Solve The Following System Of Equations Algebraically: 3x + 2y = 4 4x + 3y = 7

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Understanding how to solve systems of equations algebraically is a fundamental skill in mathematics, especially in algebra. It enables students and professionals to find the values of variables that satisfy multiple equations simultaneously. In this article, we will explore the step-by-step process to solve the system:

\[
\begin{cases}
3x + 2y = 4 \\
4x + 3y = 7
\end{cases}
\]

by employing algebraic methods such as substitution and elimination. Additionally, we will discuss the importance of these methods, their applications, and some tips for avoiding common mistakes.

Understanding Systems of Equations

A system of equations consists of two or more equations with the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

Types of Solutions

- Unique Solution: When the equations intersect at a single point, resulting in specific values for variables.
- Infinite Solutions: When the equations represent the same line, leading to infinitely many solutions.
- No Solution: When the lines are parallel and do not intersect, indicating no possible solution.

In our case, we are aiming to find the specific values of \(x\) and \(y\) that satisfy both equations.

Methods for Solving Systems of Equations Algebraically

There are primarily two methods for solving systems algebraically:

1. Substitution Method
2. Elimination Method

Each method has its advantages and is suitable depending on the structure of the system.

Substitution Method

This method involves solving one of the equations for one variable and substituting that expression into the other equation. It is particularly useful when one equation is already solved for a variable or can easily be rearranged.

Elimination Method

This technique involves adding or subtracting equations to eliminate one variable, making it easier to solve for the remaining variable. It is effective when the coefficients of one variable are the same or opposites.

Step-by-Step Solution of the System

Let's now apply these methods to our specific system:

$$
\begin{cases}
3x + 2y = 4 \quad (1) \\
4x + 3y = 7 \quad (2)
\end{cases}
$$

Using the Elimination Method

Step 1: Prepare equations for elimination

We want to eliminate one variable. To do so, we can align coefficients of either x or y. Let's eliminate x.

To do this, find the least common multiple (LCM) of the coefficients of x: 3 and 4. The LCM is 12.

Step 2: Equalize the coefficients

Multiply equation (1) by 4:

$$
4 \times (3x + 2y) = 4 \times 4 \Rightarrow 12x + 8y = 16 \quad (3)
$$

Multiply equation (2) by 3:

$$
3 \times (4x + 3y) = 3 \times 7 \Rightarrow 12x + 9y = 21 \quad (4)
$$

Step 3: Subtract equations to eliminate x

Subtract equation (3) from equation (4):

$$
(12x + 9y) - (12x + 8y) = 21 - 16
$$

$$
12x - 12x + 9y - 8y = 5
$$

$$
0x + y = 5
$$

Thus, we find:

$$
\boxed{y = 5}
$$

Step 4: Substitute $y = 5$ back into one of the original equations

Use equation (1):

$$
3x + 2(5) = 4
$$

$$
3x + 10 = 4
$$

$$
3x = 4 - 10 = -6
$$

$$
x = -2
$$

Final Solution:

$$
\boxed{
x = -2, \quad y = 5
}
$$

This point $(-2, 5)$ is the solution where both equations intersect.

Verifying the Solution

Verification ensures the solution satisfies both equations.

- Substitute $x = -2$ and $y = 5$ into equation (2):

$$
4(-2) + 3(5) = -8 + 15 = 7
$$

which matches the right side of equation (2).

- Similarly, check equation (1):

$$
3(-2) + 2(5) = -6 + 10 = 4
$$

which matches the right side of equation (1). Both equations are satisfied, confirming the solution is correct.

Additional Methods and Tips for Solving Systems

While the elimination method worked efficiently here, sometimes substitution or graphical methods are more suitable depending on the problem's structure.

Substitution Method Revisited

Suppose one of the equations is already solved for a variable. For example, if the first equation was:

$$ 3x + 2y = 4 $$

Rearranged as:

$$ y = \frac{4 - 3x}{2} $$

Substituting into the second equation:

$$ 4x + 3 \left(\frac{4 - 3x}{2} \right) = 7 $$

This approach can sometimes simplify calculations, especially when one variable is isolated.

Graphical Method (Brief Overview)

Plotting both equations on a coordinate plane visually shows the intersection point, corresponding to the solution. However, this method is less precise unless done carefully and is mainly used for approximate solutions.

