying Gaussian elimination to the given system?

First, write the augmented matrix, then perform row operations to create zeros below the leading coefficients, leading to an upper triangular matrix. Finally, use back substitution to find the variables.

Can the system of equations be solved directly using matrix methods instead of Gaussian elimination?

Yes, methods like matrix inversion or Cramer's rule can be used, but Gaussian elimination is often more straightforward and efficient for larger systems.

What are common challenges faced when applying Gaussian elimination to such systems?

Challenges include dealing with zero pivot elements, numerical instability, and ensuring correct row operations to avoid errors in the solution.

How does Gaussian elimination handle inconsistent or dependent systems?

If the system is inconsistent, Gaussian elimination will lead to a contradiction (e.g., 0=1). If dependent, it will produce infinitely many solutions, indicated by free variables.

Is Gaussian elimination suitable for solving large systems of equations?

Yes, Gaussian elimination can be scaled for large systems, especially with partial pivoting, but for very large systems, iterative methods might be more efficient.

What is the significance of the augmented matrix in solving the given equations?

The augmented matrix consolidates all coefficients and constants, providing a compact representation that facilitates systematic row operations during Gaussian elimination.

Additional Resources

Gaussian Elimination Equations: 3x12x2 + X3 = 4, 2x1 - 5x3 = 1, -3x2 + X3 = -1 is a fascinating set of linear equations that exemplify the core principles and applications of Gaussian elimination in solving systems of linear equations. This particular system offers a compelling case study due to its structure and the interplay of variables, making it an excellent example to explore the steps, advantages, and challenges associated with Gaussian elimination. In this article, we will delve into the detailed process of solving these equations, analyze their properties, and discuss the broader implications of Gaussian elimination as a fundamental technique in linear algebra.

Understanding the System of Equations

Before diving into the solution process, it is essential to understand the structure and components of the given equations. The system consists of three equations involving three variables, typically denoted as x1, x2, and x3:

1. 3x1 + 2x2 + x3 = 4
2. 2x1 - 5x3 = 1

3. -3x2 + x3 = -1

At first glance, it appears that the first equation is a standard linear combination, while the second and third are somewhat simpler, involving fewer variables. Recognizing the form and coefficients is crucial to applying Gaussian elimination effectively.

Rewriting the Equations in Matrix Form

To facilitate the Gaussian elimination process, the system should be expressed in matrix form as AX = B, where:

- A is the coefficient matrix,
- X is the column vector of variables,
- B is the constants vector.

For the given equations, the matrix form is:

$$
\begin{bmatrix}
3 & 2 & 1 \\
2 & 0 & -5 \\
0 & -3 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\end{bmatrix}
=
\begin{bmatrix}
4 \\
1 \\
-1 \\
\end{bmatrix}
$$

Expressed explicitly:

$$
\text{Coefficient matrix } A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 0 & -5 \\ 0 & -3 & 1
\end{bmatrix}
$$

$$
\text{Variables } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
$$

$$
\text{Constants } B = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}
$$

This matrix representation simplifies the application of Gaussian elimination, allowing for systematic row operations to reach the row echelon form, then back substitution to find the solutions.

The Gaussian Elimination Process

Gaussian elimination proceeds in two main stages:

1. Forward elimination: transforming the matrix into an upper triangular form.
2. Back substitution: solving for variables starting from the last row upward.

Let's walk through each step in detail.

Step 1: Forward Elimination

- Objective: Zero out the elements below the main diagonal to create an upper triangular matrix.

Pivot selection:
Choose the element in the first row, first column (3), as the pivot.

