# Solve Y-y' = 4e + 3e2x With The Conditions X = 0, Y = 0, Y'= -1 And Y" = 2. -x

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Introduction

Differential equations are fundamental tools in mathematics and engineering, used to model a wide array of phenomena—from physics and biology to economics and beyond. Among these, first and second-order differential equations are particularly prevalent, providing insights into the behavior of systems over time or space. Solving such equations involves finding functions that satisfy given relationships between the functions and their derivatives, often under specific initial or boundary conditions.

In this article, we will explore a specific differential equation:

$Y - y' = 4e + 3e^{2x}$

accompanied by the initial conditions:

- X = 0, Y = 0
- Y' = -1
- Y'' = 2
- and the condition involving negative x: -x

Our goal is to systematically analyze this differential equation, interpret the conditions, and derive the explicit solution. This problem combines several key concepts in differential equations, including linear equations, initial conditions, and the use of derivatives to find particular solutions.

Understanding how to approach and solve such equations is essential for students and professionals working in fields that require modeling dynamic systems. This article aims to provide a comprehensive, step-by-step guide to solving this problem, emphasizing clarity, depth, and practical insights.

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Context and Relevance of the Differential Equation

Before delving into the solution process, it’s crucial to understand the context and significance of the given equation and conditions.

The Differential Equation

The equation:

$Y - y' = 4e + 3e^{2x}$

is a first-order linear differential equation involving exponential functions. The presence of e and $e^{2x}$ suggests exponential growth or decay behavior, common in natural processes such as radioactive decay, population dynamics, or capacitor discharge.

The Initial and Derivative Conditions

The conditions:

- At X = 0, Y = 0
- Y' = -1 at some point (possibly at X=0)
- Y'' = 2

provide necessary data to determine the particular solution, including initial position, slope, and curvature.

The Significance of the Conditions

- Knowing Y(0) = 0 helps anchor the solution at a specific point.
- The derivative condition Y' = -1 indicates the slope of the solution at a certain point.
- The second derivative Y'' = 2 describes the concavity or acceleration of the function.
- The mention of -x may relate to boundary conditions or specific solution behavior over the domain.

Practical Applications

Such equations appear in physics when analyzing systems with rate changes influenced by exponential factors, for example:

- Cooling or heating processes
- Population models with exponential growth components
- Motion equations with exponential damping

Understanding how to solve these equations allows engineers and scientists to predict system behavior accurately.

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Step-by-Step Solution Approach

Step 1: Recognize the Type of Differential Equation

The equation:

$Y - y' = 4e + 3e^{2x}$

can be rewritten for clarity. Since Y and y' are involved, and assuming Y and y' denote the same function and its derivative, this appears to be a first-order linear differential equation:

Y - dY/dx = 4e + 3e^{2x}

which simplifies to:

$dY/dx - Y = -4e - 3e^{2x}$

Note: The sign change occurs because moving Y to the other side yields $dY/dx - Y = -(4e + 3e^{2x})$

Step 2: Write in Standard Linear Differential Equation Form

The standard form for a first-order linear differential equation is:

$$dy/dx + P(x) y = Q(x)$$

Comparing:

- $dy/dx - y = -4e - 3e^{2x}$

we identify:

- $P(x) = -1$
- $Q(x) = -4e - 3e^{2x}$

Step 3: Find the Integrating Factor (IF)

The integrating factor is:

$$\mu(x) = e^{\int P(x)\, dx} = e^{\int -1\, dx} = e^{-x}$$

Step 4: Multiply Entire Equation by the Integrating Factor

Multiplying both sides by $e^{-x}$:

$$e^{-x}\, dy/dx - e^{-x}\, y = -4e^{-x}\, e - 3e^{-x}\, e^{2x}$$

Simplify the right side:

- $-4e^{-x}\, e = -4\, e^{-x + 1}$
- $-3\, e^{-x}\, e^{2x} = -3\, e^{-x + 2x} = -3\, e^{x}$

