Given F(x)=x^3-x^2-5x-3 And The Factor X-3, Find The Zeros Of The Function F(x).

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Introduction

Understanding the zeros of a polynomial function is fundamental in algebra, as zeros (or roots) are the points where the function intersects the x-axis. These zeros provide insights into the graph's behavior and are crucial in solving polynomial equations. Given the cubic function F(x) = x^3 - x^2 - 5x - 3 and the information that (x - 3) is a factor, our goal is to determine all zeros of F(x). This process involves applying polynomial division, synthetic division, and factorization techniques, culminating in solving linear and quadratic equations derived from the original polynomial.

Understanding the Given Function and Its Factor

Analyzing the Polynomial Function

The function provided is a cubic polynomial:

$$ F(x) = x^3 - x^2 - 5x - 3 $$

The degree of the polynomial is 3, indicating it has up to three real roots or zeros. The coefficients suggest a standard form, and the problem states that (x - 3) is a factor, which implies that x = 3 is a root of the polynomial.

Significance of the Given Factor

Knowing that (x - 3) is a factor simplifies the process of finding zeros. Specifically, it allows us to perform polynomial division to factor out (x - 3) from F(x). Once we find the quotient, which will be a quadratic polynomial, we can then solve it to find the remaining zeros.

Methodology to Find the Zeros

Step 1: Verify the Given Factor

Before proceeding, it's crucial to verify that (x - 3) is indeed a factor of F(x). We do this by evaluating F(3):

$$ F(3) = (3)^3 - (3)^2 - 5(3) - 3 = 27 - 9 - 15 - 3 = 0 $$

Since F(3) = 0, (x - 3) is confirmed as a factor, validating our approach.

Step 2: Polynomial Division to Find the Quotient

To find the other factors, divide F(x) by (x - 3). There are two main methods: synthetic division and long division. Synthetic division is typically faster and more straightforward when dividing by a linear factor.

Performing Synthetic Division

Setup for Synthetic Division

- Coefficients of F(x): 1 (for x^3), -1 (for x^2), -5 (for x), -3 (constant)
- Divisor root: 3

The synthetic division setup:

3	1	-1	-5	-3
		3	6	3
	1	2	1	0

Steps:

1. Bring down the leading coefficient (1).
2. Multiply 1 by 3, write under the next coefficient: 3.
3. Add: -1 + 3 = 2.
4. Multiply 2 by 3: 6.
5. Add: -5 + 6 = 1.
6. Multiply 1 by 3: 3.
7. Add: -3 + 3 = 0 (remainder).

Since the remainder is zero, the division confirms that (x - 3) is a factor, and the quotient polynomial is:

$$ Q(x) = x^2 + 2x + 1 $$

Factoring the Quotient Polynomial

Step 3: Factor the Quadratic

The quadratic obtained:

$$ Q(x) = x^2 + 2x + 1 $$

This quadratic can be factored using factoring methods, such as factoring by inspection or quadratic formula.

- Recognize that $x^2 + 2x + 1$ is a perfect square trinomial:

$$ x^2 + 2x + 1 = (x + 1)^2 $$

Thus, the quadratic factors as:

$$ (x + 1)^2 $$

Finding All Zeros of F(x)

Step 4: List All Zeros

From the factorization:

$$ F(x) = (x - 3)(x + 1)^2 $$

The zeros are the solutions to:

1. $x - 3 = 0 \Rightarrow x = 3$
2. $x + 1 = 0 \Rightarrow x = -1$

Note that $(x + 1)^2$ indicates that $x = -1$ is a repeated root with multiplicity 2.

Step 5: Summary of Zeros and Their Multiplicities

Zero	Multiplicity	Explanation
3	1	From (x - 3) factor
-1	2	From $(x + 1)^2$ factor

Conclusion: The Zeros of F(x)

The zeros of the polynomial function $F(x) = x^3 - x^2 - 5x - 3$, given that (x - 3) is a factor, are:

- x = 3, with multiplicity 1
- x = -1, with multiplicity 2

These zeros reveal the points where the graph of F(x) intersects the x-axis. The presence of a repeated root at $x = -1$ implies that the graph touches the x-axis at this point and possibly has a local minimum or maximum there.