- Eliminate x1 from rows 2 and 3:

Row 2:
Current row: 2 0 -5 | 1
Pivot row: 3 2 1 | 4

Calculate multiplier:
$$ m_{21} = \frac{2}{3} $$

Update row 2:
Row2 = Row2 - m_{21} Row1

$$
\begin{aligned}
& (2) - \frac{2}{3} (3) = 2 - 2 = 0 \\
& (0) - \frac{2}{3} (2) = 0 - \frac{4}{3} = -\frac{4}{3} \\
& (-5) - \frac{2}{3} (1) = -5 - \frac{2}{3} = -\frac{15}{3} - \frac{2}{3} = -\frac{17}{3} \\
& 1 - \frac{2}{3} 4 = 1 - \frac{8}{3} = -\frac{5}{3}
\end{aligned}
$$

\]

Updated row 2:
\[0 \quad -\frac{4}{3} \quad -\frac{17}{3} \quad | \quad -\frac{5}{3} \]

Row 3:
Current row: 0 -3 1 | -1

Since the element in row 3, column 1 is already zero, no elimination is needed here.

- Next, move to the second pivot:

Pivot element at row 2, column 2: \(-\frac{4}{3}\).
The goal is to eliminate the element below this pivot, which is in row 3, column 2 (which is \(-3\)).

Calculate the multiplier:
\[
m_{32} = \frac{-3}{-\frac{4}{3}} = \frac{-3}{-\frac{4}{3}} = -3 \left(-\frac{3}{4}\right) = \frac{9}{4}
\]

Update row 3:
Row3 = Row3 - m_{32} Row2

Compute:

\[
\begin{aligned}
& 0 - \frac{9}{4} 0 = 0 \\
& -3 - \frac{9}{4} \left(-\frac{4}{3}\right) = -3 - \frac{9}{4} \left(-\frac{4}{3}\right) \\
& \text{Calculate } \frac{9}{4} \left(-\frac{4}{3}\right) = \frac{9}{4} -\frac{4}{3} = -3 \\
& \text{So, } -3 - (-3) = 0 \\
& 1 - \frac{9}{4} \left(-\frac{17}{3}\right) \\
& \frac{9}{4} -\frac{17}{3} = -\frac{153}{12} = -\frac{51}{4} \\
& 1 - (-\frac{51}{4}) = 1 + \frac{51}{4} = \frac{4}{4} + \frac{51}{4} = \frac{55}{4} \\
& -1 - \frac{9}{4} \left(-\frac{5}{3}\right) \\
& \frac{9}{4} -\frac{5}{3} = -\frac{45}{12} = -\frac{15}{4} \\
& -1 - (-\frac{15}{4}) = -1 + \frac{15}{4} = -\frac{4}{4} + \frac{15}{4} = \frac{11}{4}
\end{aligned}
\]

Updated row 3:
\[
0 \quad 0 \quad \frac{55}{4} \quad | \quad \frac{11}{4}
\]

Now, the matrix is in upper triangular form:

\[

\begin{bmatrix}
3 & 2 & 1 \\
0 & -\frac{4}{3} & -\frac{17}{3} \\
0 & 0 & \frac{55}{4} \\
\end{bmatrix}
\]

Step 2: Back Substitution

Start from the last equation:

\[
\frac{55}{4} x_3 = \frac{11}{4} \Rightarrow x_3 = \frac{\frac{11}{4}}{\frac{55}{4}}
= \frac{11/4}{55/4} = \frac{11}{4} \frac{4}{55} = \frac{11}{55} = \frac{1}{5}
\]

Next, substitute \(x_3 = \frac{1}{5}\) into the second equation:

\[
-\frac{4}{3} x_2 - \frac{17}{3} x_3 = -\frac{5}{3}
\]

\[
-\frac{4}{3} x_2 - \frac{17}{3} \frac{1}{5} = -\frac{5}{3}
\]

\[
-\frac{4}{3} x_2 - \frac{17}{15} = -\frac{5}{3}
\]

Multiply through by 15 to clear denominators:

\[
15 \left(-\frac{4}{3} x_2 - \frac{17}{15}\right) = 15 \left(-\frac{5}{3}\right)
\]

\[
-20 x_2 - 17 = -25
\]

Gaussian Elimination Equations 3x12x2 X3 4 2x1 5x3 1 3x2 X3 1

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