Show That The Matrix Is Invertible And Find Its Inverse. A = Sin Cos Cos Sin

Show That The Matrix Is Invertible And Find Its Inverse. A = Sin Cos Cos Sin

Understanding whether a matrix is invertible and determining its inverse is a fundamental aspect of linear algebra, with applications spanning engineering, computer science, physics, and mathematics. In this comprehensive article, we will explore the process of showing that the matrix $A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$ is invertible and how to explicitly find its inverse. Through step-by-step explanations, mathematical derivations, and detailed insights, this guide aims to enhance your understanding of matrix invertibility and inverse computation, especially for matrices involving trigonometric functions.

Understanding the Matrix $A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Before delving into invertibility and inverse computation, it is essential to understand the structure and properties of the matrix A.

Matrix Components

- The matrix A is a 2x2 matrix with entries involving sine and cosine functions of an angle θ.
- Its elements are:

$$
A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}
$$

- These entries are continuous and differentiable functions, making the matrix's properties depend on the value of θ.

Relevance of the Matrix

- Such matrices often appear in rotation transformations, reflections, and in systems involving oscillatory behaviors.
- Determining invertibility is crucial because it indicates whether the matrix transformation is reversible.

Criteria for Matrix Invertibility

In linear algebra, a matrix is invertible (non-singular) if and only if its determinant is non-zero.

Determinant of a 2x2 Matrix

For a generic 2x2 matrix:
\[
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}
\]
the determinant is:
\[
\det(A) = ad - bc
\]

Applying this to our matrix:
\[
A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}
\]

we find:
\[
\det(A) = (\sin \theta)(\sin \theta) - (\cos \theta)(\cos \theta) = \sin^2 \theta - \cos^2 \theta
\]

Using Trigonometric Identities

Recall the fundamental identity:
\[
\sin^2 \theta + \cos^2 \theta = 1
\]
and the difference of squares:
\[
\sin^2 \theta - \cos^2 \theta = - (\cos^2 \theta - \sin^2 \theta)
\]
which can be expressed as:
\[
\det(A) = \sin^2 \theta - \cos^2 \theta = - \cos 2\theta
\]
since:
\[
\cos 2\theta = \cos^2 \theta - \sin^2 \theta
\]

Key Point:
\[

$$
\boxed{
\det(A) = - \cos 2\theta
}
$$

Determining When A Is Invertible

Given the determinant:

$$
\det(A) = - \cos 2\theta
$$

The matrix A is invertible if:

$$
\det(A) \neq 0 \implies - \cos 2\theta \neq 0
$$

which simplifies to:

$$
\cos 2\theta \neq 0
$$

Values of θ for Invertibility

Since $\cos 2\theta = 0$ when:

$$
2\theta = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}
$$

we have:

$$
\theta = \frac{\pi}{4} + \frac{k\pi}{2}
$$

Conclusion:
The matrix A is invertible for all θ such that:

$$
\theta \neq \frac{\pi}{4} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}
$$

Calculating the Inverse of A

Once we establish that A is invertible, we can proceed to compute its inverse explicitly.

Standard Formula for the Inverse of a 2x2 Matrix

For:

$$
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}
$$

the inverse is:

$$
A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
$$

provided $\det(A) \neq 0$.

Applying this to our matrix:

$$
A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}
$$

we get:

$$
A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} \sin \theta & -\cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}
$$

Recall:

$$
\det(A) = - \cos 2\theta
$$

Thus:

$$
A^{-1} = \frac{1}{- \cos 2\theta} \begin{bmatrix} \sin \theta & -\cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}
$$

Expressing the Inverse in Terms of θ

The inverse matrix becomes:

$$
A^{-1} = - \frac{1}{\cos 2\theta} \begin{bmatrix} \sin \theta & -\cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}
$$

Note: This inverse exists only when $\cos 2\theta \neq 0$.

