If The Center Of A Circle Is At (5,-7)and Its Radius Is 6, Complete Itsequation:

If The Center Of A Circle Is At (5,-7)and Its Radius Is 6, Complete Itsequation: Understanding how to formulate the equation of a circle given its center and radius is fundamental in coordinate geometry. Whether you're a student preparing for exams or a math enthusiast exploring the properties of geometric figures, mastering this process is crucial. In this article, we will explore the step-by-step method for completing the equation of a circle when provided with the center point and radius, along with practical examples, tips, and related concepts to deepen your understanding.

Understanding the Standard Equation of a Circle

Before diving into the specifics of this problem, it's essential to review the general form of the circle's equation.

The Standard Form

The equation of a circle with center at (h, k) and radius r is expressed as:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

- (h, k) are the coordinates of the circle's center.
- r is the radius of the circle.

This formula is derived from the distance formula, representing all points (x, y) that are exactly r units away from the center.

Why is the Equation Important?

Knowing the standard form allows you to:
- Plot the circle accurately on a coordinate plane.
- Determine whether a given point lies inside, on, or outside the circle.
- Find the circle's graph efficiently.
- Analyze geometric properties related to the circle.

Given Data and the Goal

In our specific problem, you're provided with:

- The center at point $(5, -7)$
- The radius $r = 6$

Your task is to formulate the complete equation of this circle.

Step-by-Step Process to Complete the Equation

Let's walk through the steps systematically.

Step 1: Identify the Center Coordinates

The center is at $(h, k) = (5, -7)$.

Step 2: Note the Radius

Radius: $r = 6$.

Step 3: Square the Radius

Calculate r^2:

$$r^2 = 6^2 = 36$$

Step 4: Write the Equation

Using the standard form:

$$(x - h)^2 + (y - k)^2 = r^2$$

Substitute the known values:

$$(x - 5)^2 + (y + 7)^2 = 36$$

Note: Since the y-coordinate of the center is -7, the term becomes $(y - (-7)) = (y + 7)$.

Complete Equation of the Circle

The complete equation is:

$$(x - 5)^2 + (y + 7)^2 = 36$$

This form is the most straightforward and useful for graphing and analysis.

Additional Concepts and Variations

Understanding the core equation enables you to manipulate and analyze circles in various forms.

1. Center-Radius Form vs. General Form

The center-radius form is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

The general form expands into:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

where D, E, F are constants derived through algebraic expansion or completing the square.

Converting to General Form:
- Expand $(x - h)^2$ and $(y - k)^2$.
- Simplify and rearrange to match the general form.

2. Graphing the Circle

To graph the circle:
- Plot the center at $(5, -7)$.
- From the center, draw a radius of 6 units in all directions.
- Sketch the circle passing through points $(5 + 6, -7)$, $(5 - 6, -7)$, $(5, -7 + 6)$, and $(5, -7 - 6)$.

3. Points on the Circle

Find points that satisfy the equation:
- For $x = 5$:

$$ (5 - 5)^2 + (y + 7)^2 = 36 $$
$$ 0 + (y + 7)^2 = 36 $$
$$ (y + 7)^2 = 36 $$
$$ y + 7 = \pm 6 $$
$$ y = -7 \pm 6 $$
- $y = -7 + 6 = -1$
- $y = -7 - 6 = -13$

So, points $(5, -1)$ and $(5, -13)$ lie on the circle.

Similarly, for $y = -7$:

$$ (x - 5)^2 + (0)^2 = 36 $$
$$ (x - 5)^2 = 36 $$
$$ x - 5 = \pm 6 $$
$$ x = 5 \pm 6 $$
- $x = 11$
- $x = -1$

Points $(11, -7)$ and $(-1, -7)$ are on the circle.

Practical Applications of Circle Equations

Understanding how to complete the equation of a circle has multiple applications:
- Engineering: Design of circular components.
- Computer Graphics: Rendering circles and arcs.
- Navigation & GPS: Defining regions based on distance.
- Physics: Analyzing circular motion.

