What Is An Equation Of The Line That Passes Through The Points (3,6) And (1,2)?

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Understanding how to find the equation of a line that passes through two given points is a fundamental concept in algebra and coordinate geometry. It enables us to describe the line precisely, analyze its properties, and apply this knowledge to various mathematical and real-world problems. In this article, we will explore the step-by-step process of determining the equation of a line passing through the points (3,6) and (1,2). We will cover essential concepts such as the slope of a line, the point-slope form, and the slope-intercept form, providing detailed explanations and examples along the way.

Introduction to the Equation of a Line

A line in a two-dimensional coordinate plane can be described mathematically by an equation. The most common forms are:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

Each form provides a different perspective and utility depending on the problem at hand. When given two points, the goal is often to find the equation in slope-intercept form, which makes it easy to interpret the line's slope and y-intercept.

Understanding the Key Concepts

Before diving into the calculations, it's important to understand some fundamental concepts:

What Is the Slope of a Line?

The slope (m) measures how steep a line is. It is defined as the ratio of the change in y-coordinates to the change in x-coordinates between two points:

$$ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} $$

This value indicates how much y increases or decreases as x increases.

Why Is Slope Important?

The slope helps us understand the line's direction:

- A positive slope indicates the line rises from left to right.
- A negative slope indicates the line falls from left to right.
- A zero slope indicates a horizontal line.
- An undefined slope (division by zero) indicates a vertical line.

What Is the Equation of a Line?

The equation of a line describes all points (x, y) that lie on it. Knowing two points allows us to determine the line's slope and its equation.

Step-by-Step Process to Find the Equation of the Line

Let's now proceed step by step to find the equation of the line passing through the points (3,6) and (1,2).

Step 1: Identify the Coordinates

Given points:

- Point 1: $(x_1, y_1) = (3, 6)$
- Point 2: $(x_2, y_2) = (1, 2)$

Step 2: Calculate the Slope (m)

Using the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the known values:

$$m = \frac{2 - 6}{1 - 3} = \frac{-4}{-2} = 2$$

The slope of the line is 2.

Step 3: Choose a Point and Use the Point-Slope Form

The point-slope form is:

$$ y - y_1 = m(x - x_1) $$

Choose either point; here, we'll use (3, 6):

$$ y - 6 = 2(x - 3) $$

Step 4: Simplify to Slope-Intercept Form ($y = mx + b$)

Distribute the slope:

$$ y - 6 = 2x - 6 $$

Add 6 to both sides:

$$ y = 2x - 6 + 6 $$

$$ y = 2x $$

The equation in slope-intercept form is:

$$ y = 2x $$

This is the equation of the line passing through (3,6) and (1,2).

Verifying the Line Equation

To ensure our equation is correct, substitute the x-values of the original points into the line’s equation and verify the y-values:

- For (3,6):

$$ y = 2(3) = 6 $$

- For (1,2):

$$ y = 2(1) = 2 $$

Both points satisfy the equation, confirming its correctness.

Alternative Forms of the Line Equation

While the slope-intercept form is often most convenient, other forms may be useful:

Standard Form

Rearranged from $y = 2x$:

$$ y - 2x = 0 $$

Or:

$$ 2x - y = 0 $$

Point-Slope Form (Reiterated)

Using point (1, 2):

$$ y - 2 = 2(x - 1) $$

This form is useful when you want to write the equation directly from a point and the slope.

Applications of the Equation of a Line

Knowing the equation of a line passing through two points has several practical applications:

- Graphing lines: Visualizing relationships between variables.
- Solving geometric problems: Finding intersections, distances, or midpoints.
- Modeling real-world situations: Analyzing trends in data.
- Calculus and physics: Describing motion or rates of change.

Common Mistakes and Tips

When calculating the equation of a line through two points, watch out for:

- Incorrect slope calculation: Remember to subtract the y-values and x-values in the correct order.
- Division by zero: If $x_1 = x_2$, the line is vertical, and the equation is x = constant.
- Sign errors: Carefully handle negative signs during calculations.
- Using the wrong point: Ensure you substitute the correct point into the point-slope form.

