

What Is The Common Ratio Of The Geometric Sequence Below? 96, 48, 24, 12, 6, ...

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Understanding geometric sequences is fundamental in mathematics, especially in fields such as finance, physics, computer science, and engineering. These sequences showcase a pattern where each term is derived by multiplying the previous term by a constant value called the common ratio. Recognizing and calculating this ratio is crucial for analyzing the behavior of geometric sequences and applying this knowledge to real-world problems.

In this article, we will explore the geometric sequence: 96, 48, 24, 12, 6, ... and determine its common ratio. We will also delve into the concepts surrounding geometric sequences, how to identify the common ratio, and explore practical applications to give you a comprehensive understanding of this mathematical concept.

Understanding Geometric Sequences

What Is a Geometric Sequence?

A geometric sequence is a list of numbers where each term after the first is found by multiplying the previous term by a fixed, non-zero number called the common ratio (denoted as r). The general form of a geometric sequence is:

$$[a, \quad ar, \quad ar^2, \quad ar^3, \quad \ldots]$$

where:

- a is the first term,
- r is the common ratio,
- $r \neq 0$.

For example, in the sequence 3, 6, 12, 24, 48, ..., the common ratio is 2 because each term is multiplied by 2 to get the next.

Why Is the Common Ratio Important?

The common ratio determines the nature of the sequence:

- If $|r| > 1$, the sequence grows exponentially.
- If $0 < |r| < 1$, the sequence decreases exponentially and approaches zero.
- If $r = 1$, the sequence is constant.
- If $r = -1$, the sequence alternates between two values.

Knowing the common ratio allows mathematicians and students to predict future terms, analyze the behavior of the sequence, and solve related problems efficiently.

Analyzing the Sequence: 96, 48, 24, 12, 6, ...

Step 1: Identify the First Term

In the sequence provided:

$$\text{First term } a_1 = 96$$

This is straightforward as the sequence begins with 96.

Step 2: Find the Common Ratio

The key to understanding this sequence is to determine the constant multiplier between successive terms. To do this, divide a term by its previous term:

$$r = \frac{\text{Next term}}{\text{Previous term}}$$

Let's compute the ratios between successive terms:

- $r_1 = \frac{48}{96} = 0.5$
- $r_2 = \frac{24}{48} = 0.5$
- $r_3 = \frac{12}{24} = 0.5$
- $r_4 = \frac{6}{12} = 0.5$

Since all these ratios are equal to 0.5, we confirm that the sequence is geometric with a common ratio:

$$r = 0.5$$

This ratio indicates that each successive term is half of the previous one.

Step 3: Confirm the Pattern

Given the common ratio, the sequence can be expressed as:

$$a_n = a_1 \times r^{n-1}$$

where:

- $a_1 = 96$,
- $r = 0.5$,
- n is the term number.

Let's verify this with the third term ($n=3$):

$$a_3 = 96 \times (0.5)^{3-1} = 96 \times (0.5)^2 = 96 \times 0.25 = 24$$

which matches the sequence provided.

Properties of the Sequence with Common Ratio 0.5

Sequence Behavior

Because the common ratio is 0.5, which is less than 1 but greater than 0, the sequence decreases exponentially:

- The terms are halving with each step.
- The sequence approaches zero as $n \rightarrow \infty$.

Mathematical Significance

This sequence demonstrates exponential decay, a concept applicable in various fields such as:

- Radioactive decay in physics.
- Depreciation of assets in finance.
- Population decline models.
- Circuit damping in engineering.

Calculating the Common Ratio in Other Sequences

While the sequence given is straightforward, understanding how to find the

common ratio in more complex sequences is vital.

Method 1: Division of Successive Terms

As shown earlier, divide each term by its immediate predecessor:

$$r = \frac{a_{n+1}}{a_n}$$

This method works when the sequence is known and the terms are non-zero.

Method 2: Using the First and Nth Terms

If you know the first term a_1 , the n -th term a_n , and the sequence is geometric:

$$a_n = a_1 \times r^{n-1}$$

then,

$$r = \left(\frac{a_n}{a_1} \right)^{\frac{1}{n-1}}$$

This is useful when you have the first and the last known terms of the sequence.

Practical Applications of Geometric Sequences with Ratio 0.5

Understanding sequences with ratios like 0.5 is not just theoretical; it has real-world applications:

1. Radioactive Decay

Radioactive substances decay exponentially over time. The amount remaining after each half-life is half of the previous amount, making the common ratio 0.5.

