Select All Of The Equations That Passes Through (5,2) And (0,18)??25 Points

Select All Of The Equations That Passes Through (5,2) And (0,18)??25 Points

When dealing with coordinate geometry, one common problem involves identifying all equations of lines that pass through two specific points. In this case, the points are (5, 2) and (0, 18). Determining lines that pass through these coordinates requires understanding the concepts of slope, point-slope form, and different types of equations—such as slope-intercept, point-slope, and standard form. This article provides a comprehensive guide to selecting all equations that satisfy these conditions, helping students and enthusiasts grasp the process efficiently.

Understanding the Basics of Line Equations

Before delving into the specific equations passing through the points, it is essential to understand the foundational concepts involved in line equations.

What Is a Line Equation?

A line equation expresses the relationship between the x and y coordinates of all points lying on a straight line. Depending on the form, it can be written as:
- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

Key Concepts

- Slope (m): Measures the steepness of the line, calculated as (change in y) / (change in x).
- Points: Specific coordinates through which the line passes.
- Equation forms: Different expressions to describe a line, each useful in different contexts.

Calculating the Slope Between the Two Points

The first step in finding the line equations passing through (5, 2) and (0, 18) is to calculate the slope (m).

Formula for Slope

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

Applying the Coordinates

Given:
- Point 1: (x_1, y_1) = (5, 2)
- Point 2: (x_2, y_2) = (0, 18)

Calculating:

$$ m = \frac{18 - 2}{0 - 5} = \frac{16}{-5} = -\frac{16}{5} $$

Result: The slope of the line passing through these points is $-\frac{16}{5}$.

Formulating the Equation of the Line

Once the slope is known, the next step is to derive the equation of the line that passes through both points.

Using Point-Slope Form

The point-slope form is:

$$ y - y_1 = m(x - x_1) $$

Plugging in point (5, 2):

$$ y - 2 = -\frac{16}{5}(x - 5) $$

Simplify:

$$ y - 2 = -\frac{16}{5}x + \frac{16}{5} \times 5 $$

$$ y - 2 = -\frac{16}{5}x + 16 $$

Add 2 to both sides:

$$ y = -\frac{16}{5}x + 16 + 2 $$

$$ y = -\frac{16}{5}x + 18 $$

Equation in Slope-Intercept Form

The line passing through (5, 2) and (0, 18) can be written as:
$$ y = -\frac{16}{5}x + 18 $$

This is one of the equations passing through both points.

Listing All Possible Equations That Pass Through the Points

In some contexts, especially in exams or problem sets, the question might ask for all equations of lines passing through these points, considering different forms or variations. Let's explore various equations and forms.

1. Slope-Intercept Form

- Equation:
$$ y = -\frac{16}{5}x + 18 $$
- Features: Standard form, directly gives y as a function of x.

2. Point-Slope Form (using point (0, 18))

- Equation:
$$ y - 18 = -\frac{16}{5}(x - 0) $$
- Simplifies to:
$$ y = -\frac{16}{5}x + 18 $$
- Note: It's the same as the previous form, just emphasizing the use of the point (0, 18).

3. Standard Form

Transform the slope-intercept form into standard form:
$$ y = -\frac{16}{5}x + 18 $$
Multiply through by 5 to clear denominators:
$$ 5y = -16x + 90 $$
Bring all to one side:
$$ 16x + 5y = 90 $$
Equation:
$$ 16x + 5y = 90 $$

This is another valid form representing the same line.

4. General Form with Different Coefficients

Since lines can be expressed in multiple forms, the general form could be:
$$ Ax + By + C = 0 $$
For the line above:
$$ 16x + 5y - 90 = 0 $$

Are There Multiple Lines Passing Through These Points? A Clarification

Given two distinct points, there is exactly one unique line passing through both. Therefore, the question of selecting all equations that pass through both points typically refers to:

- The different algebraic forms of the same line.
- Equations that are scalar multiples of each other.

