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Understanding the composition of functions is a fundamental concept in mathematics, especially in algebra and calculus. When given two functions, such as $F(x) = \frac{1}{x}$ and $G(x) = x + 4$, analyzing their composition involves understanding how one function is applied to the output of the other. Specifically, in this case, we are asked to determine the graph of the composite function $(g \circ f)(x) = g(f(x))$. This article provides a comprehensive explanation of how to approach this problem, including the steps to simplify the composite function, the characteristics of the resulting graph, and visual interpretation techniques. Whether you're a student preparing for exams or a math enthusiast interested in graph transformations, this guide will clarify the process and help you identify the correct graph among multiple options.

Understanding the Functions Involved

Before diving into the composition, it’s essential to understand each function individually.

Function $F(x) = \frac{1}{x}$

- Type of Function: Rational function
- Domain: All real numbers except $x=0$ (since division by zero is undefined)
- Range: All real numbers except $y=0$
- Graph Characteristics:

- Hyperbolic shape with two branches
- Vertical asymptote at $x=0$
- Horizontal asymptote at $y=0$
- Symmetrical with respect to the origin (odd function)

Function $G(x) = x + 4$

- Type of Function: Linear function
- Domain: All real numbers
- Range: All real numbers
- Graph Characteristics:
- Straight line with slope 1
- y-intercept at $(0, 4)$
- No asymptotes or curvature

Understanding Function Composition: $(g \circ f)(x)$

The composition $(g \circ f)(x)$ means applying $f(x)$ first, then applying g to the result. Mathematically:

$$
(g \circ f)(x) = g(f(x))
$$

Given the functions:

$$
f(x) = \frac{1}{x}

\]

$$
g(x) = x + 4
$$

The composite becomes:

$$
(g \circ f)(x) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 4
$$

This simplified form provides the function's explicit expression, enabling us to analyze and graph it directly.

Analyzing the Composite Function $(g \circ f)(x) = \frac{1}{x} + 4$

Understanding the characteristics of $\frac{1}{x} + 4$ is crucial for identifying its graph.

Key Features of $\frac{1}{x} + 4$

- Type of Function: Transformed rational function
- Domain: All real numbers except $x=0$ (since $\frac{1}{x}$ is undefined at $x=0$)
- Range: All real numbers except $y=4$
- Vertical Asymptote: At $x=0$, due to the $\frac{1}{x}$ term
- Horizontal Asymptote: At $y=4$, because as $x \to \pm \infty$, $\frac{1}{x} \to 0$, so the entire

function approaches 4

Transformation Explanation

- The original $f(x)=\frac{1}{x}$ has asymptotes at $x=0$ and $y=0$.
- Adding 4 shifts the entire graph vertically upward by 4 units.
- The vertical asymptote remains at $x=0$.
- The horizontal asymptote shifts from $y=0$ to $y=4$.

Graphical Implications

- The graph consists of two branches:
- One in the first quadrant approaching $x=0^+$ and $y \to +\infty$, and approaching $y=4$ from above as $x \to +\infty$.
- The other in the third quadrant approaching $x=0^-$ and $y \to -\infty$, and approaching $y=4$ from below as $x \to -\infty$.
- The asymptotes divide the plane into regions, and the graph approaches these asymptotes but never touches them.

How to Identify the Correct Graph

Given multiple options, you can follow these steps to determine which graph correctly depicts $\frac{1}{x} + 4$:

Step 1: Check for Vertical Asymptote at $x=0$

- The graph should have a vertical asymptote at $x=0$.
- The two branches should tend towards infinity and negative infinity as they approach $x=0$.

Step 2: Check for Horizontal Asymptote at $y=4$

- As $x \to +\infty$, the graph should approach $y=4$ from above.
- As $x \to -\infty$, the graph should approach $y=4$ from below.
- The line $y=4$ should be a horizontal asymptote.

Step 3: Confirm the Shape of the Hyperbolic Branches

- The branches should resemble those of $1/x$, just shifted vertically.
- The branch in the first quadrant should decrease toward the asymptote at $y=4$ as x increases.
- The branch in the third quadrant should increase toward $y=4$ as x decreases.

Step 4: Verify the Domain and Range

- The domain should exclude $x=0$.
- The range should exclude $y=4$.

