Solve Each System Of Linear Equations Using Matrices!4x-2y=25x-2y+z=73x+4y-z=3

Solve Each System Of Linear Equations Using Matrices!4x-2y=25x-2y+z=73x+4y-z=3

Solving systems of linear equations is a fundamental skill in mathematics, especially in algebra and linear algebra, with broad applications in engineering, physics, computer science, economics, and more. When systems involve multiple variables and equations, solving them manually can become tedious and error-prone. This is where matrices come into play, offering a systematic, efficient, and elegant method for solving such systems.

In this article, we will explore how to solve the following system of linear equations using matrices:

1. $4x - 2y = 2$
2. $5x - 2y + z = 7$
3. $3x + 4y - z = 3$

By the end, you will understand the step-by-step process of representing the system in matrix form, using methods like Gaussian elimination or matrix inversion to find the solutions for variables x, y, and z.

Understanding the System of Equations

Before diving into matrix methods, it’s essential to understand the system itself:

1. Equation 1: $4x - 2y = 2$
2. Equation 2: $5x - 2y + z = 7$
3. Equation 3: $3x + 4y - z = 3$

This system contains three equations with three variables: x, y, and z. The goal is to find the values of these variables that satisfy all three equations simultaneously.

Representing the System in Matrix Form

Matrix methods require expressing the system in matrix form as $AX = B$, where:

- A is the coefficient matrix,
- X is the column vector of variables,
- B is the constants vector.

Step 1: Write the coefficients of the variables from each equation:

Equation	x	y	z	Constant

1	4	-2	0	2
2	5	-2	1	7
3	3	4	-1	3

Step 2: Formulate matrix A, vector X, and vector B:

\[
A = \begin{bmatrix}
4 & -2 & 0 \\
5 & -2 & 1 \\
3 & 4 & -1 \\
\end{bmatrix},
\quad
X = \begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix},
\quad
B = \begin{bmatrix}
2 \\
7 \\
3 \\
\end{bmatrix}
\]

The system is now written as:

\[
\mathbf{A} \times \mathbf{X} = \mathbf{B}
\]

Methods to Solve the System Using Matrices

There are several matrix-based methods to solve the system:

- Gaussian Elimination / Gauss-Jordan Elimination
- Matrix Inversion Method (if the matrix is invertible)
- Cramer's Rule (for small systems)

In this article, we'll focus on the matrix inversion method and Gaussian elimination.

Method 1: Using Matrix Inversion

The solution for X can be obtained as:

\[

$$
\mathbf{X} = \mathbf{A}^{-1} \times \mathbf{B}
$$

provided that A is invertible (i.e., has a non-zero determinant).

Step 1: Calculate the determinant of matrix A.

$$
\det(A) = 4 \times \begin{vmatrix} -2 & 1 \\ 4 & -1 \end{vmatrix} - (-2) \times \begin{vmatrix} 5 & 1 \\ 3 & -1 \end{vmatrix} + 0 \times \dots
$$

Calculating:

$$
\det(A) = 4 \times (-2 \times -1 - 1 \times 4) - (-2) \times (5 \times -1 - 1 \times 3) + 0
$$

$$
= 4 \times (2 - 4) - (-2) \times (-5 - 3)
$$

$$
= 4 \times (-2) - (-2) \times (-8)
$$

$$
= -8 - 16 = -24
$$

Since $\det(A) \neq 0$, the inverse exists.

Step 2: Find A^{-1}, the inverse of matrix A.

The inverse of a 3x3 matrix involves calculating the matrix of cofactors, then transposing it (to get the adjugate), and dividing by the determinant.

Note: Due to the complexity, in practice, you would use a calculator or computational tool (like MATLAB, NumPy in Python, or a graphing calculator) to compute the inverse.

Step 3: Multiply A^{-1} with B to get X.

Alternatively, for manual calculations, Gaussian elimination is often more straightforward.

Method 2: Gaussian Elimination

This method involves transforming the augmented matrix into row echelon form to solve for variables via back-substitution.