Why Only One Line?

In Euclidean geometry, two distinct points determine exactly one line.

Multiple Equations, Same Line

Different algebraic forms (slope-intercept, point-slope, standard) describe the same line. They are equivalent solutions expressed differently.

Additional Equations Passing Through the Points? Exploring the Possibilities

The key question is whether there are other equations besides the ones derived that pass through both points, considering different slopes or forms.

Can a Line with a Different Slope Pass Through (5,2) and (0,18)?

- No, because the slope is fixed; any line passing through these two points must have slope $-\frac{16}{5}$.
- Lines with different slopes will not pass through both points.

What about Parallel or Perpendicular Lines?

- Lines parallel to the original line do not pass through both points unless they are coincident.
- Perpendicular lines will have different slopes and won't pass through both points.

Summary

- The only line passing through both (5, 2) and (0, 18) is the one with slope $-\frac{16}{5}$ and the derived equations.
- Variations in form are just different representations of this same line.
- No other lines with different slopes pass through both points.

Practical Applications and Problem-Solving Strategies

Understanding how to find the line passing through two points is fundamental in various fields, including physics, engineering, and computer graphics.

Step-by-Step Approach for Similar Problems

1. Identify the points: Clearly note the given coordinates.
2. Calculate the slope: Use the slope formula.
3. Choose a form: Decide whether to use point-slope, slope-intercept, or standard form.
4. Plug in the point and slope: Derive the equation accordingly.
5. Simplify and convert: Express the equation in the desired form.

Common Pitfalls to Avoid

- Dividing by zero (when $x_2 = x_1$), indicating a vertical line.
- Forgetting to check if the points are the same.
- Mixing forms without proper conversion.

Special Case: Vertical and Horizontal Lines

In some problems, the given points may lead to vertical or horizontal lines.

Vertical Line Example

- For points with the same x-coordinate, e.g., (5, 2) and (5, y), the line is vertical.
- Equation: $x = 5$

Horizontal Line Example

- For points with the same y-coordinate, e.g., (x, 18) and (x, 18), the line is horizontal.
- Equation: $y = 18$

In our case, since $x_1 \neq x_2$, the line is neither vertical nor horizontal.

Conclusion: Final Summary of Equations Passing Through (5, 2) and (0, 18)

- Unique Line Equation in Slope-Intercept Form:
$$y = -\frac{16}{5}x + 18$$

- Equivalent Forms:
- Point-slope form (using either point):
$$y - 2 = -\frac{16}{5}(x - 5)$$
$$y - 18 = -$$

Frequently Asked Questions

What is the main goal when solving for equations passing through the points (5,2) and (0,18)?

The goal is to find the equations of lines or functions that pass through both given points.

How do you determine if a linear equation passes through the points (5,2) and (0,18)?

Check if the point satisfies the equation and verify that both points lie on the line by substituting their coordinates into the equation.

What is the slope of the line passing through (5,2) and (0,18)?

The slope is (18 - 2) / (0 - 5) = 16 / -5 = -16/5.

How do you find the equation of the line passing through (5,2) and (0,18)?

Use the point-slope form: $y - y_1 = m(x - x_1)$. Plug in one point and the slope to get the equation.

Which type of equations can pass through both points: linear, quadratic, or exponential?

A linear equation can pass through both points; quadratic or exponential equations generally do not unless specifically constructed.

Can the equation y = mx + b pass through both points (5,2) and (0,18)?

Yes, if the slope m and intercept b are chosen so that both points satisfy the equation.

How do you verify if a given equation passes through both points?

Substitute the x and y coordinates of each point into the equation and check if both satisfy the equation.

What are some common equations that pass through (5,2) and (0,18)?

The most common is the line y = -16/5 x + 18, derived from the two points.

Why is it important to select all equations that pass through both points for this problem?

Because multiple equations, such as different lines or functions, might pass through the points, and the problem requires identifying all valid options.