Step 5: Eliminate Graphs That Do Not Match These Features

- Any graph lacking the correct asymptotes or with asymptotes at incorrect positions should be discarded.

Graph Transformation Summary

Summarizing the transformations involved in $(g \circ f)(x) = \frac{1}{x} + 4$:

- The original graph of $F(x) = 1/x$ is shifted vertically upward by 4 units.
- Asymptotes remain at:
- Vertical: $x=0$
- Horizontal: $y=4$

This understanding helps in quickly matching the graph to the correct option among multiple choices.

Practical Tips for Sketching or Recognizing the Graph

- Identify asymptotes first: They are the key markers.
- Note the shift: Recognize that the entire $1/x$ graph moves up by 4 units.
- Check the behavior near asymptotes: The graph should tend toward infinity near $x=0$, and toward $y=4$ as $|x|$ becomes large.
- Observe the branches: Confirm the shape and position relative to axes.

Conclusion

The composite function $(g \circ f)(x) = \frac{1}{x} + 4$ is a vertically shifted version of the basic reciprocal function $1/x$. Its graph features a vertical asymptote at $x=0$ and a horizontal asymptote at $y=4$. When presented with multiple graphs, focus on these asymptotes, the shape of the hyperbolic branches, and the domain and range to select the correct illustration. Understanding how transformations affect the graph of rational functions is essential for accurate identification and analysis in both academic and real-world contexts.

Additional Resources:

- Graphing Rational Functions
- Transformations of Functions
- Asymptote Analysis
- Function Composition in Algebra

By mastering these concepts, students and enthusiasts will be better equipped to analyze complex functions and interpret their graphical representations with confidence.

Keywords: $F(x)=\frac{1}{x}$, $G(x)=x+4$, composite function, $(g \circ f)(x)$, graph of rational functions, asymptotes, transformations, hyperbolic graph, function analysis, graph identification

Frequently Asked Questions

What is the expression for (gf)(x) when F(x)=1/x and G(x)=x+4?

(gf)(x) = F(G(x)) = 1/(x+4).

How do you find the composition (gf)(x) given F(x)=1/x and G(x)=x+4?

Replace x in F(x) with G(x), so (gf)(x) = 1/(x+4).

What is the domain of (gf)(x) = 1/(x+4)?

The domain is all real numbers except x = -4, where the denominator becomes zero.

Which features does the graph of (gf)(x) = 1/(x+4) have?

It has a vertical asymptote at x = -4 and a horizontal asymptote at y = 0.

How does the graph of (gf)(x) = 1/(x+4) compare to the graph of y=1/x?

It is a shifted version of y=1/x, shifted 4 units to the left along the x-axis.

If you were to choose the correct graph for (gf)(x) = 1/(x+4), what key features should you look for?

Look for a hyperbola with a vertical asymptote at x = -4 and a horizontal asymptote at y = 0.

Additional Resources

Understanding the Composition of Functions: If F(x) = 1/x and G(x) = x + 4, Which of the Following Is the Graph of (g∘f)(x)?

When exploring the world of functions, one of the most intriguing concepts is the composition of functions, often denoted as (g∘f)(x). This process involves taking the output of one function and using it as the input for another. In our case, with the functions F(x) = 1/x and G(x) = x + 4, examining the composition $(g \circ f)(x)$ offers insight into how combining simple functions can produce complex and interesting graphs. Understanding the behavior, domain, and range of this composition reveals much about the interplay between these two functions.

What Is Function Composition?

Before diving into the specifics of our functions, it's essential to grasp what function composition entails:

- Definition: The composition of two functions, f and g, written as $(g \circ f)(x)$, means applying f to x first, then applying g to the result. Formally:

$(g \circ f)(x) = g(f(x))$

- Process: To evaluate, you replace x in g with f(x).

- Result: The outcome is a new function that reflects the combined transformation of the two original functions.

In our case:

- $f(x) = 1/x$

- $g(x) = x + 4$

Thus,

- $(g \circ f)(x) = g(f(x)) = g(1/x) = (1/x) + 4$

Step-by-Step Breakdown of (g□f)(x)

1. Understanding the inner function f(x) = 1/x

- Domain: All real numbers except x = 0, since division by zero is undefined.