Step 1: Write the augmented matrix:

$$
\left[
\begin{array}{ccc|c}
4 & -2 & 0 & 2 \\
5 & -2 & 1 & 7 \\
3 & 4 & -1 & 3 \\
\end{array}
\right]
$$

Step 2: Use row operations to simplify.

- Divide Row 1 by 4 to make leading 1:

$$
R_1 \leftarrow R_1/4
$$

$$
\left[
\begin{array}{ccc|c}
1 & -0.5 & 0 & 0.5 \\
5 & -2 & 1 & 7 \\
3 & 4 & -1 & 3 \\
\end{array}
\right]
$$

- Eliminate the x-term in rows 2 and 3:

$$
R_2 \leftarrow R_2 - 5 \times R_1
$$
$$
R_3 \leftarrow R_3 - 3 \times R_1
$$

Calculations:

- For R2:

$$
5 - 5 \times 1 = 0
$$
$$
-2 - 5 \times (-0.5) = -2 + 2.5 = 0.5
$$
$$
1 - 5 \times 0 = 1
$$

$$
7 - 5 \times 0.5 = 7 - 2.5 = 4.5
$$

- For R3:

$$
3 - 3 \times 1 = 0
$$
$$
4 - 3 \times (-0.5) = 4 + 1.5 = 5.5
$$
$$
-1 - 3 \times 0 = -1
$$
$$
3 - 3 \times 0.5 = 3 - 1.5 = 1.5
$$

Updated augmented matrix:

$$
\left[
\begin{array}{ccc|c}
1 & -0.5 & 0 & 0.5 \\
0 & 0.5 & 1 & 4.5 \\
0 & 5.5 & -1 & 1.5 \\
\end{array}
\right]
$$

- Next, eliminate y in row 3:

$$
R_3 \leftarrow R_3 - \left(\frac{5.5}{0.5}\right) \times R_2 = R_3 - 11 \times R_2
$$

Calculations:

$$
R_3:\ 0,\ 5.5 - 11 \times 0.5 = 5.5 - 5.5 = 0
$$
$$
-1 - 11 \times 1 = -1 - 11 = -12
$$
$$
1.5 - 11 \times 4.5 = 1.5 - 49.5 = -48
$$

Updated matrix:

$$
\left[
\begin{array}{ccc|c}
1 & -0.5 & 0 & 0.5 \\
0 & 0.5 & 1 & 4.5 \\
0 & 0 & -12 & -48 \\
\end{array}
\right]
$$

Step 3: Back substitution:

- From row 3:

$$
-12z = -48 \Rightarrow z = \frac{-48}{-12} = 4
$$

- From row 2:

$$
0.5 y + z = 4.5 \Rightarrow 0.5 y + 4 = 4.5 \Rightarrow 0.5 y = 0.5 \Rightarrow y = 1
$$

- From row 1:

\[
x - 0.5 y = 0.5 \

Frequently Asked Questions

What is the first step to solve the system of equations using matrices?

The first step is to write the system in matrix form as AX = B, where A is the coefficient matrix, X is the variable matrix, and B is the constants matrix.

What are the coefficient matrix, variable matrix, and constants matrix for the given system?

The coefficient matrix A is [[4, -2, 0], [1, 4, -1], [3, 2, -1]], the variable matrix X is [[x], [y], [z]], and the constants matrix B is [[25], [7], [3]].

How do you find the solution to the system using matrix

methods?

You can find the solution by calculating X = A^{-1} B, where A^{-1} is the inverse of matrix A, provided that A is invertible.

How do you verify if the matrix A is invertible?

Calculate the determinant of matrix A; if the determinant is non-zero, A is invertible, and you can proceed to find its inverse.

What is the inverse of the coefficient matrix A in this problem?

You find the inverse by using methods such as row reduction or the adjugate and determinant. Once calculated, it can be multiplied by B to find the solution.

What is the solution to the system after performing matrix calculations?

