Rewrite The Function G(x)=2x^2+16x+6 In Vertex Form By Completing The Square

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Understanding how to rewrite quadratic functions in their vertex form is a fundamental skill in algebra that provides insights into the graph's shape, vertex, and axis of symmetry. Specifically, transforming a quadratic function like $G(x)=2x^2+16x+6$ into vertex form by completing the square allows students and mathematicians to easily identify the vertex of the parabola and analyze the function's properties. This process not only enhances problem-solving capabilities but also deepens comprehension of quadratic functions' behavior.

In this comprehensive guide, we will explore the step-by-step method to convert the quadratic function $G(x)=2x^2+16x+6$ into its vertex form using the completing the square technique. We will also discuss the significance of the vertex form, provide practical tips, and include common mistakes to avoid. Whether you're preparing for tests, homework, or simply seeking to strengthen your algebra skills, this article aims to make the process clear, accessible, and SEO-optimized for anyone searching for guidance on rewriting quadratic functions.

Understanding Quadratic Functions and Their Forms

Before diving into the process of completing the square, it is essential to understand the different forms of quadratic functions and what information each provides.

Standard Form of a Quadratic Function

The standard form is written as:

$$f(x) = ax^2 + bx + c$$

where:
- $a \neq 0$,
- b and c are constants.

For $G(x)=2x^2+16x+6$, the coefficients are:
- $a=2$,
- $b=16$,
- $c=6$.

This form is useful for identifying the coefficients directly but does not readily provide information about the parabola's vertex.

Vertex Form of a Quadratic Function

The vertex form is written as:

$$f(x) = a(x - h)^2 + k$$

where:
- (h, k) is the vertex of the parabola,
- a determines the parabola's opening direction and width.

Rewriting a quadratic in vertex form makes it easier to graph the parabola, determine the vertex, and analyze key features.

Why Convert to Vertex Form?

Converting a quadratic function to vertex form offers several advantages:

- Identify the vertex quickly: The coordinates (h, k) are directly visible.
- Graphing made easier: Knowing the vertex simplifies sketching.
- Analyze transformations: Understand shifts, stretches, and reflections.
- Solve optimization problems: Find maximum or minimum values efficiently.

Step-by-Step Guide to Rewrite $G(x)=2x^2+16x+6$ in Vertex Form

Now, let's focus on transforming the given quadratic function:

$$G(x) = 2x^2 + 16x + 6$$

using the completing the square method.

Step 1: Factor out the leading coefficient from the quadratic and linear terms

Since the coefficient of x^2 is 2, factor it out from the quadratic and linear parts:

$$ G(x) = 2(x^2 + 8x) + 6 $$

This simplifies the process of completing the square inside the parentheses.

Step 2: Complete the square inside the parentheses

To complete the square for $x^2 + 8x$:

- Take half of the coefficient of x, which is 8, and divide by 2:

$$ \frac{8}{2} = 4 $$

- Square this value:

$$ 4^2 = 16 $$

- Add and subtract this square inside the parentheses to maintain equality:

$$ G(x) = 2(x^2 + 8x + 16 - 16) + 6 $$

Rewrite as:

$$ G(x) = 2[(x^2 + 8x + 16) - 16] + 6 $$

- Recognize that $x^2 + 8x + 16$ is a perfect square trinomial:

$$ (x + 4)^2 $$

So, the expression becomes:

$$ G(x) = 2[(x + 4)^2 - 16] + 6 $$

Step 3: Distribute the 2 and simplify

Distribute the 2 across the terms inside the brackets:

$$ G(x) = 2(x + 4)^2 - 2 \times 16 + 6 $$

$$ G(x) = 2(x + 4)^2 - 32 + 6 $$

Combine the constants:

$$G(x) = 2(x + 4)^2 - 26$$

Step 4: Write the function in vertex form

The final form of the quadratic function is:

$$G(x) = 2(x + 4)^2 - 26$$

This is the vertex form of the quadratic function.

Interpreting the Vertex Form

From the rewritten function:

$$G(x) = 2(x + 4)^2 - 26$$

we can identify key properties:

- Vertex: $(h, k) = (-4, -26)$

- Opening direction: Since $a=2 > 0$, the parabola opens upward.

- Width of the parabola: The larger the absolute value of a, the narrower the parabola.

- Axis of symmetry: The vertical line $x = h$, which is $x = -4$.

- Minimum value: The parabola's vertex gives the minimum value of $G(x)$, which is -26.

Additional Tips and Common Mistakes

When converting quadratic functions using completing the square, keep these tips in mind:

- Always factor out the leading coefficient first to simplify completing the square.
- Remember to add and subtract the same value inside the parentheses to keep the function equal.

