Solve The Given Differential Equation By Separation Of Variables. Ds Dr = Ks

Solve The Given Differential Equation By Separation Of Variables. Ds Dr = Ks

Differential equations are fundamental in describing various phenomena across physics, engineering, biology, and many other scientific disciplines. Among the various methods to solve differential equations, the separation of variables is particularly straightforward and elegant when applicable. In this article, we will explore how to solve the differential equation Ds Dr = Ks using the separation of variables technique. We will also delve into the underlying principles, steps involved, and practical applications of this method.

Understanding the Differential Equation Ds Dr = Ks

Before jumping into the solution, it is essential to understand the nature of the given differential equation.

Form of the Equation

The equation is written as:

$$ Ds \, Dr = Ks $$

Where:
- D represents differentiation with respect to an independent variable, often time t.
- s and r are functions of the independent variable, possibly position, displacement, or other quantities.
- K is a constant.

However, the notation $Ds \, Dr$ suggests a product of derivatives, which may need clarification. Commonly, in differential equations, the notation might indicate:

$$ \frac{ds}{dt} \times \frac{dr}{dt} = Ks $$

or similar forms. For clarity, we will interpret the equation as:

$$ \frac{ds}{dt} \times \frac{dr}{dt} = Ks $$

which is a typical form where the product of derivatives equals a function of s (or r, depending on context).

Alternatively, if the original statement intends Ds and Dr as derivatives of s and r with respect to a common variable, the equation could be:

$$\frac{ds}{dt} \times \frac{dr}{dt} = Ks$$

which simplifies to a relation between derivatives and the function s.

Note: Since the original statement is somewhat ambiguous, we will assume the most common interpretation where s and r are functions of a variable t, and the equation involves their derivatives.

Rewriting the Differential Equation for Clarity

Suppose the equation is:

$$\frac{ds}{dt} \times \frac{dr}{dt} = Ks$$

Our goal is to solve for $s(t)$ and $r(t)$ explicitly or find their relations. To do so, we must manipulate the equation into a form suitable for separation of variables.

Method of Separation of Variables

The separation of variables technique involves rearranging a differential equation so that all terms involving one variable are on one side, and all terms involving the other variable are on the other side. The general steps include:

1. Express the differential equation in a form where variables can be separated.
2. Integrate both sides with respect to their respective variables.
3. Solve the resulting equations for the unknown functions.

This method is applicable primarily when the differential equation can be written as:

$$f(s) ds = g(r) dr$$

or similar forms.

Step-by-Step Solution Process

Let's approach the original problem systematically.

Step 1: Express the Equation Clearly

Given the original:

$$ Ds Dr = Ks $$

Assuming:

$$ \frac{ds}{dt} \times \frac{dr}{dt} = Ks $$

We can write:

$$ \left(\frac{ds}{dt}\right) \left(\frac{dr}{dt}\right) = Ks $$

Suppose r is a function of s, or vice versa, to reduce the complexity.

Alternatively, if the problem involves two functions $s(t)$ and $r(t)$ with their derivatives related multiplicatively, we might seek to eliminate t by expressing dr/ds.

Step 2: Express dr/ds to Reduce the Equation

From the chain rule:

$$ \frac{dr}{dt} = \frac{dr}{ds} \times \frac{ds}{dt} $$

Plugging into the original:

$$ \frac{ds}{dt} \times \frac{dr}{dt} = Ks $$

becomes:

$$ \frac{ds}{dt} \times \left(\frac{dr}{ds} \times \frac{ds}{dt} \right) = Ks $$

which simplifies to:

$$ \left(\frac{ds}{dt} \right)^2 \times \frac{dr}{ds} = Ks $$

Rearranged:

$$ \frac{dr}{ds} = \frac{Ks}{\left(\frac{ds}{dt} \right)^2} $$

However, this introduces more unknowns unless ds/dt can be expressed in terms of s.

Alternatively, if the problem is designed to be separable, perhaps the original differential equation is:

$$\frac{ds}{dt} = f(s)$$
$$\frac{dr}{dt} = g(r)$$

and their product:

$$f(s) \times g(r) = Ks$$

In this case, the equation is separable if $f(s)$ and $g(r)$ are functions solely of their variables.

Note: For the purpose of clarity, let's consider a simplified and typical form of the differential equation suitable for separation of variables.

Step 3: Assuming a Simplified Differential Equation

Suppose the differential equation is:

$$\frac{ds}{dt} = \frac{Ks}{r}$$

and r is constant or a known function, or vice versa.

Alternatively, consider the classical form:

$$\frac{ds}{dt} = a s$$

which is separable, leading to exponential solutions.

Given the ambiguity, let's proceed with a general separable form:

$$\frac{ds}{dt} = f(s)$$

and solve accordingly.

Solving a Typical Separable Differential Equation

Let's explore how to solve a general differential equation of the form:

$$\frac{dy}{dt} = f(y)$$

by separation of variables.

Step 1: Rewrite the Equation

Expressed as:

$$ \frac{dy}{f(y)} = dt $$

This separates the variables y and t.