Common Mistakes to Avoid

When solving systems algebraically, be aware of common pitfalls:

- Sign errors: Always double-check signs during addition or subtraction.
- Incorrect multiplication: Ensure each term in an equation is multiplied accurately.
- Misaligned coefficients: When eliminating variables, properly align coefficients to avoid errors.
- Forgetting to verify solutions: Always substitute back into original equations to verify correctness.

Applications of Solving Systems of Equations

Solving systems algebraically has practical applications across various fields:

- Physics: Calculating intersection points of forces or trajectories.
- Economics: Finding equilibrium points where supply equals demand.
- Engineering: Solving systems related to circuit analysis or structural calculations.
- Computer Science: Algorithm development involving simultaneous equations.

Conclusion

Mastering algebraic methods to solve systems of equations—like the one involving \(3x +

2y = 4\) and \(4x + 3y = 7\)—is essential for advancing in mathematics and applied sciences. Understanding the elimination and substitution techniques allows for efficient problem-solving and analytical reasoning. Remember to verify solutions and be cautious with calculations to avoid common errors. With practice, solving systems algebraically becomes an intuitive process that enhances your analytical skills and mathematical understanding.

Keywords: solve system of equations algebraically, elimination method, substitution method, linear equations, algebra, solving for variables, mathematics, equations, solution, verification

Frequently Asked Questions

How do I solve the system of equations 3x + 2y = 4 and 4x + 3y = 7 algebraically?

You can use either substitution or elimination. For elimination, multiply equations to align coefficients and subtract to find one variable, then substitute back to find the other.

What is the first step to solve the system 3x + 2y = 4 and 4x + 3y = 7 algebraically?

Choose a method such as elimination or substitution; for elimination, multiply equations to match coefficients for one variable.

How can I apply the elimination method to solve 3x + 2y = 4 and 4x + 3y = 7?

Multiply the first equation by 3 and the second by 2 to align coefficients of y, then subtract the equations to eliminate y and solve for x.

Can substitution be used to solve the system 3x + 2y = 4 and 4x + 3y = 7?

Yes, solve one equation for one variable (e.g., y in terms of x) and substitute into the other to find the value of x.

What are the solutions for x and y in the system 3x + 2y = 4 and 4x + 3y = 7?

The solutions are x = 1 and y = 0.5.

How do I verify the solution to the system 3x + 2y = 4 and 4x + 3y = 7?

Substitute the found values of x and y into both original equations to check if both equations are satisfied.

What is the importance of solving systems of equations algebraically?

It allows you to find exact values of variables that satisfy multiple equations simultaneously, useful in many real-world applications.

Is there a shortcut to solve the system 3x + 2y = 4 and 4x + 3y = 7 without multiplying?

You can use substitution directly by solving one equation for a variable and substituting into the other, avoiding multiplication.

What is the solution set for the system 3x + 2y = 4 and 4x + 3y = 7?

The unique solution is x = 1 and y = 0.5, representing a single intersection point.

How can I write the solution to the system of equations in coordinate form?

The solution in coordinate form is (x, y) = (1, 0.5).

Additional Resources

Solve The Following System Of Equations Algebraically: 3x + 2y = 4 and 4x + 3y = 7

When faced with a system of equations like 3x + 2y = 4 and 4x + 3y = 7, solving algebraically involves finding the values of the variables that satisfy both equations simultaneously. This process is fundamental in algebra and forms the basis for understanding how multiple equations relate to each other. Whether you're tackling homework, preparing for exams, or just aiming to sharpen your algebra skills, understanding the methods to solve such systems is essential. In this guide, we'll explore various algebraic techniques—substitution, elimination, and graphical interpretation—in a detailed, step-by-step manner to ensure clarity and confidence in solving this system.

Understanding the System of Equations

Before diving into the methods, let's analyze what we are dealing with:

- Equation 1: $3x + 2y = 4$
- Equation 2: $4x + 3y = 7$

These are linear equations in two variables, x and y. Our goal is to find the pair (x, y) that satisfies both equations simultaneously.

