Solve The System Of Linear Equations. 4x + 6y = 12 X + 2y = 10 The Solution Is

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Understanding how to solve systems of linear equations is a fundamental skill in algebra, mathematics, engineering, and various applied sciences. In this article, we will explore the methods to solve the specific system of equations:

$$
\begin{cases}
4x + 6y = 12 \\
x + 2y = 10
\end{cases}
$$

We will discuss various strategies, including substitution, elimination, and graphical methods, to find the solutions. This comprehensive guide aims to help students, educators, and professionals grasp the concepts involved in solving such systems efficiently and accurately.

What Is a System of Linear Equations?

A system of linear equations consists of two or more equations involving the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously. The solutions to the system can be:

- A single unique solution (the equations intersect at one point)
- Infinitely many solutions (the equations are the same line)
- No solution (the equations are parallel and never intersect)

In our case, the system involves two equations with two variables, x and y, which typically has a unique solution unless the equations are dependent or inconsistent.

Methods for Solving the System

There are several methods to solve systems of linear equations:

1. Substitution Method
2. Elimination Method
3. Graphical Method
4. Matrix Method (using determinants or matrices)

Each method has its advantages and is suitable for different types of systems.

1. Substitution Method

This method involves solving one of the equations for one variable and substituting that expression into the other equation.

Step-by-step process for our system:

- Solve the second equation $x + 2y = 10$ for x:

$$
x = 10 - 2y
$$

- Substitute this expression into the first equation $4x + 6y = 12$:

$$
4(10 - 2y) + 6y = 12
$$

- Simplify:

$$
40 - 8y + 6y = 12
$$
$$
40 - 2y = 12
$$

- Isolate y:

$$
-2y = 12 - 40
$$
$$
-2y = -28
$$
$$
y = \frac{-28}{-2} = 14
$$

- Find x by substituting y back into $x = 10 - 2y$:

$$
x = 10 - 2(14) = 10 - 28 = -18
$$

Solution:
$$
x = -18, \quad y = 14
$$

This pair of values is the solution to the system.

2. Elimination Method

The elimination method involves manipulating the equations to eliminate one variable, making it easier to solve for the other.

Step-by-step process:

- The system:

$$
\begin{cases}
4x + 6y = 12 \quad (1) \\
x + 2y = 10 \quad (2)
\end{cases}
$$

- Multiply equation (2) by a factor that makes the coefficients of x in both equations equal (or opposites). Here, multiply (2) by 4:

$$
4x + 8y = 40 \quad (3)
$$

- Now, subtract equation (1) from equation (3):

$$
(4x + 8y) - (4x + 6y) = 40 - 12
$$
$$
(4x - 4x) + (8y - 6y) = 28
$$
$$
0 + 2y = 28
$$
$$
2y = 28
$$
$$
y = 14
$$

- Substitute y into equation (2):

$$
x + 2(14) = 10
$$
$$
x + 28 = 10
$$
$$
x = 10 - 28 = -18
$$

Solution:
$$
x = -18, \quad y = 14
$$

Both methods lead to the same solution, confirming its accuracy.

3. Graphical Method

Graphing involves plotting both equations on a coordinate plane and identifying the point where they intersect.

- Rewrite equations in slope-intercept form $y = mx + b$:

From $4x + 6y = 12$:

$$
6y = -4x + 12
$$
$$
y = -\frac{2}{3}x + 2
$$

From $x + 2y = 10$:

$$
2y = -x + 10
$$
$$
y = -\frac{1}{2}x + 5
$$

- Plot the two lines:

- Line 1: Slope $-\frac{2}{3}$, y-intercept 2
- Line 2: Slope $-\frac{1}{2}$, y-intercept 5

- Find the intersection point manually or using graphing tools:

Set the right sides equal to find the intersection:

$$
-\frac{2}{3}x + 2 = -\frac{1}{2}x + 5
$$

Multiply through by 6 (least common denominator) to clear fractions:

$$
6 \times \left(-\frac{2}{3}x + 2 \right) = 6 \times \left(-\frac{1}{2}x + 5 \right)
$$

$$
-4x + 12 = -3x + 30
$$

Bring variables to one side:

$$
-4x + 12 + 3x = 30
$$
$$
- x + 12 = 30
$$
$$
- x = 18
$$

\]
\[
x = -18
\]

Substitute into one of the equations to find y:

\[
y = -\frac{1}{2}(-18) + 5 = 9 + 5 = 14
\]

Graphical solution:
Point \((-18, 14)\)

This confirms the solution found via algebraic methods.

Understanding the Solution

The solution to the system:

\[
\boxed{
x = -18, \quad y = 14
}
\]

represents the unique point where both lines intersect on the coordinate plane. This point satisfies both original equations, meaning substituting these values back into the equations confirms their correctness.

Verifying the Solution

Verification is an essential step to ensure the solution is accurate.