2. Financial Depreciation

Assets such as vehicles or machinery often depreciate by a fixed percentage each year, modeled as a geometric sequence with ratio less than 1.

3. Population Decline

Certain species or populations decline exponentially due to environmental factors, with ratios indicating the proportion of the population that survives over time.

4. Signal Damping in Electronics

Electrical signals in damping circuits decay exponentially with each cycle, characterized by ratios similar to 0.5.

5. Data Compression and Algorithms

Algorithms that halve data sizes or perform recursive halving utilize geometric principles with ratios of 0.5.

Summary and Key Takeaways

- The sequence 96, 48, 24, 12, 6, ... is a geometric sequence.
- The common ratio (r) can be found by dividing any term by its immediate predecessor, resulting in $(r = 0.5)$.
- The sequence demonstrates exponential decay, halving each time.
- Recognizing the common ratio helps in predicting future terms, analyzing the sequence's behavior, and applying it in various scientific and financial contexts.
- The concept of the common ratio extends beyond simple sequences and is fundamental in understanding exponential models across disciplines.

Conclusion

Determining the common ratio of a geometric sequence is a fundamental skill in mathematics, enabling deeper analysis of exponential patterns. In the sequence 96, 48, 24, 12, 6, ..., the common ratio is clearly 0.5, illustrating a sequence of exponential decay by halving. This understanding not only helps in solving mathematical problems but also provides insight into natural phenomena, financial models, and technological processes that exhibit similar geometric behaviors.

By mastering the calculation and implications of the common ratio, students and professionals can better analyze and interpret sequences and exponential models in their respective fields.

Frequently Asked Questions

What is the common ratio of the geometric sequence 96, 48, 24, 12, 6, ...?

The common ratio is found by dividing any term by the previous term. For this sequence, $48 \div 96 = 0.5$, so the common ratio is 0.5.

How do you determine the common ratio in a geometric sequence?

You determine the common ratio by dividing any term by the term immediately before it. For example, in the sequence 96, 48, 24, ..., $48 \div 96 = 0.5$.

What is the value of the common ratio in the sequence 96, 48, 24, ...?

The common ratio is 0.5 because each term is half of the previous one.

Is the common ratio in this sequence positive or negative?

The common ratio is positive, specifically 0.5.

Can the common ratio be a fraction in a geometric sequence?

Yes, the common ratio can be a fraction. In this case, it is 0.5.

What pattern does the sequence 96, 48, 24, 12, 6, ... follow?

It follows a geometric pattern where each term is multiplied by 0.5 to get the next term.

If the first term of the sequence is 96 and the common ratio is 0.5, what is the third term?

The third term is 24, calculated as $96 \times 0.5 \times 0.5 = 24$.

How can you verify the common ratio of a geometric sequence?

By dividing any term by its preceding term; if the quotient is consistent

across the sequence, that's the common ratio.

What is the significance of the common ratio in a geometric sequence?

The common ratio determines the rate at which the sequence grows or shrinks; in this sequence, it indicates the sequence is decreasing by half each time.

Is the common ratio in the sequence 96, 48, 24, 12, 6, ... greater than, less than, or equal to 1?

The common ratio is less than 1, specifically 0.5, indicating a decreasing sequence.

Additional Resources

What Is The Common Ratio Of The Geometric Sequence Below? 96, 48, 24, 12, 6, ...

Understanding the common ratio of a geometric sequence is fundamental in grasping the characteristics of such sequences. In the sequence 96, 48, 24, 12, 6, ..., identifying the common ratio allows us to analyze the pattern, predict future terms, and explore the sequence's behavior mathematically. This article aims to thoroughly examine what the common ratio is, how to compute it, and what implications it has for the given sequence.

What Is a Geometric Sequence?

Before delving into the specifics of the sequence provided, it is essential to understand what a geometric sequence entails.

Definition of a Geometric Sequence

A geometric sequence is a sequence of numbers where each term after the first is obtained by multiplying the previous term by a constant value called the common ratio. Formally, a geometric sequence can be expressed as:

$[a, ar, ar^2, ar^3, \dots]$

where:

- a is the first term,
- r is the common ratio,
- and n indicates the position of the term in the sequence.