Additional Resources

Select All Of The Equations That Passes Through (5,2) And (0,18)??25 Points

---

Introduction

In the realm of algebra and coordinate geometry, the challenge of determining which equations pass through specific points is both fundamental and intellectually stimulating. The problem of identifying all equations that pass through given points—here, (5, 2) and (0, 18)—serves as a profound illustration of how algebraic principles, geometric intuition, and analytical methods intertwine. This task, often presented in academic settings, tests one's understanding of linear and nonlinear equations, their general forms, and the conditions that satisfy multiple points simultaneously.

This article delves deeply into the process of selecting equations passing through these two points. We will explore various types of equations—linear, quadratic, exponential, and others—and analyze their compatibility with the points. The discussion aims to be comprehensive, analytical, and accessible, providing clarity on the underlying mathematics and their implications. Whether you're a student seeking to grasp the concepts or a professional reviewing methodologies, this review offers a detailed journey through the problem.

Understanding the Problem: Points and Equations

Coordinates and Their Significance

The points in question are:

- Point A: (5, 2)
- Point B: (0, 18)

In coordinate geometry, each point is represented as (x, y), where x is the horizontal coordinate and y is the vertical coordinate. The problem asks us to identify all equations that pass through both points, implying that substituting the x-values into these equations should yield the corresponding y-values.

The challenge is to determine whether these equations are linear (straight lines), quadratic (parabolas), exponential, or of other forms, and to understand the conditions each must satisfy.

Types of Equations and Their Suitability

1. Linear Equations

The simplest and most common type of equations passing through two points is linear. The general form is:

$$ y = mx + c $$

where:

- m is the slope,
- c is the y-intercept.

Deriving the Equation for Points (5, 2) and (0, 18):

To find the specific line passing through these two points, we perform the following steps:

- Calculate the slope:

$$
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18 - 2}{0 - 5} = \frac{16}{-5} = -\frac{16}{5}
$$

- Determine the y-intercept c:

Using the point (0, 18):

$$
y = mx + c \Rightarrow 18 = -\frac{16}{5} \times 0 + c \Rightarrow c = 18
$$

Equation of the line:

$$
\boxed{ y = -\frac{16}{5}x + 18 }
$$

This line passes through both points, as verified by substitution.

Key Takeaways:

- The unique linear equation passing through (5, 2) and (0, 18) is $y = -\frac{16}{5}x + 18$.
- Any other linear equation passing through both points would be a multiple of this line, which isn't meaningful unless considering parametric forms or scaling.

2. Quadratic Equations

Quadratic equations have the form:

$$ y = ax^2 + bx + c $$

Given two points, infinitely many quadratics can pass through both points unless additional constraints are provided. To determine the quadratic that passes through (5, 2) and (0, 18), we need at least one more point or a specific condition.

Without an additional condition, the set of quadratics passing through these points is infinite. However, assuming we want the quadratic with specific properties or particular constraints (e.g., passing through a third point or having a specific vertex), we can derive particular solutions.

Example:

Suppose we want a quadratic passing through the points with the form:

$$
y = ax^2 + bx + c
$$

Substituting the points:

- At (0, 18):

$$
18 = a \times 0^2 + b \times 0 + c \Rightarrow c = 18
$$

- At (5, 2):

$$
2 = a \times 25 + b \times 5 + 18
$$

$$
2 - 18 = 25a + 5b
$$

$$
-16 = 25a + 5b
$$

Dividing both sides by 5:

$$
-3.2 = 5a + b
$$

Expressing b:

$$
b = -3.2 - 5a
$$

As a is arbitrary, the quadratic's form depends on parameter a. For each choice of a, b adjusts accordingly, yielding a family of quadratic equations passing through both points.

---

3. Exponential and Other Nonlinear Equations

Beyond linear and quadratic forms, one can consider exponential, logarithmic, or trigonometric functions, though their passage through specific points depends on their parameters.