- Substitute \(x = -18 \) and \(y = 14 \) into the first equation:

\[
4(-18) + 6(14) = -72 + 84 = 12
\]

- Substitute into the second equation:

\[
(-18) + 2(14) = -18 + 28 = 10
\]

Both equations are satisfied, confirming that \((-18, 14)\) is the solution.

Additional Tips for Solving Systems of Equations

- Always check whether the system has a unique solution, infinitely many solutions, or no solution.
- When equations are in different forms, convert to the same form for easier comparison.
- Use graphing tools or software for complex systems or when visual understanding helps.
- Practice with different methods to become proficient and recognize which method is most efficient for a given system.

Applications of Solving Systems of Linear Equations

Understanding how to solve systems of equations is vital in various real-world applications:

- Engineering: Designing circuits, analyzing forces in structures.
- Economics: Equilibrium analysis, cost-profit optimization.
- Physics: Kinematic equations, motion analysis.
- Computer Science: Algorithm development, graphics programming.
- Business: Budget planning, resource allocation.

Conclusion

Solving systems of linear equations like:

$$
\begin{cases}
4x + 6y = 12 \\
x + 2y = 10
\end{cases}
$$

is a fundamental skill that can be approached through substitution, elimination, or graphical methods. In our case, all methods yield the solution:

$$
\boxed{
x = -18, \quad y = 14
}
$$

Mastering these techniques enables problem-solvers to analyze and interpret complex systems across various disciplines efficiently. Whether working with simple algebraic systems or advanced matrix methods, the core principles remain consistent and essential for mathematical literacy and practical problem-solving.

If you want to enhance your understanding further, consider practicing with similar systems, exploring matrix solutions, or applying these methods in real-world scenarios to see their effectiveness firsthand.

Frequently Asked Questions

How do you solve the system of equations 4x + 6y = 12 and x + 2y = 10?

You can use substitution or elimination methods. For example, from the second equation x + 2y = 10, express x as 10 - 2y and substitute into the first equation to find y, then find x.

What is the solution to the system 4x + 6y = 12 and x + 2y = 10?

The solution is x = 4 and y = 3.

Can you solve the system of equations using the elimination method?

Yes. Multiply the second equation by 4 to align with the x coefficient, then subtract to eliminate x and solve for y, followed by substituting back to find x.

What is the value of y in the system 4x + 6y = 12 and x + 2y = 10?

Y equals 3.

How do I verify the solution to the system of equations?

Substitute the found values of x and y into both original equations to see if they satisfy both equations.

Is the system 4x + 6y = 12 and x + 2y = 10 consistent and independent?

Yes, the system has a unique solution, making it consistent and independent.

What is the graphical interpretation of solving this system?

The solution (x=4, y=3) is the point where the two lines represented by the equations intersect.

Can this system be solved using matrix methods?

Yes, you can write the system in matrix form and use methods like Gaussian elimination or inverse matrices to find the solution.

What are common mistakes to avoid when solving this system?

Common mistakes include incorrect substitution, arithmetic errors during elimination, and forgetting to check the solution in both equations.

Additional Resources

Solve The System Of Linear Equations: A Comprehensive Guide

Understanding how to solve systems of linear equations is a fundamental skill in algebra, essential for students, educators, engineers, scientists, and anyone dealing with mathematical modeling. This detailed review explores the process step-by-step, focusing on the specific system:

$$
\begin{cases}
4x + 6y = 12 \\
x + 2y = 10
\end{cases}
$$

We will analyze various methods to solve such systems, interpret the solutions, and understand the underlying principles. By the end of this piece, you'll have a robust grasp of solving systems of linear equations and be able to apply these techniques confidently.

Understanding the Basics of Systems of Linear Equations

What Is a System of Linear Equations?

A system of linear equations comprises two or more equations involving the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

For example, the given system:

$$
\begin{cases}
4x + 6y = 12 \quad (1) \\
x + 2y = 10 \quad (2)
\end{cases}
$$

has two equations with two variables, x and y.

Why Solve Systems of Equations?

Solving systems of equations helps in:

- Finding intersection points in geometry.
- Determining unknowns in physical, economic, or engineering models.
- Optimizing solutions in linear programming.
- Analyzing relationships between variables.

Methods for Solving the System

Several techniques are available, each suited to different types of problems or preferences.

1. Graphical Method

Plotting each equation on a coordinate plane and identifying the point(s) of intersection.

Advantages:
- Visual understanding.
- Easy to comprehend for simple systems.

Limitations:
- Less precise.
- Not suitable for complex or non-integer solutions.

Application to Our System:
- Plot \(4x + 6y = 12\).
- Plot \(x + 2y = 10\).
- Find the intersection point.

Note: While illustrative, for exact solutions, algebraic methods are preferred.

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2. Substitution Method

This involves solving one equation for one variable and substituting it into the other.

Step-by-step:

- Solve one of the equations for one variable.
- Substitute this expression into the other equation.
- Solve for the remaining variable.
- Back-substitute to find the other variable.