- Be careful with signs—adding and subtracting squares can be confusing.
- Double-check your perfect square trinomial to ensure correctness.
- Simplify constants carefully after distributing.

Common mistakes include:

- Forgetting to factor out a initially.
- Omitting the subtraction of the square term.
- Miscalculating half of the coefficient of x.
- Forgetting to adjust the constant term after completing the square.

Practical Applications of Rewriting in Vertex Form

Transforming quadratic functions into vertex form is not just an academic exercise. It has practical applications in various fields:

- Physics: Analyzing projectile motion and finding maximum height.
- Economics: Modeling profit functions to find maximum profit.
- Engineering: Designing parabolic reflectors and antennas.
- Statistics: Understanding quadratic regression models.

Conclusion

Rewriting the quadratic function $G(x)=2x^2+16x+6$ in vertex form through completing the square provides valuable insights into the parabola's properties. This process involves factoring out the leading coefficient, completing the square inside the parentheses, and simplifying to reveal the vertex's coordinates and the parabola's shape. Mastering this technique enhances your algebra skills and enables you to analyze quadratic functions more effectively.

Whether you're preparing for exams, working on a project, or just seeking a deeper understanding of quadratic functions, practicing completing the square will empower you to manipulate and interpret these functions with confidence. Remember to follow the step-by-step process carefully, watch for common errors, and recognize the significance of the vertex form in various real-world applications.

By mastering the conversion of quadratic functions into their vertex form, you open the door to a richer understanding of algebra, functions, and the beautiful geometry behind parabolas.

Meta Description: Learn how to rewrite the quadratic function $G(x)=2x^2+16x+6$ into vertex form by completing the square. Follow our step-by-step guide to understand the process, identify the vertex, and analyze the parabola's properties.

Frequently Asked Questions

How do I rewrite the quadratic function $G(x) = 2x^2 + 16x + 6$ in vertex form by completing the square?

First, factor out 2 from the quadratic and linear terms: $G(x) = 2(x^2 + 8x) + 6$. Then, complete the square inside the parentheses by adding and subtracting $(8/2)^2 = 16$. So, $G(x) = 2(x^2 + 8x + 16 - 16) + 6$. Simplify to $G(x) = 2((x + 4)^2 - 16) + 6$, then distribute the 2: $G(x) = 2(x + 4)^2 - 32 + 6$, resulting in $G(x) = 2(x + 4)^2 - 26$.

What is the vertex form of the quadratic function $G(x) = 2x^2 + 16x + 6$?

The vertex form of $G(x)$ is $G(x) = 2(x + 4)^2 - 26$.

How can I find the vertex of the quadratic function $G(x) = 2x^2 + 16x + 6$ after rewriting in vertex form?

In the vertex form $G(x) = 2(x + 4)^2 - 26$, the vertex is at the point $(-4, -26)$.

Why is completing the square useful for rewriting quadratic functions in vertex form?

Completing the square allows you to rewrite the quadratic in a form that clearly reveals the vertex's coordinates, making graphing and analyzing the function easier.

Can I use the same method to convert any quadratic function to vertex form?

Yes, completing the square is a general method applicable to any quadratic function to rewrite it in vertex form, provided the quadratic coefficient is not zero.

What are the key steps involved in completing the square for G(x) = 2x^2 + 16x + 6?

The steps are: factor out the leading coefficient from the quadratic and linear terms, complete the square inside the parentheses, adjust the constant term accordingly, and then simplify to obtain the vertex form.

How does rewriting G(x) = 2x^2 + 16x + 6 in vertex form help in graphing the parabola?

The vertex form clearly shows the vertex's position and the parabola's orientation, simplifying the graphing process and understanding the function's maximum or minimum point.

What is the importance of the coefficient 2 in the quadratic function when converting to vertex form?

The coefficient 2 affects the width and orientation of the parabola. When completing the square, it must be factored out first to correctly find the vertex form and identify the vertex.

Additional Resources

Rewrite The Function G(x)=2x^2+16x+6 In Vertex Form By Completing The Square

Mathematics often reveals its most elegant properties when algebraic expressions are rewritten into more insightful forms. One such transformation involves converting quadratic functions into their vertex form—a process that not only simplifies understanding the graph's key features but also enhances problem-solving efficiency. In this article, we will explore how to rewrite the quadratic function $G(x) = 2x^2 + 16x + 6$ into its vertex form by completing the square. This technique is fundamental in algebra and calculus, offering a window into the parabola’s vertex, axis of symmetry, and direction of opening.