Common Mistakes to Avoid

- Forgetting to square the radius.
- Confusing the signs when substituting the center coordinates.
- Mixing up the coefficients when expanding or converting between forms.
- Not simplifying the equation fully.

Summary

To complete the equation of a circle given its center and radius:
1. Identify the center coordinates (h, k).
2. Square the radius to find r^2.
3. Substitute into the standard form:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

4. Write the final equation with the given data.

In our specific case, the complete equation is:

$$ (x - 5)^2 + (y + 7)^2 = 36 $$

Mastering this process allows you to confidently work with circles in various contexts, from basic geometry problems to complex real-world applications.

Conclusion

Formulating the equation of a circle from its center and radius is a fundamental skill in coordinate geometry. By understanding the standard form and practicing substitution, you can effortlessly derive the complete equation for any circle. Remember to carefully handle signs and perform accurate calculations, and you'll master this concept in no time. Whether for academic purposes or practical applications, this knowledge forms a cornerstone of understanding geometric relationships in the coordinate plane.

Frequently Asked Questions

What is the general form of a circle's equation given its center at (h, k) and radius r?

The general form is $(x - h)^2 + (y - k)^2 = r^2$.

How do you write the equation of a circle with center at (5, -7) and radius 6?

Substitute h=5, k=-7, and r=6 into the formula: $(x - 5)^2 + (y + 7)^2 = 36$.

What is the completed equation of the circle with center at (5, -7) and radius 6?

The equation is $(x - 5)^2 + (y + 7)^2 = 36$.

How can I verify if a point lies on this circle?

Plug the point's coordinates into the equation; if the equation holds true, the point lies on the circle.

What is the significance of the values (5, -7) and 6 in the circle's equation?

They represent the center coordinates (5, -7) and the radius length 6, defining the size and position of the circle.

Can the equation be rewritten in standard form for this circle?

Yes, the standard form is $(x - 5)^2 + (y + 7)^2 = 36$.

How do I find the diameter of the circle?

Multiply the radius by 2; for a radius of 6, the diameter is 12.

If I wanted to graph this circle, what key points should I plot?

Plot the center at (5, -7), then mark points 6 units in all directions: (5 ± 6, -7), (5, -7 ± 6).

What are some real-world examples of circles with similar equations?

Examples include a round table with a known center point and radius, or a circular fountain positioned at specific coordinates.

Additional Resources

If The Center Of A Circle Is At (5,-7) and Its Radius Is 6, Complete Its Equation

Understanding the fundamental properties of circles is essential in both academic and real-world applications, ranging from engineering and design to navigation and computer graphics. One of the core concepts involves representing circles mathematically through equations. When given specific parameters such as the circle’s center and radius, the task often involves crafting its standard or general equation accurately. This article delves into the process of completing the equation of a circle when the center and radius are known, with a detailed exploration of the underlying principles, formula derivation, and practical examples.

The Significance of Circle Equations in Mathematics and Beyond

Before detailing the specific problem at hand, it’s crucial to appreciate why understanding circle equations matters. Circles are among the most fundamental shapes in geometry, characterized by all points equidistant from a fixed point called the center. Their equations form the backbone for various applications:
- Designing mechanical parts with circular symmetry
- Plotting orbits in astronomy
- Creating graphic designs and animations
- Developing algorithms in computer graphics and image processing

Mathematically, representing a circle via an algebraic equation allows precise calculations, transformations, and analysis.

Understanding the Standard Equation of a Circle

The starting point in solving the problem is understanding the general form of a circle's equation. The most common and straightforward form is the standard form:

$(x - h)^2 + (y - k)^2 = r^2$

Where:
- (h, k) are the coordinates of the circle’s center.
- r is the radius of the circle.

This equation states that any point (x, y) lying on the circle will satisfy the relationship that the distance from (x, y) to (h, k) equals r.

Deriving the Equation with Known Center and Radius

Given:
- Center (h, k) = (5, -7)
- Radius r = 6

Substituting into the standard form:

$(x - 5)^2 + (y + 7)^2 = 6^2$

Simplifies to:

$(x - 5)^2 + (y + 7)^2 = 36$

This is the complete equation of the circle in its standard form.