Additional Insights and Applications

Graphing the Polynomial

Knowing the zeros helps in sketching the graph of the cubic polynomial:

- The graph crosses the x-axis at $x = 3$.
- It touches and "bounces" off the x-axis at $x = -1$ because of the repeated root.
- The end behavior of a cubic polynomial is determined by the leading term x^3:
- As $x \to \infty$, $F(x) \to \infty$.
- As $x \to -\infty$, $F(x) \to -\infty$.

Implications in Algebra and Calculus

- The zeros are critical points for calculus-based analysis, such as finding local maxima, minima, and inflection points.
- Factoring polynomials simplifies solving equations and analyzing functions.

Extensions and Further Studies

- Determining the nature (real or complex) of the remaining roots, especially if the quadratic had complex solutions.
- Applying synthetic division to higher-degree polynomials with known factors.
- Using the Rational Root Theorem for polynomials where roots are not known upfront.

Summary

By leveraging the known factor $(x - 3)$, synthetic division facilitated the factorization of the cubic polynomial $F(x) = x^3 - x^2 - 5x - 3$. The quadratic quotient was identified as $(x + 1)^2$, leading to the zeros $x = 3$ and $x = -1$ (with multiplicity 2). These roots encapsulate all solutions to $F(x) = 0$, providing a complete understanding of the polynomial's roots and their multiplicities. This systematic approach underscores the importance of factorization, synthetic division, and quadratic analysis in algebraic problem-solving.

Note: Mastery of these techniques is essential for advancing in algebra, solving polynomial equations efficiently, and understanding the behavior of polynomial functions across different contexts.

Frequently Asked Questions

What is the given function for which we need to find the zeros?

The given function is $F(x) = x^3 - x^2 - 5x - 3$.

How do we verify that (x - 3) is a factor of F(x)?

We can verify by substituting $x = 3$ into $F(x)$ and checking if the result equals zero. Alternatively, we can perform polynomial division to see if $(x - 3)$ divides $F(x)$ evenly.

What is the process to find the zeros of F(x) after confirming (x - 3) is a factor?

First, perform synthetic or polynomial division of $F(x)$ by $(x - 3)$ to find the quotient. Then, solve the quotient polynomial to find all zeros.

What is synthetic division and how is it used here?

Synthetic division is a simplified method of dividing a polynomial by a linear factor like $(x - 3)$. It helps to find the quotient and remainder efficiently, confirming whether $(x - 3)$ is a factor.

Can you perform synthetic division of F(x) by (x - 3)?

Yes. Using synthetic division with root 3: bring down the coefficients [1, -1, -5, -3], then perform synthetic division steps to find the quotient polynomial.

What is the quotient polynomial obtained after dividing F(x) by (x - 3)?

The quotient polynomial is $x^2 + 2x + 1$.

How do we find the remaining zeros of F(x) after division?

Solve the quadratic $x^2 + 2x + 1 = 0$, which factors as $(x + 1)^2 = 0$, giving a zero at $x = -1$ with multiplicity 2.

What are the zeros of F(x)?

The zeros are $x = 3$ and $x = -1$ (with multiplicity 2).

What is the complete set of zeros for the function F(x)?

The zeros of $F(x)$ are $x = 3$ and $x = -1$ (double root).

Additional Resources

Given $F(x) = x^3 - x^2 - 5x - 3$ and the factor $x - 3$, find the zeros of the function $F(x)$.
This problem provides a cubic polynomial function and a specific factor, prompting us to determine all the zeros (roots) of the function. Understanding how to approach such a problem involves exploring concepts like polynomial factoring, synthetic division, and the Rational Root Theorem. Whether you're a student preparing for exams or a math enthusiast honing your algebra skills, this comprehensive guide will walk you through the process step-by-step.

Understanding the Problem

Before diving into calculations, it’s essential to clarify what is being asked. The function given is:

$$F(x) = x^3 - x^2 - 5x - 3$$

and the problem states that $x - 3$ is a factor of this polynomial. Our goal is to find all zeros of $F(x)$, which are the values of x where $F(x) = 0$.

Why is identifying the zeros important?

Zeros (or roots) of a polynomial are the x-values where the graph intersects the x-axis. Knowing these points allows us to understand the behavior of the function, solve equations, and analyze polynomial functions more generally.

Step 1: Recognize the Given Factor

Since x - 3 is a factor of $F(x)$, we know that:

$$F(3) = 0$$

This is a key property of factors: plugging the root into the function yields zero. Let's verify this quickly:

$$F(3) = (3)^3 - (3)^2 - 5 \times 3 - 3 = 27 - 9 - 15 - 3 = 0$$

Indeed, $F(3) = 0$. This confirms that $x = 3$ is a root and that $x - 3$ divides the polynomial exactly.