After computing, the solution is approximately $x \approx 4$, $y \approx 3$, and $z \approx -1$.

Can this system be solved using other methods besides matrices?

Yes, other methods include substitution and elimination, but matrices provide a systematic approach, especially for larger systems.

Why is it important to check the determinant before solving with matrices?

Checking the determinant ensures that the coefficient matrix is invertible; if it's zero, the system may have no unique solution or infinitely many solutions.

What are the practical applications of solving systems of equations using matrices?

This method is widely used in engineering, physics, computer science, and economics to analyze and solve real-world problems involving multiple variables and constraints.

Additional Resources

Solve Each System of Linear Equations Using Matrices! is a fundamental skill in algebra that allows us to handle complex systems efficiently and systematically. Whether you're a student trying to master the basics or a professional applying these techniques in engineering or data science, understanding how to solve systems of equations using matrices is invaluable. In this article, we'll walk through a detailed, step-by-step guide to solving the following system using matrix methods:

\[
\begin{cases}
4x - 2y = 25 \\
x - 2y + z = 7 \\
3x + 4y - z = 3
\end{cases}
\]

(Note: The original example had some inconsistencies in the equations; here, we'll clarify and correct them for clarity and consistency.)

Introduction to Solving Systems of Equations with Matrices

When dealing with multiple equations involving multiple variables, traditional substitution or elimination methods can become cumbersome. Matrices provide an organized and scalable approach, especially suited for larger systems.

Why Use Matrices?

- Efficiency: Matrices streamline calculations, especially with the aid of calculators or software.
- Clarity: They organize equations systematically.
- Scalability: Handling larger systems becomes manageable.

Basic Concepts

- Coefficient Matrix (A): Encodes the coefficients of variables.
- Variable Matrix (X): Contains variables.
- Constants Matrix (B): Contains the constants on the right side.

The system can be written in matrix form as:

\[
A \mathbf{x} = \mathbf{b}
\]

Where:

- \(A\) is the coefficient matrix.
- \(\mathbf{x}\) is the column vector of variables.
- \(\mathbf{b}\) is the constants vector.

Step 1: Write the System in Matrix Form

Given the corrected system:

\[
\begin{cases}

4x - 2y + 0z = 25 \\
1x - 2y + 1z = 7 \\
3x + 4y - 1z = 3
\end{cases}
\]

Identify the coefficients:

Equation	x	y	z	Constant
1	4	-2	0	25
2	1	-2	1	7
3	3	4	-1	3

Now, construct the matrices:

- Coefficient matrix A:

$$
A = \begin{bmatrix}
4 & -2 & 0 \\
1 & -2 & 1 \\
3 & 4 & -1
\end{bmatrix}
$$

- Variables vector $\mathbf{x}$:

$$
\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
$$

- Constants vector $\mathbf{b}$:

$$
\mathbf{b} = \begin{bmatrix} 25 \\ 7 \\ 3 \end{bmatrix}
$$

Step 2: Determine the Method to Solve

The main matrix methods include:

- Gaussian Elimination / Row Reduction: Systematic process to reduce the augmented matrix to row-echelon form.
- Inverse Matrix Method: If A is invertible, then

$$
\mathbf{x} = A^{-1} \mathbf{b}
$$

For this guide, we'll focus on the inverse matrix method for clarity, but note that Gaussian elimination is equally valid and often more practical for larger systems.

Step 3: Calculate the Inverse of the Coefficient Matrix

3.1: Check if \(A\) is invertible

Calculate the determinant \(\det(A)\):

\[
\det(A) = 4 \times \begin{vmatrix} -2 & 1 \\ 4 & -1 \end{vmatrix}
- (-2) \times \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}
+ 0 \times \text{(remaining minor)}
\]

Calculate minors:

\[
\begin{aligned}
\begin{vmatrix} -2 & 1 \\ 4 & -1 \end{vmatrix} &= (-2)(-1) - (1)(4) = 2 - 4 = -2 \\
\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} &= (1)(-1) - (1)(3) = -1 - 3 = -4
\end{aligned}
\]

Compute determinant:

\[
\det(A) = 4 \times (-2) - (-2) \times (-4) + 0 \times (\text{something}) = -8 - 8 + 0 = -16
\]

Since \(\det(A) \neq 0\), the matrix is invertible.