Understanding the Importance of Vertex Form

Before delving into the mechanics of rewriting the quadratic, it’s crucial to understand why vertex form matters. The standard quadratic form, $ax^2 + bx + c$, is straightforward but doesn’t immediately reveal the parabola’s most essential features. Conversely, the vertex form, $y = a(x - h)^2 + k$, explicitly displays the vertex (h, k), the point at which the parabola reaches its maximum or minimum.

Why Convert to Vertex Form?

- Identify the vertex quickly: The vertex form directly shows the parabola's highest or lowest point.
- Determine the axis of symmetry: The line $x = h$ passes through the parabola's center.
- Graphing ease: It simplifies plotting the parabola accurately.
- Analyze transformations: It helps understand shifts, stretches, and reflections.

Starting with the Standard Quadratic Function

Our initial function is:

$G(x) = 2x^2 + 16x + 6$

This quadratic is in standard form. The goal is to manipulate it into the vertex form, which will look like:

$G(x) = a(x - h)^2 + k$

where (h, k) is the vertex of the parabola.

Step 1: Factor out the leading coefficient from the quadratic terms

Since the coefficient of x^2 is 2, factor it out from the terms involving x:

$G(x) = 2(x^2 + 8x) + 6$

This step simplifies the process of completing the square inside the parentheses.

Completing the Square: The Core Technique

Completing the square transforms a quadratic expression into a perfect square trinomial, which can be expressed as $(x + m)^2$. The process involves:

- Taking the coefficient of x in the parentheses.
- Halving it.
- Squaring the result.
- Adjusting the expression accordingly to maintain equality.

Step-by-step process:

1. Identify the coefficient of x inside the parentheses

Inside the parentheses, the quadratic part is $x^2 + 8x$.

2. Divide the coefficient of x by 2

$8 \div 2 = 4$

3. Square this result

$4^2 = 16$

4. Add and subtract this square inside the parentheses

To complete the square, we add 16 and subtract 16 to keep the expression equivalent:

$G(x) = 2(x^2 + 8x + 16 - 16) + 6$

Which simplifies to:

$G(x) = 2[(x + 4)^2 - 16] + 6$

5. Distribute the 2 across the terms

$G(x) = 2(x + 4)^2 - 2 \times 16 + 6$

$G(x) = 2(x + 4)^2 - 32 + 6$

6. Combine like terms

$G(x) = 2(x + 4)^2 - 26$

Final Form and Interpretation

Having completed the square, the quadratic function is now expressed in vertex form:

$G(x) = 2(x + 4)^2 - 26$

Identifying the Vertex:

- The vertex’s x-coordinate (h) is -4 (note the sign change inside the parentheses).
- The y-coordinate (k) is -26.

Therefore, the vertex of the parabola is at (-4, -26).

Additional features:

- The parabola opens upward because the coefficient of the squared term, 2, is positive.
- The axis of symmetry is the vertical line $x = -4$.
- The minimum value of $G(x)$ is -26, achieved at $x = -4$.

Applications and Insights from the Vertex Form

Converting a quadratic into vertex form isn't just an algebraic exercise—it provides immediate insights into the function's behavior:

- Graphing: With the vertex at (-4, -26), sketching the parabola becomes straightforward. Plot the vertex, then use the parabola's symmetry to find additional points.
- Transformation understanding: The form reveals that the graph is a parabola shifted 4 units left and 26 units down from the origin, with a vertical stretch factor of 2.
- Optimization problems: In real-world scenarios, such as maximizing profit or minimizing cost modeled by a quadratic, the vertex form makes it easier to identify optimal values directly.

Summary of the Process

Transforming $G(x) = 2x^2 + 16x + 6$ into vertex form involves a systematic approach:

1. Factor out the leading coefficient from the quadratic terms.
2. Complete the square inside the parentheses:
- Take half of the linear coefficient.
- Square it.
- Add and subtract this square to maintain equality.
3. Simplify the expression, distribute, and combine constants.
4. Recognize the vertex form and interpret the vertex.

This method, known as completing the square, is a cornerstone technique in algebra that unlocks deeper understanding of quadratic functions.

Concluding Remarks

Mastering the transformation of quadratic functions into vertex form enhances both theoretical understanding and practical problem-solving skills. Whether used to analyze the properties of a parabola or to simplify calculations in optimization problems, completing the square remains an invaluable tool in the mathematician’s toolkit. The process, though algebraically intensive at first, becomes intuitive with practice, providing a clear pathway from standard form to the elegant symmetry of the vertex form.

In conclusion, rewriting $G(x) = 2x^2 + 16x + 6$ in vertex form by completing the square reveals the parabola's vertex at (-4, -26), exposes its symmetry, and provides a foundation for graphing and analysis. This technique exemplifies the beauty of algebra—transforming complex expressions into insightful, manageable forms that deepen our understanding of mathematical relationships.

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