Step 2: Integrate Both Sides

Integrate:

$$ \int \frac{dy}{f(y)} = \int dt $$

The integral on the left depends on the specific form of $f(y)$.

Step 3: Solve for $y(t)$

After integration, solve the resulting equation for $y(t)$, possibly involving initial conditions.

Application to the Original Equation

Suppose the original differential equation is:

$$ \frac{ds}{dt} = \frac{K s}{r(t)} $$

and $r(t)$ is known or can be expressed as a function of s or t. If $r(t)$ is proportional to $s(t)$, then the equation becomes:

$$ \frac{ds}{dt} = \frac{K s}{c s} = \frac{K}{c} $$

which leads to straightforward integration:

$$ \frac{ds}{dt} = \text{constant} $$

and the solution:

$$ s(t) = \frac{K}{c} t + C $$

For more complex scenarios, the method involves integrating with respect to the relevant variables.

Practical Examples and Applications

To illustrate, let's consider some practical examples where the separation of variables is applied to similar equations.

Example 1: Exponential Growth

Equation:

$$\frac{ds}{dt} = r s$$

Solution:

$$\frac{1}{s} ds = r dt$$
$$\int \frac{1}{s} ds = \int r dt$$
$$\ln |s| = r t + C$$
$$s(t) = S_0 e^{r t}$$

where $S_0 = e^{C}$ is the initial value.

Example 2: Radioactive Decay

Equation:

$$\frac{dN}{dt} = -\lambda N$$

Solution:

$$\frac{1}{N} dN = -\lambda dt$$
$$\int \frac{1}{N} dN = -\lambda \int dt$$
$$\ln |N| = -\lambda t + C$$
$$N(t) = N_0 e^{-\lambda t}$$

Summary and Key Takeaways

- The separation of variables is effective when the differential equation can be expressed as a product of functions of individual variables.
- The method involves algebraic manipulation to isolate variables on opposite sides of the equation.
- Integration of both sides then yields the solution, often involving arbitrary constants determined by initial conditions.
- Not all differential equations are separable; recognizing when the method applies is crucial.
- The approach provides explicit solutions for many common differential

equations encountered in science and engineering.

Conclusion

Solving the differential equation

Frequently Asked Questions

What is the general method to solve the differential equation Ds Dr = Ks using separation of variables?

The method involves rewriting the equation as Ds Dr = Ks, then separating the variables so that all Ds terms are on one side and all Dr terms are on the other, followed by integrating both sides to find the solution.

How do you separate variables in the differential equation Ds Dr = Ks?

Rewrite the equation as Ds / K = dr / s, then rearrange to get Ds / D = K dr / s, allowing you to integrate both sides separately with respect to their variables.

What are the steps to integrate both sides after separating variables in this differential equation?

Integrate the left side with respect to D and the right side with respect to r: ∫ (1/D) dD = ∫ K / s dr, then solve these integrals to find the relation between D and r.

What is the general solution form for the differential equation Ds Dr = Ks after separation of variables?

The solution typically takes the form D = C e^{K r} or a similar exponential relation, depending on the integration constants and initial conditions.

Can the separation of variables method be applied if the differential equation is not explicitly separable, like Ds Dr = Ks?

Yes, as long as the equation can be rearranged into a form where all D terms are on one side and all r terms on the other, separation of variables is applicable.

What are common challenges faced when solving Ds Dr = Ks by separation of variables?

Challenges include correctly rearranging the equation to isolate variables, performing integrals involving functions like 1/D or 1/s, and applying initial conditions to determine integration constants.

Additional Resources

Solve The Given Differential Equation By Separation Of Variables: Ds Dr = Ks

Differential equations are fundamental tools in mathematics and engineering, describing how quantities change concerning each other. Among various methods for solving differential equations, separation of variables stands out as one of the most straightforward and widely applicable techniques, especially for first-order equations that can be expressed as a product of a function of x and a function of y. In this article, we explore the process of solving the differential equation $\frac{dy}{dx} = Ks$ by separation of variables, examining the underlying principles, step-by-step solutions, and practical considerations.

Understanding the Differential Equation: Ds Dr = Ks

Before delving into the solution method, it is essential to understand the form and significance of the differential equation:

What does $\frac{dy}{dx} = Ks$ represent?

- Here, y is a dependent variable, while x is the independent variable.
- K and s are constants or functions depending on the context. Typically, K is a constant, and s could be a parameter or a function related to y or x.
- The equation indicates that the rate of change of y with respect to x is proportional to some parameter s.

Clarifying the notation

- If the notation $Ds \, Dr = Ks$ is intended, it suggests a differential relationship involving s and r. However, for the purpose of this discussion, assuming the core differential equation is:

$$
\frac{dy}{dx} = Ks
$$

where s could be a known function of x or y.

- If s is a constant, then the equation simplifies to:

$$
\frac{dy}{dx} = K s
$$

which is directly integrable.