Alternative Expression Using Double-Angle Identities

Since the determinant involves $\cos 2\theta$, it's often helpful to express the inverse in terms of double-angle formulas to simplify or analyze its properties.

Recall:

$$
\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta
$$

Therefore, the inverse can be written as:

$$
A^{-1} = - \frac{1}{2 \cos^2 \theta - 1} \begin{bmatrix} \sin \theta & -\cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}
$$

which provides a direct relationship between the inverse entries and the original angle θ.

Properties of the Inverse Matrix

Understanding the properties of A^{-1} helps recognize how the inverse transforms vectors and how it relates to the original matrix.

Key Properties:

- Linearity: A^{-1} preserves linear transformations, reversing the effect of A.
- Determinant: $\det(A^{-1}) = 1 / \det(A)$, so:

$$
\det(A^{-1}) = - \frac{1}{\cos 2\theta}
$$

- Symmetry: The original matrix A is symmetric if $\sin \theta = \cos \theta$, but A^{-1} generally is not symmetric unless additional conditions are met.

Practical Applications of Invertibility and Inverse Matrices

The process of verifying invertibility and calculating the inverse of matrices involving sine and cosine functions has numerous applications across different fields.

Applications Include:

- Rotation Transformations: In 2D graphics and robotics, matrices similar to \(A\) are used to rotate vectors. Knowing the inverse helps in undoing rotations.
- Signal Processing: In Fourier analysis, trigonometric matrices often appear, and invertibility is crucial for signal reconstruction.
- Physics: In systems involving oscillatory motion, matrices with sinusoidal entries model transformations, and invertibility ensures solvability.
- Control Theory: Invertible matrices are essential for system controllability and observability analyses.

Summary: Key Takeaways

- The matrix \(A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}\) is invertible if and only if \(\cos 2\theta \neq 0\).
- The determinant of \(A\) is \(\det(A) =

Frequently Asked Questions

How do I determine if the matrix A = [[$\sin\theta$, $\cos\theta$], [$\cos\theta$, $\sin\theta$]] is invertible?

To check if A is invertible, compute its determinant. For A = [[$\sin\theta$, $\cos\theta$], [$\cos\theta$, $\sin\theta$]], the determinant is $(\sin\theta)(\sin\theta) - (\cos\theta)(\cos\theta) = \sin^2\theta - \cos^2\theta$. If this determinant is non-zero, A is invertible.

What is the formula for the inverse of a 2x2 matrix, and how can it be applied to matrix A?

For a 2x2 matrix [[a, b], [c, d]], the inverse is (1/det) [[d, -b], [-c, a]], provided det $\neq$ 0. Applying this to A, compute the determinant first, then substitute the elements into the formula.

How do I compute the determinant of A = [[$\sin\theta$, $\cos\theta$], [$\cos\theta$, $\sin\theta$]]?

The determinant is calculated as $(\sin\theta)(\sin\theta) - (\cos\theta)(\cos\theta) = \sin^2\theta - \cos^2\theta$. This simplifies to $-\cos(2\theta)$ using the double angle identity.

For which values of θ is the matrix A invertible?

A is invertible when its determinant is not zero, i.e., when $\cos(2\theta) \neq 0$. This occurs when $2\theta \neq \pi/2 + k\pi$, so $\theta \neq \pi/4 + k\pi/2$ for any integer k.

How do I explicitly find the inverse of matrix A = [[sinθ, cosθ], [cosθ, sinθ]]?

First, find the determinant $D = \sin^2\theta - \cos^2\theta$. Then, the inverse is (1/D) [[sinθ, -cosθ], [-cosθ, sinθ]]. Ensure $D \neq 0$ before computing.

Can you provide the inverse matrix A^{-1} for specific θ values?

Yes. For example, if $\theta = 0$, then A = [[0, 1], [1, 0]], and its inverse is the same matrix. For general θ, plug in the value into the formula: $A^{-1} = (1/(-\cos(2\theta)))$ [[sinθ, -cosθ], [-cosθ, sinθ]], provided $\cos(2\theta) \neq 0$.

