Lim 0 K=1n(x K) 2x K;[2,4] The Limit, Expressed As A Definite Integral, Is

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Understanding limits is fundamental in calculus, especially when dealing with sequences, functions, and their behaviors as variables approach specific points or infinity. In this comprehensive article, we explore the limit involving the sequence $a_K = \ln(x_K) \cdot 2x_K$ over the interval $[2,4]$, and how this limit can be expressed as a definite integral. We will analyze the problem step-by-step, interpret the notation, and demonstrate the process of transforming the limit into a meaningful integral expression, providing clarity and insight into the mathematical concepts involved.

Understanding the Expression and Notation

Before delving into the limit's evaluation and its representation as a definite integral, it's essential to clarify the notation and the components involved in the expression.

Deciphering the Sequence and Notation

- Sequence x_K: The notation suggests a sequence x_K indexed by K, with values possibly within the interval $[2,4]$. Often, in calculus, sequences are constructed to approximate a function or to facilitate Riemann sums.

- The Logarithmic Term $\ln(x_K)$: The natural logarithm function, which is continuous and increasing for $x > 0$.

- The Term $2x_K$: A linear term scaled by 2, multiplied by the sequence element x_K.

- The Limit $K \to 0$: Typically, limits involve $K \to \infty$ or $K \to a$. Here, the notation "Lim 0 K=1n(x_K) 2x_K" appears to be a condensed or shorthand expression. It might represent the limit as $K \to 0$ of the sum or an expression involving $\ln(x_K)$ and $2x_K$.

Given the context, it seems plausible that the expression aims to evaluate the limit:

$$
\lim_{K \to 0} \sum_{i} \ln(x_K) \cdot 2x_K
$$

or perhaps a Riemann sum involving these terms over the interval $[2,4]$.

Interpreting the Expression as a Riemann Sum

Considering the mention of the interval $[2,4]$, the goal may be to express the limit as a Riemann sum over this interval, which in turn can be represented as a definite integral.

In particular, the expression:

$$
\lim_{K \to 0} \sum_{i} \ln(x_K) \cdot 2x_K
$$

could correspond to a Riemann sum for the integral:

$$
\int_{2}^{4} \ln(x) \cdot 2x \, dx
$$

where the partition points x_K approximate points within $[2,4]$, and the sum over these points approximates the integral as the partition gets finer.

Transforming the Limit into a Riemann Sum

To understand how the limit relates to a definite integral, we need to formalize the process of approximating an integral via Riemann sums.

Constructing the Riemann Sum

- Partition of the interval $[2,4]$: Divide the interval into n subintervals with points $x_0=2, x_1, x_2, ..., x_n=4$.

- Sample points x_K: Choose sample points within each subinterval, often right endpoints, left endpoints, or midpoints.

- Width of subintervals: $\Delta x = \frac{4-2}{n} = \frac{2}{n}$.

- Sample points x_K: For example, if selecting right endpoints, $x_K = 2 + K \Delta x$.

- The Riemann sum:

$$
S_n = \sum_{K=1}^{n} f(x_K) \Delta x
$$

where

$$
f(x) = \ln(x) \cdot 2x
$$

and the sum approximates the integral as $n \to \infty$.

In our case:

$$
\lim_{n \to \infty} \sum_{K=1}^{n} \ln(x_K) \cdot 2x_K \Delta x
$$

which equates to the definite integral:

$$
\int_{2}^{4} \ln(x) \cdot 2x \, dx
$$

Computing the Definite Integral

Having established that the limit corresponds to the integral

$$
I = \int_{2}^{4} 2x \ln(x) \, dx
$$

we proceed to evaluate it.

Method: Integration by Parts

The integral involves the product of $2x$ and $\ln(x)$, suggesting integration by parts as an effective method.

Recall the formula:

$$
\int u\, dv = uv - \int v\, du
$$

Choose:

- $u = \ln(x)$ $\Rightarrow du = \frac{1}{x} dx$

- $dv = 2x\, dx$ $\Rightarrow v = x^2$

Apply integration by parts:

$$
I = \int_{2}^{4} 2x \ln(x) \, dx = [x^2 \ln(x)]_{2}^{4} - \int_{2}^{4} x^2 \cdot \frac{1}{x} dx
$$

Simplify the second integral:

$$
\int_{2}^{4} x^2 \cdot \frac{1}{x} dx = \int_{2}^{4} x\, dx
$$

Compute both parts:

- First term:

$$
[x^2 \ln(x)]_{2}^{4} = (4^2 \ln 4) - (2^2 \ln 2) = (16 \ln 4) - (4 \ln 2)
$$

- Second term:

$$
\int_{2}^{4} x\, dx = \left[\frac{x^2}{2}\right]_2^4 = \frac{4^2}{2} - \frac{2^2}{2} = \frac{16}{2} - \frac{4}{2} = 8 - 2 = 6
$$

Putting it together:

$$
I = (16 \ln 4) - (4 \ln 2) - 6
$$

Express $\ln 4$ in terms of $\ln 2$:

$$
\ln 4 = \ln (2^2) = 2 \ln 2
$$

So:

$$
I = 16 \times 2 \ln 2 - 4 \ln 2 - 6 = (32 \ln 2) - (4 \ln 2) - 6 = (28 \ln 2) - 6
$$

Final Expression of the Limit as a Definite Integral

Based on the above derivation, the original limit can be expressed as the value of the definite integral:

$$
\boxed{
\lim_{K \to 0} \text{(expression involving } \ln(x_K) \text{ and } 2x_K) = \int_{2}^{4} 2x \ln(x) \, dx = 28 \ln 2 - 6
}
$$

This result encapsulates the convergence of the Riemann sum to the integral, providing a precise value for the limit.

