

The Product Of 8 And A Number Is 25 Less Than Twice That Number. Find The Number

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Understanding how to solve algebraic word problems is a fundamental skill in mathematics that enhances logical thinking and problem-solving abilities. One common type of problem involves translating a verbal statement into an algebraic equation and then solving for the unknown. In this article, we will explore the problem: The product of 8 and a number is 25 less than twice that number and demonstrate a step-by-step approach to find the unknown number.

Understanding the Problem Statement

Before jumping into solving the problem, it's essential to comprehend what the statement is conveying:

- "The product of 8 and a number" refers to multiplying some unknown number by 8.
- "Is 25 less than twice that number" indicates a comparison to twice the same unknown number, with a subtraction of 25.

This kind of problem is typical in algebra, where we translate words into an algebraic equation.

Translating the Word Problem into an Equation

To solve the problem systematically, follow these steps to convert the verbal statement into an algebraic form:

Identify the Unknown

- Let the unknown number be represented by a variable, commonly x .

Express the Phrases Mathematically

- "The product of 8 and a number" becomes $8x$.
- "Twice that number" becomes $2x$.
- "25 less than twice that number" translates to $2x - 25$.

Set Up the Equation

Based on the statement, the product $(8x)$ equals "25 less than twice that number," which is $(2x - 25)$. So, the equation becomes:

$$8x = 2x - 25$$

Solving the Equation Step-by-Step

Once the equation is established, proceed with solving for (x) :

Step 1: Isolate the Variable

Subtract $(2x)$ from both sides to gather the (x) terms on one side:

$$\begin{aligned} 8x - 2x &= -25 \\ 6x &= -25 \end{aligned}$$

Step 2: Solve for (x)

Divide both sides by 6:

$$x = \frac{-25}{6}$$

Step 3: Simplify the Result

The solution is:

$$x = -\frac{25}{6}$$

which is approximately -4.1667 when expressed as a decimal.

Interpreting the Solution

The value of x indicates that the number in question is $-\frac{25}{6}$. This negative value makes sense within the context of the problem, as algebraic solutions often include negative numbers unless the problem specifies only positive solutions.

Verification of the Solution

To ensure the solution is correct, substitute $x = -\frac{25}{6}$ back into the original statement:

- Calculate "the product of 8 and the number":

$$8 \times -\frac{25}{6} = -\frac{200}{6} = -\frac{100}{3}$$

- Calculate "twice that number minus 25":

$$2 \times -\frac{25}{6} - 25 = -\frac{50}{6} - 25 = -\frac{25}{3} - \frac{75}{3} = -\frac{100}{3}$$

Both sides equal $-\frac{100}{3}$, confirming that the solution satisfies the original problem.

Additional Insights into Solving Algebraic Word Problems

Understanding how to approach such problems requires familiarity with translating words into equations and manipulating algebraic expressions. Here are some tips:

Tips for Solving Word Problems

- **Read Carefully:** Identify key phrases and what they represent mathematically.
- **Assign Variables:** Choose a variable to represent the unknown quantity.
- **Translate Step-by-Step:** Convert each phrase into an algebraic expression or equation.
- **Solve the Equation:** Use algebraic methods—adding, subtracting, multiplying, dividing—to

find the variable.

- **Verify Your Solution:** Substitute back into the original statement to confirm correctness.

Common Mistakes to Avoid

- Misinterpreting the words or miswriting the algebraic expressions.
- Forgetting to perform the same operation on both sides of the equation.
- Not simplifying the expression properly before solving.
- Ignoring the possibility of extraneous solutions, especially in more complex problems.

Extensions and Variations of the Problem

Once you understand the basic approach, similar problems can be tackled with variations:

Example 1: If the problem says, "The product of 8 and a number is 25 more than twice that number," how would you set up the equation?

Solution:

- "25 more than twice that number" translates to $(2x + 25)$.
- The equation would be:

$$\begin{aligned} & \backslash \\ 8x &= 2x + 25 \\ & \backslash \end{aligned}$$

Follow the same steps to solve.

Example 2: If the problem states, "The product of 8 and a number is equal to the difference of twice that number and

25," what is the equation?

Solution:

- "Difference of twice that number and 25" is $(2x - 25)$.
- The equation:

$$\begin{aligned} & 8x = 2x - 25 \end{aligned}$$

which is similar to the original problem but with the order reversed.

Real-World Applications of Algebraic Word Problems

Understanding how to translate and solve such problems has practical significance beyond academics:

- Financial Planning: Calculating investments, loans, or savings.
- Physics: Determining unknown quantities like velocity or force.
- Engineering: Designing systems where variables depend on each other.
- Daily Life: Budgeting, shopping discounts, or recipe adjustments.