Application to Our System:

Equation (2): $x + 2y = 10$

- Solve for x:

$$
x = 10 - 2y
$$

- Substitute into equation (1):

$$
4(10 - 2y) + 6y = 12
$$

- Simplify:

$$
40 - 8y + 6y = 12
$$

$$
40 - 2y = 12
$$

- Solve for y:

$$
-2y = 12 - 40
$$

$$
-2y = -28
$$

$$
y = \frac{-28}{-2} = 14
$$

- Find x:

$$
x = 10 - 2(14) = 10 - 28 = -18
$$

Solution:

$$
x = -18, \quad y = 14
$$

---

3. Elimination Method (Addition/Subtraction Method)

This technique involves manipulating the equations to eliminate one variable, simplifying the process.

Step-by-step:

- Multiply equations if necessary to align coefficients.
- Add or subtract equations to eliminate one variable.
- Solve for the remaining variable.
- Back-substitute to find the other.

Application to Our System:

Equation (2): $x + 2y = 10$

Equation (1): $4x + 6y = 12$

- To eliminate x, multiply (2) by -4:

$$
-4x - 8y = -40
$$

- Add to equation (1):

$$
(4x + 6y) + (-4x - 8y) = 12 - 40
$$

$$
(4x - 4x) + (6y - 8y) = -28
$$

$$
0x - 2y = -28
$$

- Simplify:

$$
-2y = -28
$$
$$
y = 14
$$

- Find x using equation (2):

$$
x + 2(14) = 10
$$
$$
x + 28 = 10
$$
$$
x = 10 - 28 = -18
$$

Solution:

$$
x = -18, \quad y = 14
$$

This matches the substitution method's result, confirming consistency.

Interpreting the Solution

The solution to the system:

$$
x = -18, \quad y = 14
$$

represents the point of intersection of the two lines described by the equations. In the coordinate plane, this point is:

$$
(-18, 14)
$$

Implications:

- The point satisfies both equations simultaneously.
- The lines represented by the equations intersect at this single point, indicating the system is consistent and independent with a unique solution.

Verifying the Solution

Verification ensures the accuracy of the solution.

- Substitute $x = -18$ and $y = 14$ into the original equations:

Equation (1):

$$
4(-18) + 6(14) = -72 + 84 = 12 \quad \checkmark
$$

Equation (2):

$$
-18 + 2(14) = -18 + 28 = 10 \quad \checkmark
$$

Both equations are satisfied, confirming the correctness.

Alternative Approaches and Advanced Techniques

While substitution and elimination are straightforward for 2x2 systems, more complex systems require advanced methods:

4. Matrix Method (Using Inverse Matrices)

Express the system in matrix form:

\[
\mathbf{A} \mathbf{x} = \mathbf{b}
\]

Where:

\[
\mathbf{A} = \begin{bmatrix}
4 & 6 \\
1 & 2
\end{bmatrix},
\quad
\mathbf{x} = \begin{bmatrix}
x \\
y
\end{bmatrix},
\quad
\mathbf{b} = \begin{bmatrix}
12 \\
10
\end{bmatrix}
\]

If \(\mathbf{A}\) is invertible (determinant non-zero), the solution is:

\[
\mathbf{x} = \mathbf{A}^{-1} \mathbf{b}
\]

Calculate the determinant:

\[
\det(\mathbf{A}) = 4 \times 2 - 6 \times 1 = 8 - 6 = 2 \neq 0
\]

Compute the inverse:

\[
\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix}
2 & -6 \\
-1 & 4
\end{bmatrix}
= \frac{1}{2} \begin{bmatrix}
2 & -6 \\
-1 & 4
\end{bmatrix}
= \begin{bmatrix}
1 & -3 \\
-0.5 & 2
\end{bmatrix}
\]

Multiply:

\[

$$
\mathbf{x} = \mathbf{A}^{-1} \mathbf{b} = \begin{bmatrix}
1 & -3 \\
-0.5 & 2
\end{bmatrix}
\begin{bmatrix}
12 \\
10
\end{bmatrix}
$$

Calculate:

- $x = 1 \times 12 + (-3) \times 10 = 12 - 30 = -18$
- $y = -0.5 \times 12 + 2 \times 10 = -6 + 20 = 14$

Matching previous results.

Practical Applications of Solving Systems of Equations

Understanding and solving such systems have real-world implications:

- Economics: To find equilibrium prices and quantities.
- Physics: To determine forces or velocities in mechanics.
- Engineering: For circuit analysis, signal processing, and control systems.
- Computer Science: In algorithms involving linear models or graphics.

Common Challenges and Troubleshooting

While solving systems, some issues may arise:

- No solution (Inconsistent system): Equations represent parallel lines. For example, if the lines do not intersect.
- Infinite solutions (Dependent system): Equations represent the same line, leading to infinitely many solutions.
- Mistakes in algebraic manipulation: Always double-check calculations.
- Incorrect application of methods: Choose the most suitable method based on the system's complexity.

Summary

Solve The System Of Linear Equations 4x 6y 12 X 2y 10 The Solution Is

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