Maximize The Function X=2x + 3y Subject To The Conditions:{ X 4{ Y 5{ 3x + 2y 52

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Understanding how to optimize a function within given constraints is a fundamental aspect of operations research, economics, and decision-making processes. In this article, we will thoroughly explore how to maximize the linear function $X = 2x + 3y$ subject to the constraints $X \leq 4$, $Y \leq 5$, and $3x + 2y \leq 52$. We will delve into the methods of solving such problems, interpret the feasible region, and identify the optimal solution.

Introduction to Linear Programming and Optimization

Linear programming (LP) is a mathematical technique used to find the best outcome in a model with linear relationships. The goal is to maximize or minimize a linear objective function, subject to a set of linear inequalities or equations, known as constraints.

For this problem, our objective function is:

$$
Z = 2x + 3y
$$

which we aim to maximize.

The constraints provided set the boundaries within which the solution must lie:

$$
X \leq 4
$$
$$
Y \leq 5
$$
$$
3x + 2y \leq 52
$$

It is common to interpret X and Y as functions of x and y, or sometimes as variables themselves. Given the context, it appears X and Y are variables, so for clarity, we'll treat the constraints as involving variables x and y.

Understanding the Constraints

Let's interpret each constraint carefully.

Constraint 1: $x \leq 4$

This restricts the value of x to be less than or equal to 4. It establishes an upper boundary on x.

Constraint 2: $y \leq 5$

Similarly, this constraint limits y to be at most 5.

Constraint 3: $3x + 2y \leq 52$

This is a linear inequality that defines a boundary line in the xy-plane. The feasible solutions must satisfy this inequality.

Graphical Method for Solving the LP

Since the problem involves two variables, a graphical approach provides an intuitive way to find the optimal solution.

Step 1: Plotting the Constraints

- Plot $x \leq 4$: A vertical line at $x=4$. The feasible region is to the left of this line.
- Plot $y \leq 5$: A horizontal line at $y=5$. The feasible region is below this line.
- Plot $3x + 2y \leq 52$: Rewrite as $2y \leq 52 - 3x$, or $y \leq \frac{52 - 3x}{2}$.

This line intersects the axes at:

- When $x=0$, $y = \frac{52}{2} = 26$.
- When $y=0$, $x = \frac{52}{3} \approx 17.33$.

Since y is restricted to be at most 5, the feasible region is capped at $y=5$.

Step 2: Identifying the Feasible Region

The feasible region is the intersection of all the inequalities:

- To the left of $x=4$,
- below $y=5$,
- and below the line $3x + 2y = 52$.

Plotting these constraints, the feasible region is a polygon bounded by the lines:

- $x=0$,
- $x=4$,

- $y=0$,
- $y=5$,
- and $3x + 2y = 52$.

The feasible region is therefore a polygon within these bounds.

Step 3: Finding the Corner Points

The maximum value of the objective function occurs at one of the vertices (corner points) of the feasible region, as per the fundamental theorem of linear programming.

Calculate the intersection points:

1. Intersection of $x=0$ and $3x + 2y=52$:

At $x=0$,

$$
3(0) + 2y = 52 \Rightarrow 2y=52 \Rightarrow y=26
$$

But since $y \leq 5$, this point is outside the feasible region (because $y=26$ exceeds the limit). Therefore, discard this point.

2. Intersection of $x=4$ and $3x + 2y=52$:

$$
3(4) + 2y=52 \Rightarrow 12 + 2y=52 \Rightarrow 2y=40 \Rightarrow y=20
$$

Check if $y \leq 5$. Since 20 > 5, this point is outside the feasible region. Discard.

3. Intersection of $y=5$ and $3x + 2y=52$:

$$
3x + 2(5) = 52 \Rightarrow 3x + 10=52 \Rightarrow 3x=42 \Rightarrow x=14
$$

But since $x \leq 4$, the point $(14,5)$ is outside the feasible region. Discard.

4. Intersection of $x=4$ and $y=5$:

The point $(4,5)$ is within the constraints.

5. Intersection of $x=0$ and $y=0$:

$$
(0,0)
$$

Clearly feasible.

6. Intersection of $x=0$ and $y=5$:

$$
(0,5)
$$

Feasible, as it satisfies all constraints.

7. Intersection of $x=4$ and $y=0$:

$$
(4,0)
$$

Feasible.

8. Intersection of x and y along the line $3x + 2y=52$ within bounds $x \leq 4$, $y \leq 5$.

Since previous points outside the feasible region, the relevant corner points are:

- $(0,0)$
- $(0,5)$
- $(4,0)$
- $(4,5)$

The point $(4,5)$ is within the feasible region, and the others are on the axes.

Calculating the Objective Function at Corner Points

Evaluate $Z=2x+3y$ at these points:

Point	x	y	$Z=2x+3y$
(0,0)	0	0	0
(0,5)	0	5	$20 +35=15$
(4,0)	4	0	$24 + 0=8$
(4,5)	4	5	$24 + 35=8+15=23$

Among these, the maximum value of Z is 23 at the point $(4,5)$.

Conclusion: Optimal Solution

The optimal solution to maximize $X=2x+3y$ under the given constraints is at $(x,y) = (4,5)$, with the maximum value:

$$
Z_{max} = 23
$$

This indicates that choosing $x=4$ and $y=5$ yields the highest value of the objective function within the feasible region.

Implications and Practical Uses

This type of linear programming problem is common in various fields such as:

- Manufacturing: Maximizing profit given resource constraints.
- Logistics: Optimizing routes or distribution plans.
- Finance: Portfolio optimization within risk limits.
- Marketing: Allocating budgets across channels for maximum impact.

Understanding how to set up and solve such problems enables decision-makers to optimize outcomes efficiently.

Additional Considerations

- Feasibility: Always verify if the candidate solutions satisfy all constraints.
- Multiple solutions: Sometimes, multiple corner points yield the same maximum or minimum; in such cases, any convex combination of these points is also optimal.
- Non-linear constraints: If constraints are non-linear, more advanced techniques are necessary.
- Software Tools: For larger or more complex problems, software such as Excel Solver, LINDO, or MATLAB can be used.

Summary