Practice Problems

To solidify your understanding, try solving these:

1. Find the equation of the line passing through (0, 0) and (4, 8).

2. Determine the line passing through (-2, 5) and (2, -1).

3. Write the equation of a vertical line passing through x = 7.

4. Find the slope and equation of the line passing through (5, -3) and (5, 4).

Answers:

1. Slope m = (8 - 0)/(4 - 0) = 2; Equation: y = 2x
2. Slope m = (-1 - 5)/(2 - (-2)) = (-6)/4 = -3/2; Using point (-2, 5): y - 5 = -3/2(x + 2); slope-intercept form: y = -3/2 x + 2
3. Equation: x = 7
4. Slope is undefined; vertical line x = 5

Conclusion

Determining the equation of a line that passes through specific points is a foundational skill in algebra and coordinate geometry. By understanding how to calculate the slope and apply the point-slope or slope-intercept forms, you can accurately describe any line given two points. This knowledge is essential for solving geometric problems, graphing functions, and applying mathematical concepts across various disciplines.

Remember, practice is key to mastering these techniques. With consistent effort, you'll be able to quickly and accurately find the equations of lines in many different contexts, unlocking a deeper understanding of the relationships within the coordinate plane.

Frequently Asked Questions

How do I find the equation of a line passing through the points (3,6) and (1,2)?

First, find the slope using $(y_2 - y_1) / (x_2 - x_1)$, then use point-slope form with one of the points to write the equation.

What is the slope of the line passing through (3,6) and (1,2)?

The slope is $(6 - 2) / (3 - 1) = 4 / 2 = 2$.

How do I write the equation of the line using the slope and a point?

Use the point-slope form: $y - y_1 = m(x - x_1)$. For example, with point (3,6) and slope 2, it becomes $y - 6 = 2(x - 3)$.

What is the slope-intercept form of the line passing through (3,6) and (1,2)?

After finding the slope, simplify the point-slope form to $y = mx + b$. For this line, the equation is $y = 2x + 0$, or simply $y = 2x$.

Is the equation of the line passing through (3,6) and (1,2) unique?

Yes, for two points, there is exactly one straight line passing through both, and thus a unique equation.

Can I verify the equation passes through both points?

Yes, by plugging in the x-values of each point into the equation and checking if the y-values match.

What is the final equation of the line passing through (3,6) and (1,2)?

The equation is $y = 2x$, since the slope is 2 and it passes through the origin when simplified.

How do I convert the point-slope form to slope-intercept form?

Solve for y in the point-slope form $y - y_1 = m(x - x_1)$ to get $y = mx + (y_1 - m x_1)$. For these points, it simplifies to $y = 2x$.

Additional Resources

Equation of a Line

Understanding the equation of a line that passes through specific points is fundamental in algebra and analytic geometry. Whether you're a student mastering the basics or a seasoned mathematician revisiting core concepts, grasping how to derive the line's equation from given points is essential. In this article, we will thoroughly explore the process of finding the equation of the line passing through the points (3, 6) and (1, 2), providing detailed explanations, step-by-step procedures, and practical insights.

Introduction to the Equation of a Line

Before delving into the specific points, it’s crucial to understand what an equation of a line represents and the different forms it can take.

What Is an Equation of a Line?

An equation of a line is a mathematical expression that describes all the points (x, y) lying on that line. It provides a relationship between the x-coordinates and y-coordinates of these points. The most common forms include:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

Here, m represents the slope of the line, while (x_1, y_1) is a specific point the line passes through. Choosing the appropriate form depends on the given data and the context of the problem.

Understanding the Points (3, 6) and (1, 2)

Given the points:

- Point A: (3, 6)
- Point B: (1, 2)

Our goal is to determine the equation of the straight line that passes through both these points.