Now, the left side simplifies to the derivative of $(e^{-x} y)$:

$$d/dx\, [e^{-x} y] = -4\, e^{-x + 1} - 3\, e^{x}$$

Step 5: Integrate Both Sides

Integrate both sides with respect to x:

$$\int d/dx\, [e^{-x} y]\, dx = \int [ -4\, e^{-x + 1} - 3\, e^{x} ]\, dx$$

which yields:

$$e^{-x} y = \int [ -4\, e^{-x + 1} - 3\, e^{x} ]\, dx + C$$

Compute the integrals separately:

1. $\int -4 e^{ -x + 1 } dx$

Since $e^{ -x + 1 } = e^{1} e^{ -x } = e e^{ -x }$

Thus:

$\int -4 e e^{ -x } dx = -4 e \int e^{ -x } dx = -4 e ( - e^{ -x } ) + C_1 = 4 e e^{ -x } + C_1$

2. $\int -3 e^{ x } dx = -3 \int e^{ x } dx = -3 e^{ x } + C_2$

Putting it all together:

$e^{-x} y = 4 e e^{ -x } - 3 e^{ x } + C$

where $C$ combines $C_1$ and $C_2$.

Step 6: Solve for Y(x)

Multiply both sides by $e^{x}$:

$Y = e^{x} [ 4 e e^{ -x } - 3 e^{ x } + C ]$

Simplify:

- $e^{x} 4 e e^{ -x } = 4 e e^{x} e^{ -x } = 4 e$

- $e^{x} ( -3 e^{ x } ) = -3 e^{ 2x }$

- $e^{x} C = C e^{ x }$

Thus, the general solution is:

$Y(x) = 4 e - 3 e^{ 2x } + C e^{ x }$

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Applying Initial Conditions to Find the Particular Solution

Given the initial conditions:

- $Y(0) = 0$
- $Y'$ at some point $= -1$
- $Y'' = 2$

we'll use these to find the constant C and verify the solution.

Step 7: Calculate Derivatives

First derivative:

$Y'(x) = d/dx [ 4 e - 3 e^{2x} + C e^{x} ]$

- Derivative of $4 e$ is 0.
- Derivative of $-3 e^{2x}$ is $-3 \, 2 e^{2x} = -6 e^{2x}$
- Derivative of $C e^{x}$ is $C e^{x}$

So,

$Y'(x) = -6 e^{2x} + C e^{x}$

Second derivative:

$Y''(x) = d/dx [ -6 e^{2x} + C e^{x} ]$

- Derivative of $-6 e^{2x}$ is $-6 \, 2 e^{2x} = -12 e^{2x}$
- Derivative of $C e^{x}$ is $C e^{x}$

Thus,

$Y''(x) = -12 e^{2x} + C e^{x}$

Step 8: Apply the Conditions

At $x = 0$:

- $Y(0) = 0$

Compute:

$Y(0) = 4 e - 3 e^{0} + C e^{0} = 4 e - 3 \, 1 + C \, 1 = 4 e - 3 + C$

Set equal to 0:

$4 e - 3 + C = 0$

Solve for C:

$C = -4 e + 3$

Step 9: Verify Y' and Y'' Conditions

At $x = 0$:

- Y' = -6 e^{ 0

# Frequently Asked Questions

## What is the general form of the differential equation given in the problem?

The differential equation is (Y - y) = 4e + 3e^{2x}.

## What are the initial conditions provided for the differential equation?

The initial conditions are X = 0, Y = 0, Y' = -1, and Y'' = 2.

## How can we interpret the condition Y' = -1 at the initial point?

It indicates that the slope of the function Y at x = 0 is -1.

## Given Y'' = 2, what does this tell us about the concavity of Y?

It indicates that Y is concave upward since the second derivative is positive.

## How do the conditions X=0 and Y=0 help in solving the differential equation?

They provide initial point data that can be used to determine the particular solution of the differential equation.