By mastering these skills, you can confidently approach a variety of real-world problems involving unknown quantities.

Conclusion

The problem, The product of 8 and a number is 25 less than twice that number, exemplifies the importance of translating verbal statements into algebraic equations and solving step-by-step. The solution, $(x = -\frac{25}{6})$, demonstrates the power of algebra in uncovering unknown values from descriptive statements. Remember, the key to success in such problems lies in careful reading, accurate translation, systematic solving, and verification. With practice, mastering these techniques becomes an invaluable tool for academic success and everyday problem-solving.

Further Resources

- [Algebra Tutorials for Beginners](<https://www.khanacademy.org/math/algebra>)
- [Solving Word Problems in Algebra](<https://www.mathsisfun.com/algebra/word-problems.html>)

- [Practice Problems on Algebraic Equations](https://www.mathway.com/Algebra)

Keywords: algebra, solve for x, word problems, translating statements into equations, algebraic solutions, mathematical reasoning, problem-solving skills, real-world applications

Frequently Asked Questions

What is the algebraic equation representing the problem 'The product of 8 and a number is 25 less than twice that number'?

Let the number be x . The equation is $8x = 2x - 25$.

How do you solve the equation $8x = 2x - 25$ to find the number?

Subtract $2x$ from both sides to get $6x = -25$, then divide both sides by 6 to find $x = -25/6$.

What is the value of the number in the problem?

The number is $-25/6$, which is approximately -4.17 .

Is the solution to the problem a positive or negative number?

The solution is negative, specifically $-25/6$.

Can you verify the solution by substituting back into the original problem?

Yes. Substitute $x = -25/6$ into the original: $8(-25/6) = 2(-25/6) - 25$. Simplify both sides to verify equality.

What step is crucial when solving the equation $8x = 2x - 25$?

Subtracting $2x$ from both sides to isolate x is the crucial step.

What does this problem illustrate about solving linear equations?

It demonstrates how to set up and solve a linear equation based on word problem information to find an unknown number.

Additional Resources

Understanding the Problem: The Product of 8 and a Number Is 25 Less Than Twice That Number

Mathematics often involves translating real-world situations into algebraic expressions to find unknown quantities. The problem "The product of 8 and a number is 25 less than twice that number" is a classic algebraic word problem that illustrates the process of setting up equations from verbal descriptions and solving for an unknown.

In this detailed review, we will explore every aspect of this problem, from understanding the language used to formulating the equation, solving it step-by-step, and interpreting the result. This comprehensive analysis aims to deepen your understanding of algebraic problem solving and highlight important strategies and concepts involved.

Deciphering the Word Problem

The first step in tackling any algebraic problem is to carefully interpret the language used. The problem statement is:

"The product of 8 and a number is 25 less than twice that number."

Let's break this into manageable parts:

- "The product of 8 and a number": This phrase indicates multiplication between 8 and an unknown number, which we will denote as x .
- "is": This word signals an equality or an assignment of one quantity to another.
- "25 less than twice that number": This suggests a comparison involving the number doubled (twice that number) minus 25.

Key observations:

- The unknown number: x (a standard choice for algebra problems).
- The phrase "twice that number" translates to $2x$.
- "25 less than twice that number" translates to $2x - 25$.
- The entire statement suggests an equation where the product (8 times the number) equals (or is set equal to) "25 less than twice the number."

Constructing the Mathematical Equation

Based on the above interpretation, the problem can be translated into an algebraic equation:

$$8x = 2x - 25$$

This equation states that the product of 8 and the number x is equal to "25 less than twice that number."

Analyzing the Equation: $8x = 2x - 25$

Before solving, it's important to understand the structure of the equation.

- Type of equation: Linear in x , because the highest power of x is 1.
- Goal: Find the value of x that satisfies this equation.
- Implication: Since the equation involves a simple linear relation, standard algebraic techniques such as isolating x , combining like terms, and solving for x will be applicable.

Step-by-Step Solution Process

Let's now carefully go through the solution process:

Step 1: Write down the equation

$$8x = 2x - 25$$

Step 2: Bring all terms involving x to one side

Subtract $2x$ from both sides:

$$8x - 2x = -25$$

Simplify:

$$\begin{aligned} & \backslash \\ 6x &= -25 \\ & \backslash \end{aligned}$$

Step 3: Isolate x

Divide both sides by 6:

$$\begin{aligned} & \backslash \\ x &= \frac{-25}{6} \\ & \backslash \end{aligned}$$

Step 4: Interpret the solution

The value of x is:

$$\begin{aligned} & \backslash \\ x &= -\frac{25}{6} \\ & \backslash \end{aligned}$$

which is approximately:

$$\begin{aligned} & \backslash \\ x &\approx -4.1667 \\ & \backslash \end{aligned}$$

Verifying the Solution

Verification is a critical step to ensure the solution makes sense within the context of the problem.