Significance of These Points

- Both points are in the Cartesian plane, with their respective x and y coordinates.
- The points are distinct, ensuring the line is well-defined (not a vertical or horizontal line unless the points happen to share the same x or y coordinate).
- These points can be used to calculate the slope, which is fundamental in deriving the line's equation.

Step-by-Step Process to Find the Equation of the Line

To determine the line's equation passing through the points (3, 6) and (1, 2), we follow a systematic approach:

1. Calculating the Slope (m)

The slope indicates the steepness of the line and is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying the coordinates:

$$
m = \frac{2 - 6}{1 - 3} = \frac{-4}{-2} = 2
$$

Interpretation:
The slope m is 2, meaning for every one unit increase in x, y increases by 2 units.

2. Choosing a Point for the Equation

Once the slope is known, we can use either of the original points to write the equation. Typically, the point-slope form is convenient:

\[

y - y_1 = m(x - x_1)

\]

Selecting point (3, 6):

\[

y - 6 = 2(x - 3)

\]

3. Deriving the Equation in Slope-Intercept Form

Expanding and simplifying:

\[

y - 6 = 2x - 6

\]

\[

y = 2x - 6 + 6

\]

\[

y = 2x

\]

Result: The equation of the line in slope-intercept form is:

\[

\boxed{y = 2x}

\]

Verification and Additional Considerations

Checking the Equation with the Second Point

To ensure accuracy, substitute (1, 2) into y = 2x:

$$
y = 2(1) = 2
$$

which matches the y-coordinate of the second point. This confirms that the derived equation is correct.

Special Cases and Variations

- Vertical Line: If the two points had the same x-coordinate (e.g., (2, 3) and (2, 7)), the line would be vertical, and its equation would be x = 2.
- Horizontal Line: If the y-coordinates were identical (e.g., (4, 5) and (7, 5)), the line would be horizontal: y = 5.

In this case, neither occurs, so the line is neither vertical nor horizontal.

Alternative Forms of the Equation

While y = 2x is the most straightforward form, other representations can be useful depending on context.

Point-Slope Form

Using point (1, 2):

$$
y - 2 = 2(x - 1)
$$

This form is particularly handy when the line must pass through a specific point, and the slope is known.

Standard Form

Rearranging y = 2x:

$$
y - 2x = 0
$$

or

$$
2x - y = 0
$$

Standard form is often used in algebraic contexts or when analyzing the coefficients.

Practical Applications and Insights

Understanding how to derive the equation of a line from two points isn't just an academic exercise; it has numerous practical applications:

- Engineering and Physics: Calculating trajectories, slopes of ramps, or forces.
- Computer Graphics: Rendering lines between points in visualizations.
- Data Analysis: Fitting linear models to data points.
- Navigation and Mapping: Plotting routes or boundaries.

Furthermore, mastering this process enhances problem-solving skills and deepens comprehension of geometric relationships.

Summary and Key Takeaways

- The slope of the line passing through (3, 6) and (1, 2) is 2.
- Using the point-slope form with either point leads to the same final equation.
- The equation of the line in slope-intercept form is:

$$
\boxed{y = 2x}
$$

- Verification confirms the correctness of the derived equation.

- The process underscores the importance of calculating the slope accurately and choosing an appropriate point for substitution.

Final Thoughts

Deriving the equation of a line through two points is a foundational skill that combines algebraic manipulation with geometric intuition. By understanding the steps—calculating the slope, selecting a point, and applying the point-slope form—you can confidently articulate the precise relationship between x and y for any two distinct points.

In the specific case of points (3, 6) and (1, 2), the simplicity of the resulting equation $y = 2x$ reflects the direct proportionality between the variables. This example showcases the elegance of algebraic methods in translating geometric concepts into precise mathematical expressions—a vital competency across numerous scientific and mathematical disciplines.

Whether for academic pursuits, professional applications, or personal curiosity, mastering this process unlocks a deeper understanding of the interconnectedness of points, lines, and equations in the Cartesian plane.

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