Exponential Function:

$$
y = A \times e^{kx}
$$

To find A and k such that the function passes through both points:

- At (0, 18):

$$
18 = A \times e^{k \times 0} = A \Rightarrow A=18
$$

- At (5, 2):

$$
2 = 18 \times e^{5k} \Rightarrow e^{5k} = \frac{2}{18} = \frac{1}{9}
$$

$$
5k = \ln \left(\frac{1}{9}\right) = -\ln 9
$$

$$
k = -\frac{\ln 9}{5}
$$

Thus, the exponential function passing through both points is:

$$
\boxed{ y = 18 \times e^{-\frac{\ln 9}{5} \times x} }
$$

which simplifies to:

$$
y = 18 \times 9^{-\frac{x}{5}}
$$

---

4. Other Functional Forms

Other functions, such as logarithmic or sinusoidal, can be tailored to pass through the points with appropriate parameter choices, but their practical relevance depends on the context.

Validating Equations: Practical Approach

In identifying all equations passing through (5, 2) and (0, 18), the key steps involve:

- Substituting the points into the general form of the equation.
- Solving for the parameters (m, c; a, b, c; \(A \), \(k \); etc.).
- Ensuring the equations satisfy both points simultaneously.

This process often results in a family of solutions, especially for nonlinear equations, unless additional constraints are specified.

Mathematical Significance

The problem elucidates several core concepts:

- Uniqueness of linear equations: Given two points, exactly one line passes through both, represented by the derived linear equation.
- Parameterization of nonlinear equations: Multiple nonlinear functions can be designed to pass through two points, showing the flexibility but also the ambiguity without further constraints.
- The importance of additional data: To uniquely determine a quadratic or higher-order polynomial, at least three points are typically required, unless specific conditions are imposed.

Applications and Real-World Relevance

Understanding how to select equations passing through given points has myriad applications:

- Data fitting: In regression analysis, selecting models that fit specific data points.
- Engineering design: Ensuring certain constraints are met by the equations describing physical systems.
- Physics: Modeling phenomena where specific initial conditions (points) are known, and the equations describing the system are to be determined.
- Economics and finance: Forecasting models that pass through observed data points.

Challenges and Pitfalls

While the mathematics is straightforward, practical challenges include:

- Overfitting: Choosing overly complex equations that pass through points but lack predictive power.
- Ambiguity: Multiple equations can pass through the same points; selecting the most appropriate requires additional criteria.
- Parameter sensitivity: Small changes in parameters can significantly alter the behavior of nonlinear functions.

Conclusion

The exploration of equations passing through the points (5, 2) and (0, 18) reveals the versatility and depth of algebra and coordinate geometry. From the straightforward linear equation to more complex nonlinear forms, the set of solutions showcases the spectrum of mathematical modeling possibilities. The key takeaway is that the linear equation:

$$
\boxed{ y = -\frac{16}{5}x + 18 }
$$

is uniquely determined by these points, serving as a foundational example. In contrast, quadratic, exponential, and other nonlinear functions can also be tailored to pass through both points, illustrating the richness of mathematical relationships.

This analysis underscores the importance of additional constraints in selecting a specific solution, the significance of understanding the form of the equations involved, and the practical implications in various scientific and engineering fields. Ultimately, the problem exemplifies the elegance of mathematics in describing and modeling our world—showing that with just two points, an entire universe of equations can be explored, each with its own significance and application.

Select All Of The Equations That Passes Through 5 2 And 0 18 25 Points

Select All Of The Equations That Passes Through 5 2 And 0 18 25 Points

Related Articles

- The Sum Of Two Integers Is 41 And Their Difference Is 5. Find The Integers
- 32. Find A Homomorphism F From U(30) To U(30) With Kernel {1, 11} And F(7) 5 7.
- Find The Perimeter Of JKL. Round Your Answer To The Nearest Tenth If Necessary

Back to Home