Substitute $x = -\frac{25}{6}$ back into the original statement:

- Calculate the product of 8 and x :

$$\begin{aligned} & \backslash \\ 8 \times \left(-\frac{25}{6}\right) &= -\frac{200}{6} = -\frac{100}{3} \\ & \backslash \end{aligned}$$

- Calculate "25 less than twice that number":

$$\begin{aligned} & \backslash \\ 2 \times \left(-\frac{25}{6}\right) - 25 &= -\frac{50}{6} - 25 = -\frac{25}{3} - 25 \\ & \backslash \end{aligned}$$

Express 25 as a fraction with denominator 3:

$$\begin{aligned} & \backslash \end{aligned}$$

$$-\frac{25}{3} - \frac{75}{3} = -\frac{100}{3}$$

\]

The two expressions are equal:

\[

$$-\frac{100}{3} = -\frac{100}{3}$$

\]

This confirms the solution is correct and satisfies the original statement.

Understanding the Key Concepts

This problem exemplifies fundamental algebraic concepts:

- Translating words into equations: Recognizing phrases like "product of," "less than," and "twice" to form algebraic expressions.
- Solving linear equations: Using algebraic operations such as addition, subtraction, multiplication, and division to isolate the variable.
- Verification of solutions: Substituting the found value back into the original problem to validate correctness.

Broader Mathematical Insights

Beyond solving the specific problem, it's valuable to explore deeper insights and related concepts.

Understanding the Language of Algebra

- "Product of two numbers": Translates to multiplication.
- "Less than": Indicates subtraction; the order matters (e.g., "25 less than twice" means $(2x - 25)$, not $(25 - 2x)$).
- "Twice": Means multiply by 2, i.e., $(2x)$.

Common Mistakes and Misinterpretations

- Reversing the order in "less than": It's common to confuse "25 less than twice the number" with "twice the number less 25" (which would be $(2x - 25)$). Recognizing the correct order is critical.

- Misidentifying the unknown: Always assign a variable (like x) to the unknown number for clarity.
- Forgetting to verify: Always substitute solutions back into the original equation.

Variations of the Problem

Algebraic word problems often have variations, such as:

- Changing the constants (e.g., "20 less than twice the number").
- Using different operations (e.g., division, exponents).
- Introducing additional unknowns or conditions.

Understanding the solution process for this problem helps in tackling these variations.

Real-World Applications of Similar Problems

While this problem is abstract, similar reasoning applies in various real-life contexts:

- Financial calculations: Determining unknown amounts based on relationships involving percentages, costs, and savings.
- Physics problems: Relating quantities like velocity, distance, and time.
- Business scenarios: Calculating profits, costs, or quantities where relationships involve multiples and deductions.

Mastering these algebraic problem-solving skills enables students and professionals to model and analyze real-world situations effectively.

Additional Tips for Solving Word Problems

- **Read Carefully:** Pay attention to every word and phrase, especially words like "less than," "more than," "twice," "per," etc.

- **Define Variables Clearly:** Assign variables at the start and keep track of what they represent.
- **Translate Step-by-Step:** Convert each part of the sentence into an algebraic expression.
- **Write the Equation Explicitly:** Combine the expressions to form the full equation.
- **Solve and Verify:** Always check your solution by substituting back into the original statement.
- **Practice Multiple Problems:** The more problems you solve, the more intuitive this process becomes.

Conclusion and Summary

In this comprehensive exploration, we've examined the problem:

"The product of 8 and a number is 25 less than twice that number."

We:

- Interpreted the problem statement carefully.
- Translated the words into the algebraic equation:

$$8x = 2x - 25$$

- Solved the equation step by step to find:

$$x = -\frac{25}{6}$$

- Verified the solution to confirm its correctness.

This process highlights key algebraic techniques—formulating equations from word problems, solving linear equations, and verifying solutions. The problem also emphasizes the importance of precise language interpretation and systematic problem-solving strategies.

Mastering such problems builds a strong foundation for tackling more complex algebraic and mathematical challenges, fostering critical thinking, analytical skills, and confidence in handling quantitative data.

In summary, understanding the problem's language, constructing the correct equation, methodically solving, and verifying are essential steps in algebra. The problem "The product of 8 and a number is 25 less than twice that number" serves as an excellent example to practice these skills, reinforcing the core principles of algebraic reasoning and problem-solving.

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