Special cases

- When s is a function of x, say $s = s(x)$, the equation becomes:

$$
\frac{dy}{dx} = K s(x)
$$

- When s is a function of y, say $s = s(y)$, the equation becomes:

$$
\frac{dy}{dx} = K s(y)
$$

In this article, we focus on the general case where the differential equation is of the form:

$$
\frac{dy}{dx} = K s(y)
$$

and demonstrate how separation of variables applies.

Separation of Variables: Concept and Principles

What is separation of variables?

Separation of variables is a method used to solve certain first-order differential equations by rewriting the equation so that all terms involving y are on one side, and all terms involving x are on the other. This technique allows for straightforward integration, leading to the solution.

Basic idea

Given an equation:

$$
\frac{dy}{dx} = f(x) g(y)
$$

if it can be expressed as:

$$
\frac{dy}{g(y)} = f(x) dx
$$

then we can integrate both sides separately:

$$
\int \frac{1}{g(y)} dy = \int f(x) dx + C
$$

where C is the constant of integration.

Features and advantages

- Simplicity: The method reduces the differential equation to two integrals.
- Applicability: Works well for equations where the variables can be separated explicitly.
- Intuitive: Provides a clear pathway from the differential equation to the solution.

Limitations

- Not applicable if the variables cannot be separated algebraically.
- Some equations require substitution or other methods before separation is possible.
- May lead to implicit solutions that need further manipulation.

Step-by-Step Solution of $\frac{dy}{dx} = Ks$ by Separation of Variables

Step 1: Express the equation in separable form

Assuming s is a function of y, the differential equation:

$$
\frac{dy}{dx} = K s(y)
$$

can be rewritten as:

$$
\frac{dy}{s(y)} = K dx
$$

This step isolates the variables, preparing for integration.

Step 2: Integrate both sides

Perform the integrations:

$$
\int \frac{1}{s(y)} dy = \int K dx
$$

- The integral on the right is straightforward:

$$
K x + C_1
$$

- The integral on the left depends on the form of $s(y)$. For example:

- If $s(y) = y$, then:

$$
\int \frac{1}{y} dy = \ln |y|
$$

- If $s(y)$ is a polynomial, rational function, or exponential, the integral varies accordingly.

Step 3: Solve the integral

Compute the integral based on the specific form of $s(y)$. For illustration, consider an example where $s(y) = y$:

$$
\int \frac{1}{y} dy = \ln |y| + C
$$

and

$$
\int K dx = K x + C_2
$$

Combine constants:

$$
\ln |y| = K x + C
$$

where $C = C_2 - C_1$.

Step 4: Express the solution

Exponentiate both sides to solve for y:

$$
|y| = e^{K x + C} = e^{C} e^{K x} = A e^{K x}
$$

where $A = e^{C} > 0$ is an arbitrary positive constant. The general solution:

$$
y = \pm A e^{K x}
$$

or simply:

$$
y = C' e^{K x}
$$

with C' being an arbitrary constant.

Examples and Applications

Example 1: Solving $\frac{dy}{dx} = 3 y$

This is a classic first-order linear differential equation, separable as:

$$
\frac{dy}{y} = 3 dx
$$

Integrating:

$$
\ln |y| = 3x + C
$$

Solution:

$$
y = C' e^{3x}
$$

where C' is an arbitrary constant.

Example 2: Solving $\frac{dy}{dx} = \sin y$

Separate variables:

$$
\frac{dy}{\sin y} = dx
$$

Recall $\int \frac{dy}{\sin y} = \int \csc y \, dy$, which integrates to:

$$
\int \csc y \, dy = - \ln |\csc y + \cot y| + C
$$

Thus:

$$
- \ln |\csc y + \cot y| = x + C
$$

or

$$
|\csc y + \cot y| = e^{-x + C'} = D e^{-x}
$$

where $D = e^{C'}$. This implicit solution can be further manipulated depending on initial conditions.

Features, Pros, and Cons of Separation of Variables

Features

- Suitable for first-order differential equations where variables can be separated algebraically.
- Often yields explicit solutions, especially for standard functions.
- Provides a straightforward integration process once variables are isolated.

Pros

- Intuitive and straightforward: Simple to implement with basic integration.
- Widely applicable: Suitable for many physical models involving exponential growth, decay, or other processes.
- Foundation for more advanced methods: Serves as a stepping stone toward solving more complex differential equations.

Cons

- Limited scope: Not applicable if variables cannot be separated algebraically.
- Implicit solutions: Sometimes the result is implicit, requiring additional steps to express explicitly.
- Complicated integrals: When $s(y)$ is complex, integrals may be difficult or impossible to evaluate analytically.

Conclusion

Solving differential equations by separation of variables is a fundamental technique in mathematical analysis, offering clarity and simplicity for a broad class of problems. When applied to equations like $\frac{dy}{dx} = Ks$, where s is a function of y, the method allows for the direct integration of both sides and the derivation of general solutions. Its effectiveness hinges on the ability to isolate variables and evaluate integrals, making it a powerful tool in the mathematician’s arsenal. Whether modeling exponential growth, radioactive decay, or other natural phenomena, separation of variables provides a clear pathway from differential equations to explicit or implicit solutions, deepening our understanding of dynamic systems.

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