What role do trigonometric identities play in finding the inverse of A?

Trigonometric identities, especially the double angle formulas like $\cos(2\theta) = \cos^2\theta - \sin^2\theta$, simplify the determinant calculation and help express the inverse explicitly.

Are there cases where the matrix A becomes non-invertible, and what does that imply?

Yes, when $\cos(2\theta) = 0$, the determinant is zero, and A is not invertible. This indicates the matrix is singular, meaning it does not have an inverse, often corresponding to specific angles where the rows are linearly dependent.

How can I verify the correctness of the inverse matrix once I compute it?

Multiply the original matrix A by its computed inverse A^{-1}. If the result is the identity matrix, then the inverse is correct. This confirms the inverse satisfies $A A^{-1} = I$.

Additional Resources

Show That The Matrix Is Invertible And Find Its Inverse: A Step-by-Step Guide for the Matrix A = Sin Cos Cos Sin

Understanding whether a matrix is invertible and, if so, how to find its inverse is a fundamental aspect of linear algebra. This process is crucial in solving systems of linear equations, transforming vectors, and analyzing linear transformations. In this article, we will explore how to show that the matrix A = Sin Cos Cos Sin is invertible and find its inverse through a detailed, step-by-step approach. Although the notation might seem abstract at first glance, we will clarify each step to build a comprehensive understanding of the process.

Introduction to Matrix Invertibility

Before diving into calculations, it's essential to understand what it means for a matrix to be invertible.

What is an Invertible Matrix?

A square matrix A is invertible (or non-singular) if there exists another matrix, A-1, such that:

$$ AA^{-1} = A^{-1}A = I $$

where I is the identity matrix of the same size as A.

Why is Invertibility Important?

- It guarantees the unique solution of the linear system $ Ax = b $, for any vector b.
- It indicates that the linear transformation represented by A is bijective (one-to-one and onto).
- In many applications, the inverse allows us to "undo" transformations, making it a powerful tool.

Criteria for Invertibility

- For a 2×2 matrix, A is invertible if and only if its determinant is non-zero.
- For larger matrices, the same principle applies: if $\det(A) \neq 0$, then A is invertible.

Clarifying the Matrix A = Sin Cos Cos Sin

The notation A = Sin Cos Cos Sin appears to be shorthand or symbolic. To proceed with the analysis, we need to interpret this notation as a concrete matrix.

Interpreting the Notation

- The phrase suggests a 2×2 matrix with elements involving sine and cosine functions.
- It could be representing:

$$
A = \begin{bmatrix}
\sin \theta & \cos \theta \\
\cos \theta & \sin \theta
\end{bmatrix}
$$

where θ is some real parameter.

Assumption for the Analysis

Based on common patterns, we will assume that:

$$
A = \begin{bmatrix}
\sin \theta & \cos \theta \\
\cos \theta & \sin \theta
\end{bmatrix}

\]

This is a standard form of a 2×2 matrix involving sine and cosine functions, often related to rotation matrices or similar transformations.

Step 1: Verify that A is a 2×2 Matrix

Given the assumed form:

\[
A = \begin{bmatrix}
\sin \theta & \cos \theta \\
\cos \theta & \sin \theta
\end{bmatrix}
\]

- It is a 2×2 matrix.
- The entries are real-valued functions of \(\theta\).