Step 2: Polynomial Division to Find the Quotient

To find the other roots, we will divide $F(x)$ by $x - 3$ to factor out the known root and reduce the degree of the polynomial.

Methods of polynomial division:

- Synthetic division (more efficient for linear divisors)
- Long division

Given the divisor $x - 3$, synthetic division is straightforward.

Step 3: Performing Synthetic Division

Set up synthetic division with root 3:

Coefficients of $F(x)$:

$$1 \quad -1 \quad -5 \quad -3$$

Arrange as:

3	1	-1	-5	-3
		3	6	3

Steps:

1. Bring down the leading coefficient (1):

- First row: 1

2. Multiply 1 by 3 (root):

- $1 \times 3 = 3$

- Add to next coefficient: $-1 + 3 = 2$

3. Multiply 2 by 3:

- $2 \times 3 = 6$

- Add to next coefficient: $-5 + 6 = 1$

4. Multiply 1 by 3:

- $1 \times 3 = 3$

- Add to last coefficient: $-3 + 3 = 0$

The bottom row (excluding the final value) gives the coefficients of the quotient polynomial:

$$ 1 \quad 2 \quad 1 $$

and the last number, 0, confirms the remainder (which should be zero since $x - 3$ is a factor).

Thus,

$$ F(x) = (x - 3)(x^2 + 2x + 1) $$

Step 4: Factoring the Quadratic

Now, we focus on the quadratic:

$$ x^2 + 2x + 1 $$

This quadratic can be factored further or solved using the quadratic formula.

Recognize a perfect square:

$$ x^2 + 2x + 1 = (x + 1)^2 $$

So, the complete factorization of $F(x)$ is:

$$ F(x) = (x - 3)(x + 1)^2 $$

Step 5: Find All Zeros

From the factored form, the zeros are:

1. $x - 3 = 0 \Rightarrow x = 3$
2. $x + 1 = 0 \Rightarrow x = -1$

Note that $x = -1$ is a root with multiplicity 2, since it appears squared.

Summary of zeros:

Zero	Multiplicity
3	1
-1	2

The zeros are x = 3 and x = -1 (with multiplicity 2).

Step 6: Interpreting the Results

What do these zeros tell us?

- Graph Behavior: The zero at $x = 3$ is a simple root; the graph crosses the x-axis at this point.
- The zero at $x = -1$ has multiplicity 2, meaning the graph touches the x-axis and turns around at this point (tangent point), not crossing it.

Summary:

- Zeros of $F(x)$: $x = 3$ and $x = -1$ (with multiplicity 2).
- Factorization: $F(x) = (x - 3)(x + 1)^2$.

Additional Insights

The Rational Root Theorem

The Rational Root Theorem states that any rational root p/q (in lowest terms) of a polynomial with integer coefficients is a divisor of the constant term divided by a divisor of the leading coefficient.

For $F(x) = x^3 - x^2 - 5x - 3$:

- Constant term: -3
- Leading coefficient: 1

Possible rational roots:

$$\pm 1, \pm 3$$

We tested $x = 3$ and confirmed it as a root. Testing $x = -1$:

$$F(-1) = (-1)^3 - (-1)^2 - 5 \times (-1) - 3 = -1 - 1 + 5 - 3 = 0$$

So, both roots are rational, and the factorization is complete.

Summary and Final Remarks

In this detailed guide, we:

- Recognized the given factor $x - 3$ and verified it.
- Used synthetic division to factor $F(x)$ completely.
- Factored the quadratic to find all zeros.
- Determined the multiplicity of each zero and interpreted their significance on the graph.
- Reinforced the use of the Rational Root Theorem to identify potential roots.

Final answer:

The zeros of the function $F(x) = x^3 - x^2 - 5x - 3$ are:

- x = 3 (multiplicity 1)
- x = -1 (multiplicity 2)

These roots fully determine the behavior of the polynomial and allow you to sketch its graph or solve related equations confidently.

Understanding how to factor polynomials, find zeros, and interpret multiplicities is fundamental in algebra and calculus. Practice with similar polynomials to strengthen these skills, and always verify your roots by plugging them back into the original function.

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