Key observations:
- The signs in the equation directly relate to the coordinates of the center.
- The radius squared appears on the right side of the equation.

Transforming the Equation Into the General Form

While the standard form offers clarity and ease for plotting, the general form of a circle’s equation is often used in algebraic analysis:

$x^2 + y^2 + Dx + Ey + F = 0$

Transforming from the standard form involves expanding the squared binomials and collecting like terms.

Step-by-step process:

1. Start with the standard form:

$$(x - 5)^2 + (y + 7)^2 = 36$$

2. Expand each binomial:

- $(x - 5)^2 = x^2 - 10x + 25$
- $(y + 7)^2 = y^2 + 14y + 49$

3. Sum the expanded forms:

$$x^2 - 10x + 25 + y^2 + 14y + 49 = 36$$

4. Combine like terms:

$$x^2 + y^2 - 10x + 14y + (25 + 49) = 36$$

$$x^2 + y^2 - 10x + 14y + 74 = 36$$

5. Bring all to one side to set equal to zero:

$$x^2 + y^2 - 10x + 14y + 74 - 36 = 0$$

Simplifies to:

$$x^2 + y^2 - 10x + 14y + 38 = 0$$

This is the general form of the circle’s equation with the given parameters.

Understanding the Components of the Equation

Breaking down the general form:

$$x^2 + y^2 + Dx + Ey + F = 0$$

- D and E relate to the center coordinates:
- $h = -D/2$
- $k = -E/2$

- F influences the size and position of the circle.

Using the completed general form:

$x^2 + y^2 - 10x + 14y + 38 = 0$

- $D = -10$, so $h = -(-10)/2 = 5$, matching the center's x-coordinate.
- $E = 14$, so $k = -14/2 = -7$, matching the y-coordinate.
- The constant term $F = 38$ helps determine the radius through a different formula, which will be discussed next.

Calculating the Radius From the Equation

While the radius was initially given as 6, understanding how to extract it from the general form is valuable.

In the general form, the radius r can be found via the formula:

$$r = \sqrt{h^2 + k^2 - F}$$

Given:
- $h = 5$
- $k = -7$
- $F = 38$

Calculating:

$$r = \sqrt{5^2 + (-7)^2 - 38}$$
$$r = \sqrt{25 + 49 - 38}$$
$$r = \sqrt{74 - 38}$$
$$r = \sqrt{36}$$
$$r = 6$$

This confirms the radius matches the original given value, providing a consistency check for the algebraic work.

Practical Applications and Visual Representation

Knowing the equation of a circle enables precise plotting and analysis in various fields.

Applications include:
- Engineering Design: Ensuring components fit within specified circular boundaries.
- Navigation: Calculating positions relative to a fixed point.

- Computer Graphics: Rendering circles accurately in digital images.
- Physics: Analyzing orbits and circular motion paths.

Visualizing the circle:
- The center at (5, -7) is plotted on the coordinate plane.
- The radius of 6 units means the circle extends 6 units in all directions from the center.
- The equation confirms the shape’s size and position, allowing for accurate rendering.

Summary and Key Takeaways

- The standard form of a circle’s equation is $(x - h)^2 + (y - k)^2 = r^2$.
- Given the center (5, -7) and radius 6, the complete standard form is $(x - 5)^2 + (y + 7)^2 = 36$.
- Expanding this yields the general form $x^2 + y^2 - 10x + 14y + 38 = 0$.
- The relationship between the algebraic coefficients and the geometric properties (center and radius) is straightforward and can be used for conversions and validations.
- Understanding these equations is vital for applications across science, engineering, and technology.

In conclusion, transforming the given parameters into a complete circle equation involves understanding the standard form, performing algebraic expansions, and recognizing the relationship between algebraic coefficients and geometric features. Mastery of these concepts enhances problem-solving skills and provides a foundation for more advanced geometric and algebraic analyses.

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