- Range: All real numbers except 0, because 1/x never equals 0; it approaches infinity or negative infinity as x approaches zero.

- Graph behavior: The graph of f(x) = 1/x is a hyperbola with two branches:

- In the first quadrant: approaching positive infinity as x □ 0□, and approaching zero as x □ □.

- In the third quadrant: approaching negative infinity as x □ 0□, and approaching zero as x □ -□.

2. Applying g to f(x):

- g(f(x)) = (1/x) + 4

This means for any value of x (except zero), the output is (1/x) + 4.

Graphing (g□f)(x): Step-by-Step Analysis

1. Understanding the transformed function

- The expression (1/x) + 4 is a vertical shift of the basic reciprocal function 1/x upward by 4 units.

- The original hyperbola f(x) = 1/x is shifted vertically, resulting in a similar hyperbola but centered around y = 4 instead of y = 0.

2. Domain and Range of (g□f)(x)

- Domain: Same as f(x), since g is defined for all real inputs, but f(x) is undefined at x=0.

- Therefore, domain of (g□f)(x): x □ □ \ {0}

- Range: The values of (1/x) + 4 cover all real numbers except 4, since 1/x can be any real number except zero, and adding 4 shifts the entire range accordingly.

- As x □ 0□, (1/x) + 4 □ +□

- As x □ 0□, (1/x) + 4 □ -□

- As x □ ±□, (1/x) + 4 □ 4

- Key point: The horizontal asymptote of the graph is y = 4, because as x approaches infinity or negative infinity, the function approaches 4.

Visualizing the Graph of (g□f)(x)

1. Shape and Asymptotes

- The graph resembles a hyperbola similar to the original 1/x but shifted vertically.

- Vertical asymptote: x = 0 (where the function is undefined).

- Horizontal asymptote: y = 4 (approach as $x \to \pm\infty$).

2. Quadrant Behavior

- When x > 0:

- (1/x) + 4 is positive and decreases toward 4 as x increases.

- The graph approaches y = 4 from above as $x \to \infty$.

- Near $x = 0^+$, the function shoots up toward $+\infty$.

- When x < 0:

- (1/x) + 4 is negative and approaches 4 from below as $x \to -\infty$.

- Near $x = 0^-$, the function plunges toward $-\infty$.

Interpreting Multiple Choice Graphs

When presented with multiple graphs, key features to look for include:

- Vertical asymptote at x = 0.

- Horizontal asymptote at y = 4.

- The two branches of the graph should mirror the hyperbolic shape of 1/x, but shifted vertically.

- The behavior near the asymptotes:

- Approaching $+\infty$ as $x \to 0^+$.

- Approaching $-\infty$ as $x \to 0^-$.

- The parts of the graph in each quadrant:

- Quadrant I: $x > 0$, $(g \circ f)(x) > 4$.

- Quadrant III: $x < 0$, $(g \circ f)(x) < 4$.

Summary of Key Points

Aspect	Description
Function	$(g \circ f)(x) = (1/x) + 4$
Domain	$\mathbb{R} \setminus \{0\}$
Range	$\mathbb{R} \setminus \{4\}$
Vertical asymptote	$x = 0$
Horizontal asymptote	$y = 4$
Behavior near asymptotes	Approaches $\pm\infty$ depending on side
Shape	Hyperbolic branches shifted upward by 4

Practical Tips for Recognizing the Correct Graph

- Identify the asymptotes:

- Look for a vertical line at $x = 0$.

- Look for a horizontal line at y = 4.

- Check the behavior as x approaches zero:

- Does the graph tend toward infinity on one side and negative infinity on the other?

- Verify the shape in each quadrant:

- Does the graph resemble the reciprocal hyperbola shifted vertically?

- Assess the limits at infinity:

- Are the branches approaching y = 4?

Final Thoughts

The composition $(g \circ f)(x) = (1/x) + 4$ demonstrates how combining simple functions results in a hyperbolic graph shifted vertically. Recognizing the asymptotic behavior, domain, and range is crucial for accurately identifying its graph among multiple options. By focusing on these features, you can confidently select the correct graph that visualizes the composition of F(x) = 1/x and G(x) = x + 4.

Understanding this process not only enhances your grasp of function composition but also prepares you to analyze more complex transformations in algebra, calculus, and beyond.

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