## What is the significance of the term 4e + 3e^{2x} in the differential equation?

It represents a nonhomogeneous term that influences the particular solution of the differential equation.

## What steps are involved in solving this differential equation with the given conditions?

First, rewrite the equation in a standard form, find the complementary solution, determine particular solutions considering the nonhomogeneous term, and then apply initial conditions to find specific constants.

# Additional Resources

Solve Y - y' = 4e + 3e^{2x} With The Conditions X = 0, Y = 0, Y' = -1 And Y'' = 2 - x

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## Introduction

The differential equation $Y - y' = 4e + 3e^{2x}$ presents an intriguing challenge for mathematicians and engineers alike. Its structure embodies the hybrid nature of linear nonhomogeneous equations, where the solution involves both integrating factors and particular solutions. The given initial conditions—$X = 0$, $Y = 0$, $Y' = -1$, and $Y'' = 2 - x$—add layers of complexity, demanding a meticulous approach that combines analytical methods with a thorough understanding of differential calculus.

The purpose of this comprehensive review is to dissect the problem systematically, elucidate the solution process, and interpret the implications of the initial conditions. This exploration will serve as a detailed guide for mathematicians, students, and professionals interested in advanced differential equations.

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## Overview of the Differential Equation

The equation:

$$Y - y' = 4e + 3e^{2x}$$

can be viewed as a first-order linear differential equation with a nonhomogeneous term. Recasting it into standard form yields:

$$y' - Y = -4e - 3e^{2x}$$

where $y'$ signifies the derivative of $y$ with respect to $x$.

Note: The notation suggests a potential typographical inconsistency—possibly, the uppercase $Y$ and lowercase $y$ are used interchangeably. For clarity, we will assume the standard convention where lowercase $y$ is the primary function, and uppercase $Y$ was perhaps a notation slip. If the original notation intended $Y$ as the function, the same approach applies; otherwise, assuming the conventional lowercase notation simplifies the analysis.

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## Fundamental Solution Strategy

### 1. Recognize the Type of Equation

Given the form:

$$y' - y = \text{Nonhomogeneous Part}$$

This is a linear first-order ordinary differential equation (ODE). The general solution comprises two parts:

- The complementary (homogeneous) solution $y_h$, solving $y' - y = 0$.
- A particular solution $y_p$, solving the entire nonhomogeneous equation.

2. Homogeneous Equation

$$ y' - y = 0 $$

has the characteristic solution:

$$ y_h = Ce^{x} $$

where $C$ is an arbitrary constant.

3. Particular Solution

Given the nonhomogeneous term:

$$ 4e + 3e^{2x} $$

which involves exponential functions, the method of undetermined coefficients suggests a particular solution of the form:

$$ y_p = A + Be^{2x} $$

since the right-hand side contains a constant term and an exponential with a different exponent.

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Step-by-Step Solution

1. Write the Differential Equation in Standard Form

Assuming the original equation is:

$$ y' - y = -4e - 3e^{2x} $$

then:

$$ y' = y - 4e - 3e^{2x} $$

or, more conventionally:

$$ y' - y = -4e - 3e^{2x} $$

2. Find the Integrating Factor

The integrating factor $\mu(x)$ is:

$$ \mu(x) = e^{-\int 1 dx} = e^{-x} $$

Multiplying both sides by $e^{-x}$:

$$ e^{-x} y' - e^{-x} y = -4e^{-x} e - 3e^{2x} e^{-x} $$

Simplify the right side:

- $-4e^{-x} e = -4 e^{1 - x}$
- $-3 e^{2x} e^{-x} = -3 e^{2x - x} = -3 e^{x}$

So the equation becomes:

$$\frac{d}{dx} (e^{-x} y) = -4 e^{1 - x} - 3 e^{x}$$

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3. Integrate to Find $y$

Integrate both sides:

$$e^{-x} y = \int \left( -4 e^{1 - x} - 3 e^{x} \right) dx + C$$

Break down the integral:

$$\int -4 e^{1 - x} dx + \int -3 e^{x} dx$$

First integral:

$$-4 e^{1 - x}$$

Let’s perform substitution:

- $u = 1 - x \Rightarrow du = -dx$

Thus:

$$\int -4 e^{u} dx = \int -4 e^{u} (-du) = 4 \int e^{u} du = 4 e^{u} + D$$

Back-substituting:

$$4 e^{1 - x}$$

Second integral:

$$-3 e^{x} \Rightarrow \int -3 e^{x} dx = -3 e^{x} + D$$

Putting it all together:

$$e^{-x} y = 4 e^{1 - x} - 3 e^{x} + C$$

Multiply through by $e^{x}$:

$$y = e^{x} \left( 4 e^{1 - x} - 3 e^{x} + C \right)$$

Simplify each term:

- $e^{x} \times 4 e^{1 - x} = 4 e^{x + 1 - x} = 4 e^{1} = 4 e$
- $e^{x} \times -3 e^{x} = -3 e^{2x}$
- $e^{x} \times C = C e^{x}$

Thus, the general solution:

$$y(x) = 4 e - 3 e^{2x} + C e^{x}$$

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4. Incorporating Initial Conditions

Given:

- $x = 0$
- $y(0) = 0$
- $y'(0) = -1$
- $y''(x) = 2 - x$

Step 1: Find $C$ from $y(0) = 0$

At $x=0$:

$$y(0) = 4 e - 3 e^{0} + C e^{0} = 4 e - 3 + C$$

Set equal to zero:

$$0 = 4 e - 3 + C \Rightarrow C = -4 e + 3$$

Step 2: Find $y'(x)$

Differentiate $y(x)$:

$$y' = \frac{d}{dx} \left( 4 e - 3 e^{2x} + C e^{x} \right)$$

$$y' = 0 - 6 e^{2x} + C e^{x}$$

At $x=0$:

$$y'(0) = -6 e^{0} + C e^{0} = -6 + C$$

Given $y'(0) = -1$:

$$-1 = -6 + C \Rightarrow C = 5$$

But previously, $C = -4 e + 3$. Equate:

$$5 = -4 e + 3 \Rightarrow -4 e = 2 \Rightarrow e = -\frac{1}{2}$$

which is impossible since $e$ is Euler's number ($\approx 2.718$). This inconsistency indicates that the initial assumptions or the problem's initial conditions are incompatible with the general solution derived.

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5. Addressing the Inconsistency and Additional Conditions

The inconsistency suggests that the initial conditions provided—specifically $y(0) = 0$, $y'(0) = -1$, and the second derivative condition $y'' = 2 - x$—are derived from a different context or that they refer to a different function $Y$, possibly indicating a second-order differential equation.

Given the initial statement:

> Y'' = 2 - x

which suggests a second-order differential equation, the entire problem might be about solving a second-order ODE with initial conditions, rather than the first-order equation initially presented.

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6. Reinterpreting the Problem as a Second-Order Differential Equation

Suppose the actual problem involves:

$$Y'' = 2 - x$$

with initial conditions:

$$Y(0) = 0, \quad Y'(0) = -1, \quad Y'' = 2 - x$$

and the original expression involving $Y - y' = 4e + 3e^{2x}$ is a separate relation or perhaps an additional constraint.

Solving the second-order ODE:

$$Y'' = 2 - x$$

Integrate twice:

- First integration:

$$Y' = \int (2 - x) dx = 2x - \frac{1}{2} x^{2} + C_1$$

- Applying initial condition $Y'(0) = -1$:

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# Solve Y Y 4e 3e2x With The Conditions X 0 Y 0 Y 1 And Y 2 X

Solve Y Y 4e 3e2x With The Conditions X 0 Y 0 Y 1 And Y 2 X

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