Additional Insights and Applications

Understanding how limits relate to definite integrals through Riemann sums is crucial in calculus, especially in the context of:

- Fundamental Theorem of Calculus: Connecting derivatives and integrals.

- Approximate Calculations: Using Riemann sums to approximate areas under curves.

- Analyzing Function Behavior: Studying the limiting behavior of sequences and sums.

Applications include:

- Computing areas and volumes.

- Evaluating complex integrals involving logarithmic and polynomial functions.

- Developing numerical methods for integration.

Summary and Conclusion

In this article, we dissected the expression involving the sequence x_K, interpreted it as a Riemann sum over the interval $[2,4]$, and demonstrated how the limit as the partition gets finer converges to the definite integral:

$$
\int_{2}^{4} 2x \ln(x) \, dx
$$

which evaluates to $28 \ln 2 - 6$. This process highlights the fundamental connection between limits, sums, and integrals in calculus, emphasizing the importance of understanding the transition from discrete sums to continuous integrals. Whether applied in theoretical mathematics or practical computations, these concepts form the backbone of analysis and approximation methods.

Keywords: calculus, limit, definite integral, Riemann sum, integration by parts, logarithmic function, interval $[2,4]$, mathematical analysis, integral

Frequently Asked Questions

What is the limit of the function lim_{x→1} ln(x) /

(2x) as x approaches 1?

The limit is 0 because as x approaches 1, ln(x) approaches 0 and 2x approaches 2, so the entire expression approaches 0/2 = 0.

How can the limit lim_{x→2} ln(x) / (2x) be expressed as a definite integral?

It can be expressed as the integral of the derivative of ln(x) over the interval [2, a], but in this specific case, it relates to the Fundamental Theorem of Calculus, representing an integral that approaches the limit as x approaches 2.

What is the significance of representing limits as definite integrals in calculus?

Representing limits as definite integrals allows for the evaluation of limits involving functions that can be integrated, facilitating the calculation of areas, accumulated quantities, or the behavior of functions over intervals.

In the interval [2,4], how is the limit of ln(x) / (2x) related to the area under a curve?

While the limit itself is a pointwise value, expressing it as a definite integral over [2,4] involves integrating the function ln(x) / (2x), which can represent the area under the curve between these points.

Why is understanding the connection between limits and definite integrals important in calculus?

Understanding this connection helps in solving problems involving accumulation, area calculations, and the evaluation of functions as limits, which are fundamental concepts in calculus analysis.

Additional Resources

Limit: An In-Depth Exploration of Its Expression as a Definite Integral

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Introduction

In the realm of calculus, limits serve as the foundational building blocks that allow us to understand the behavior of functions as they approach specific points or infinity. They not only underpin the concept of derivatives and integrals but also provide insights into the continuity, convergence, and overall behavior of mathematical functions. Among the myriad of limit problems encountered, the expression:

$$
\lim_{n \to \infty} K=1n(x K) 2x K; \quad [2,4]
$$

may seem cryptic at first glance, but when dissected carefully, reveals a fascinating connection to definite integrals, especially in the context of Riemann sums.

This article aims to unravel this complex limit, examining its structure, interpretation, and how it can be expressed as a definite integral. We will explore each component meticulously, equipping readers with a comprehensive understanding of the transition from sequences and sums to the elegant framework of integrals.

Deciphering the Limit Expression

The Notation and Components

The given expression appears to be:

$$
\lim_{n \to \infty} K=1n(x K) 2x K; \quad [2,4]
$$

While the notation is somewhat ambiguous at first glance, it likely involves:

- n: An integer tending to infinity.
- x: A variable within the interval $[2,4]$.
- K: Possibly a constant or a function of x or n.
- The phrase "1n(x K) 2x K": Possibly shorthand or typographical representation of a sum or product involving x and K.

Given the context and the mention of expressing the limit as a definite integral, it is reasonable to interpret this as a limit of a sum involving a function over the interval $[2,4]$.

In particular, the notation resembles the structure of a Riemann sum, which is a sum of function values multiplied by subinterval widths, approaching a definite integral as the number of subdivisions tends to infinity.

The Connection to Riemann Sums and Definite Integrals

Understanding Riemann Sums

A Riemann sum approximates the area under a curve $f(x)$ over an interval $[a, b]$:

$$
S_n = \sum_{i=1}^n f(x_i^) \Delta x
$$

where:

- $\Delta x = \frac{b - a}{n}$: The width of each subinterval.
- $x_i^$: A sample point in the i-th subinterval.
- $f(x_i^)$: The function evaluated at the sample point.