Methods to Solve the System Algebraically

There are primarily three methods to solve such systems:

1. Substitution Method
2. Elimination Method
3. Graphical Method (more visual, but often complemented by algebraic methods)

For the purpose of this guide, we'll focus on the first two algebraic techniques, as they are most effective for exact solutions.

Step-by-Step Solution Using the Substitution Method

Step 1: Solve one equation for one variable

Choose either equation, and solve for one variable in terms of the other.

Let's solve Equation 1 for y:

$3x + 2y = 4$
$\rightarrow 2y = 4 - 3x$
$\rightarrow y = (4 - 3x) / 2$

Step 2: Substitute into the other equation

Plug this expression for y into Equation 2:

$4x + 3y = 7$

Substituting y:

$4x + 3 [(4 - 3x) / 2] = 7$

Step 3: Simplify and solve for x

Multiply through to clear the denominator:

$4x + (3/2) (4 - 3x) = 7$

Distribute:

$4x + (3/2)4 - (3/2)3x = 7$

Calculate each term:

$4x + (12/2) - (9/2)x = 7$

Simplify:

$4x + 6 - (9/2)x = 7$

Now, combine like terms. To combine the x-terms, convert 4x to a fraction with denominator 2:

$(8/2)x + 6 - (9/2)x = 7$

Combine the x-terms:

$[(8/2) - (9/2)]x + 6 = 7$
$(-1/2)x + 6 = 7$

Subtract 6 from both sides:

$(-1/2)x = 1$

Multiply both sides by -2 to solve for x:

$x = -2\ 1$
$x = -2$

Step 4: Find y using the value of x

Recall $y = (4 - 3x) / 2$

Plug in $x = -2$:

$y = (4 - 3 (-2)) / 2$
$= (4 + 6) / 2$
$= 10 / 2$
$= 5$

Solution:

$x = -2, y = 5$

Step-by-Step Solution Using the Elimination Method

Step 1: Align the equations for elimination

Write the equations:

Equation 1: $3x + 2y = 4$
Equation 2: $4x + 3y = 7$

Step 2: Equalize coefficients for one variable

To eliminate one variable, we can multiply the equations to have the same coefficient (but opposite signs). Let's eliminate y:

Multiply Equation 1 by 3:

$(3x + 2y)\ 3 \rightarrow 9x + 6y = 12$

Multiply Equation 2 by -2:

$(4x + 3y)\ -2 \rightarrow -8x - 6y = -14$

Now, add the two equations:

$(9x + 6y) + (-8x - 6y) = 12 + (-14)$

Simplify:

$(9x - 8x) + (6y - 6y) = -2$

$x + 0 = -2$

Therefore:

$x = -2$

Step 3: Substitute x back into one of the original equations

Using Equation 1:

$3x + 2y = 4$

Plug in $x = -2$:

$3\ (-2) + 2y = 4$
$-6 + 2y = 4$

Add 6 to both sides:

$2y = 10$

Divide both sides by 2:

$y = 5$

Final Solution:

- $x = -2$
- $y = 5$

Both methods lead to the same solution, confirming its accuracy.

Additional Insights and Tips

Verifying the Solution

Always verify the solution by substituting the values into both original equations:

- For Equation 1:

$3(-2) + 2(5) = -6 + 10 = 4$ ✓

- For Equation 2:

$4(-2) + 3(5) = -8 + 15 = 7$ ✓

Since both equations are satisfied, the solution is correct.

Handling More Complex Systems

While this system is straightforward, more complex systems may involve:

- Larger coefficients
- Fractions
- Nonlinear terms

In such cases, methods like substitution and elimination can be adapted or combined with matrix methods (like Gaussian elimination) for efficiency.

Practice Problems

To deepen understanding, try solving these systems:

1. $2x + y = 5$ and $x - y = 1$
2. $x/2 + y/3 = 4$ and $3x - 2y = 6$
3. $5x + 2y = 10$ and $3x + 4y = 12$

Summary

Solving a system of equations algebraically involves choosing the most suitable method—substitution or elimination—to find the common solution for the variables. The process requires careful algebraic manipulation, attention to detail, and verification of results. Practice with different systems enhances problem-solving skills and builds confidence in applying these techniques to real-world scenarios or more advanced mathematical problems.

By mastering these methods, you'll be well-equipped to tackle a wide variety of linear systems, paving the way for success in algebra and beyond.

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