Step 2: Calculate the Determinant of A

The key to determining invertibility is calculating the determinant:

\[
\det(A) = (\sin \theta)(\sin \theta) - (\cos \theta)(\cos \theta)
\]

which simplifies to:

\[
\det(A) = \sin^2 \theta - \cos^2 \theta
\]

Recall the trigonometric identity:

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

and the difference of squares:

\[
\sin^2 \theta - \cos^2 \theta = -(\cos^2 \theta - \sin^2 \theta) = -\cos 2\theta
\]

Thus,

\[
\det(A) = - \cos 2\theta
\]

Step 3: Determine When A is Invertible

- The matrix A is invertible if and only if \(\det(A) \neq 0\).
- That is:

\[
- \cos 2\theta \neq 0
\]

which simplifies to:

\[
\cos 2\theta \neq 0
\]

- The cosine function equals zero at:

\[
2\theta = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}
\]

Therefore, the matrix A is invertible for all \(\theta\) such that:

\[
2\theta \neq \frac{\pi}{2} + k\pi \quad \Rightarrow \quad \theta \neq \frac{\pi}{4} + \frac{k\pi}{2}
\]

- Conversely, A is not invertible when:

\[
\theta = \frac{\pi}{4} + \frac{k\pi}{2}
\]

Step 4: Find the Inverse of A

Assuming \(\det(A) \neq 0\), the inverse of a 2×2 matrix:

\[
A = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]

is given by:

\[
A^{-1} = \frac{1}{\det(A)} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

Applying to Our Matrix

Recall:

\[
A = \begin{bmatrix}
\sin \theta & \cos \theta \\
\cos \theta & \sin \theta
\end{bmatrix}
\]

- \(a = \sin \theta\)
- \(b = \cos \theta\)
- \(c = \cos \theta\)
- \(d = \sin \theta\)

The inverse is:

\[
A^{-1} = \frac{1}{-\cos 2\theta} \begin{bmatrix}
\sin \theta & -\cos \theta \\
-\cos \theta & \sin \theta
\end{bmatrix}
\]

which simplifies to:

\[
A^{-1} = -\frac{1}{\cos 2\theta} \begin{bmatrix}
\sin \theta & -\cos \theta \\
-\cos \theta & \sin \theta
\end{bmatrix}
\]

Summary of Results

- Invertibility condition:

\[
\boxed{

\text{A is invertible if } \cos 2\theta \neq 0
}
\]

- Inverse matrix:

\[
\boxed{
A^{-1} = -\frac{1}{\cos 2\theta} \begin{bmatrix}
\sin \theta & -\cos \theta \\
-\cos \theta & \sin \theta
\end{bmatrix}
}
\]

Additional Insights and Applications

Connection to Rotation Matrices

Interestingly, the matrix resembles a symmetric form involving sine and cosine, but it is not a standard rotation matrix unless specific conditions are met. For example, a standard rotation matrix:

\[
R(\phi) = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix}
\]

has determinant 1 and is always invertible. Our matrix's invertibility depends on \(\cos 2\theta\), which connects to double-angle identities.

Special Cases

- When \(\theta = 0\):

\[
A = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

which has determinant \(-1\), so invertible, with inverse:

\[
A^{-1} = -\frac{1}{-1} \begin{bmatrix}
0 & -1 \\
-1 & 0

\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

- When \(\theta = \pi/4\):

\[
\cos 2\theta = \cos \frac{\pi}{2} = 0
\]

so A is not invertible at this angle.

Final Remarks

Determining whether a matrix is invertible involves calculating its determinant and analyzing when it is non-zero. In our case, the matrix involving sine and cosine functions hinges on the double-angle identity, which succinctly captures the invertibility condition. Once confirmed, the inverse can be explicitly computed using the standard formula for 2×2 matrices.

Understanding these concepts and steps provides a solid foundation for tackling more complex matrices and transformations in linear algebra. Whether dealing with rotation matrices, scaling, or other linear transformations, the principles outlined here—calculating determinants, checking invertibility conditions, and applying inverse formulas—are universally applicable.

Key Takeaways

- The invertibility of a 2×2 matrix depends on its determinant being non-zero.
- For matrices involving sine and cosine functions, identities like \(\sin^2 \theta + \cos^2 \theta = 1\) and \(\cos 2\theta = \cos^2 \theta - \sin^2 \theta\) simplify calculations.
- The inverse matrix, when it exists, can be explicitly written and involves the reciprocal of the determinant

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