As $n \to \infty$, the sum S_n approaches the definite integral:

\[

$$
\int_a^b f(x) \, dx
$$

This fundamental limit bridges the discrete sum with the continuous area.

Reconstructing the Limit

Given the clues, a plausible interpretation of the expression is:

$$
\lim_{n \to \infty} \sum_{i=1}^n f(x_i^) \Delta x
$$

over the interval $[2, 4]$.

Suppose the sum is of the form:

$$
\sum_{i=1}^n x_i^ \cdot \delta
$$

where:

- $x_i^ = 2 + (i - 0.5)\Delta x$, i.e., midpoints.
- $\delta = \frac{4 - 2}{n} = \frac{2}{n}$.

If the function $f(x)$ is proportional to x (say, $f(x) = x$), then the sum approximates the integral:

$$
\lim_{n \to \infty} \sum_{i=1}^n x_i^ \delta = \int_2^4 x \, dx
$$

which computes the area under the linear function $f(x) = x$ from 2 to 4.

Formal Expression as a Definite Integral

The core idea is that the limit of the Riemann sum as $n \to \infty$ converges to the definite integral:

$$
\boxed{
\lim_{n \to \infty} \sum_{i=1}^n f(x_i^) \Delta x = \int_{a}^{b} f(x) \, dx
}
$$

where:

- $a = 2$,
- $b = 4$,
- $f(x)$ is the function that the sum approximates.

Applying the Concept: An Example

Suppose we consider the function $f(x) = x$. The Riemann sum over $[2,4]$ with n subintervals is:

$$
S_n = \sum_{i=1}^n f(x_i^) \Delta x
$$

Using midpoints:

$$
x_i^ = 2 + \left(i - \frac{1}{2}\right) \Delta x
$$
$$
\Delta x = \frac{4 - 2}{n} = \frac{2}{n}
$$

So:

$$
S_n = \sum_{i=1}^n \left[2 + \left(i - \frac{1}{2}\right) \frac{2}{n} \right] \times \frac{2}{n}
$$

Expanding:

$$
S_n = \sum_{i=1}^n \left[2 + \frac{2i - 1}{2n} \right] \times \frac{2}{n}
$$

which simplifies to:

$$
S_n = \sum_{i=1}^n \left(2 + \frac{2i - 1}{2n} \right) \times \frac{2}{n}
$$

As $n \to \infty$, this sum approaches:

$$
\int_2^4 x \, dx
$$

Calculating this integral:

$$
\int_2^4 x \, dx = \left[\frac{x^2}{2}\right]_2^4 = \frac{4^2}{2} - \frac{2^2}{2} = \frac{16}{2} - \frac{4}{2} = 8 - 2 = 6
$$

Significance of the Limit as a Definite Integral

Expressing a limit as a definite integral offers several advantages:

- Analytical Precision: Converts a potentially complex sum into a manageable integral.
- Computational Efficiency: Facilitates exact calculation using integral techniques.
- Theoretical Insight: Reveals the underlying continuous behavior of

sequences or sums.

In our case, the original limit (though somewhat cryptic) aligns with the fundamental theorem of calculus, showing that the sum of function values over a partition converges to the area under the curve $f(x)$ over $[2, 4]$.

Broader Context and Applications

In Physics and Engineering

Limits expressed as integrals are essential in calculating quantities such as:

- Work done by a force: $W = \int_a^b F(x) dx$
- Probability distributions: Integrals over density functions.
- Signal processing: Area under a curve representing signal energy.

In Economics and Social Sciences

- Consumer surplus: Integral of demand functions.
- Cost calculations: Integrating marginal costs over quantity.

Mathematical Significance

Transforming sums into integrals:

- Simplifies complex calculations.
- Provides a bridge between discrete and continuous models.
- Enables the application of calculus tools to real-world problems.

Final Thoughts: The Power of Limits and Integrals

The exploration of the limit:

$$
\lim_{n \to \infty} K=1n(x K) 2x K; \quad [2,4]
$$

serves as a compelling reminder of the elegance of calculus. It exemplifies how seemingly complicated sequences or sums can be distilled into a simple, elegant integral expression, revealing deep insights into the behavior of functions over intervals.

By understanding the transition from Riemann sums to definite integrals, students and practitioners can unlock a powerful method for analyzing continuous phenomena across numerous scientific disciplines.

Summary

- The limit involving sums over an interval can often be expressed as a definite integral.
- Riemann sums approximate the area under a function; as the number of partitions increases, they converge to the integral.

- For the interval \([2, 4]\), and assuming the sum involves a linear function \(f(x) = x\), the integral evaluates to 6.
- Expressing limits as integrals simplifies calculations and provides deeper understanding of the underlying functions.

Final Note

While the initial notation was somewhat ambiguous, the core concept remains clear: limits of sums over an interval are fundamentally connected to definite integrals. Recognizing this allows mathematicians, scientists, and engineers to analyze complex systems with elegance and precision, harnessing the full power of calculus.

Lim 0 K 1n X K 2x K 2 4 The Limit Expressed As A Definite Integral Is

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