3.2: Find \(A^{-1}\)

The inverse of a 3x3 matrix involves calculating the matrix of cofactors, then the adjugate, and dividing by the determinant.

Step-by-step:

1. Find all minors of each element.
2. Calculate cofactors by applying signs.
3. Transpose the cofactor matrix to get the adjugate.
4. Divide the adjugate by \(\det(A)\).

Due to complexity, here is the direct calculation of \(A^{-1}\):

After detailed computation (which can be done with calculator or software), the inverse matrix is:

\[

A^{-1} = \frac{1}{-16} \times
\begin{bmatrix}
-6 & -2 & 2 \\
-11 & 2 & 3 \\
-2 & -3 & 4
\end{bmatrix}
\]

Simplify:

\[
A^{-1} = \begin{bmatrix}
\frac{6}{16} & \frac{2}{16} & -\frac{2}{16} \\
\frac{11}{16} & -\frac{2}{16} & -\frac{3}{16} \\
\frac{2}{16} & \frac{3}{16} & -\frac{4}{16}
\end{bmatrix}
= \begin{bmatrix}
\frac{3}{8} & \frac{1}{8} & -\frac{1}{8} \\
\frac{11}{16} & -\frac{1}{8} & -\frac{3}{16} \\
\frac{1}{8} & \frac{3}{16} & -\frac{1}{4}
\end{bmatrix}
\]

Step 4: Multiply \(A^{-1}\) by \(\mathbf{b}\) to Find \(\mathbf{x}\)

Now, compute:

\[
\mathbf{x} = A^{-1} \mathbf{b}
\]

Perform the matrix multiplication:

\[
\begin{bmatrix}
x \\ y \\ z
\end{bmatrix}
=
\begin{bmatrix}
\frac{3}{8} & \frac{1}{8} & -\frac{1}{8} \\
\frac{11}{16} & -\frac{1}{8} & -\frac{3}{16} \\
\frac{1}{8} & \frac{3}{16} & -\frac{1}{4}
\end{bmatrix}
\begin{bmatrix}
25 \\ 7 \\ 3
\end{bmatrix}
\]

4.1: Compute each component

For \(x\):

\[
x = \left(\frac{3}{8} \times 25\right) + \left(\frac{1}{8} \times 7\right) + \left(-\frac{1}{8} \times 3\right)
\]

Calculations:

\[
x = \frac{75}{8} + \frac{7}{8} - \frac{3}{8} = \frac{75 + 7 - 3}{8} = \frac{79}{8} = 9.875
\]

For \(y\):

\[
y = \left(\frac{11}{16} \times 25\right) + \left(-\frac{1}{8} \times 7\right) + \left(-\frac{3}{16} \times 3\right)
\]

Calculations:

\[
\frac{275}{16} - \frac{7}{8} - \frac{9}{16}
\]

Express all with denominator 16:

\[
\frac{275}{16} - \frac{14}{16} - \frac{9}{16} = \frac{275 - 14 - 9}{16} = \frac{252}{16} = 15.75
\]

For \(z\):

\[
z = \left(\frac{1}{8} \times 25\right) + \left(\frac{3}{16} \times 7\right) + \

Solve Each System Of Linear Equations Using Matrices 4x 2y 25x 2y Z 73x 4y Z 3

Solve Each System Of Linear Equations Using Matrices 4x 2y 25x 2y Z 73x 4y Z 3

Related Articles

- A Story That End With Statement: I Had Never Felt So Embarrassed In My Life
- 10 Points For Two Questions Please Dont Write Anything Just To Take My Points
- What Is Technology Commercialization? Explain Briefly How It Can Be Achieved

Back to Home