Characteristics of a Geometric Sequence

- The ratio between consecutive terms remains constant.
- The sequence can either grow exponentially (if $|r| > 1$) or decay exponentially (if $|r| < 1$).
- Useful in modeling real-world phenomena such as population growth, radioactive decay, and financial calculations involving compound interest.

Understanding the Given Sequence

Given sequence:

$[96, 48, 24, 12, 6, \dots]$

This sequence appears to follow a pattern where each term is derived by multiplying the previous term by a specific ratio. To find out what this ratio is, we need to analyze the relationship between successive terms.

Step-by-Step Analysis

- The first term $(a_1 = 96)$.
- The second term $(a_2 = 48)$.
- The third term $(a_3 = 24)$.
- The fourth term $(a_4 = 12)$.
- The fifth term $(a_5 = 6)$.

To find the common ratio (r) , we can divide any term by its preceding term:

$[r = \frac{a_{n+1}}{a_n}]$

Calculating the ratios:

- $(r = \frac{48}{96} = 0.5)$
- $(r = \frac{24}{48} = 0.5)$
- $(r = \frac{12}{24} = 0.5)$
- $(r = \frac{6}{12} = 0.5)$

The consistent ratio indicates that the common ratio (r) is 0.5.

Features and Significance of the Common Ratio in This Sequence

Understanding the common ratio's value helps in analyzing the sequence's behavior and predicting future terms.

Features of the Sequence with $(r = 0.5)$

- Decreasing Sequence: Since (r) is less than 1, the sequence decreases exponentially.
- Halving Pattern: Each term is exactly half of the previous term.
- Predictability: Future terms can be easily calculated using the common ratio.

Mathematical Implications

- The sequence can be expressed explicitly as:

$$[a_n = a_1 \times r^{n-1}]$$

- For this sequence:

$$[a_n = 96 \times (0.5)^{n-1}]$$

- For example, the 6th term:

$$[a_6 = 96 \times (0.5)^5 = 96 \times \frac{1}{32} = 3]$$

which aligns with the pattern.

How to Calculate the Common Ratio

Calculating the common ratio is straightforward in most cases, but understanding the method helps reinforce the concept.

Method 1: Divide Successive Terms

- Take two consecutive terms (a_{n+1}) and (a_n) .
- Compute $(r = \frac{a_{n+1}}{a_n})$.

In the case of the sequence:

$$r = \frac{a_2}{a_1} = \frac{48}{96} = 0.5$$

Consistency across all pairs confirms the ratio.

Method 2: Use the First and nth Terms

- If you know the first term (a_1) and the (n) th term (a_n) :

$$r^{n-1} = \frac{a_n}{a_1}$$

- Solve for (r) :

$$r = \left(\frac{a_n}{a_1}\right)^{\frac{1}{n-1}}$$

Applying this to find (r) using the 5th term:

$$r = \left(\frac{6}{96}\right)^{\frac{1}{4}} = (0.0625)^{0.25} \approx 0.5$$

Applications and Real-World Significance

The concept of the common ratio extends beyond pure mathematics into various practical fields.

Financial Calculations

- Compound interest calculations often involve geometric sequences.
- Understanding the ratio helps in predicting investment growth or decay.

Physics and Natural Sciences

- Radioactive decay follows a geometric pattern with a ratio less than 1.
- Population models with decreasing populations can be analyzed similarly.

Engineering and Technology

- Signal processing and digital filters utilize exponential decay models.

Pros and Cons of Using Geometric Sequences

While geometric sequences are powerful tools, they come with their strengths and limitations.

Pros

- Simple to analyze: Once the common ratio is known, future terms are easy to compute.
- Applicable in many fields: From finance to physics.
- Predictive Power: Helps in forecasting trends and behaviors.

Cons

- Limited to multiplicative patterns: Not suitable for sequences that follow additive or more complex rules.
- Assumes constant ratio: Real-world data may involve variability, making the model less accurate.
- Exponential decay or growth: Can lead to very rapid decrease or increase, which may not always be realistic.

Summary and Final Thoughts

In conclusion, the common ratio of the sequence 96, 48, 24, 12, 6, ... is 0.5. This ratio indicates that each term is half of the previous one, exemplifying exponential decay. Recognizing the common ratio is crucial in understanding the sequence's nature, predicting subsequent terms, and applying the concept in real-world scenarios. Whether in finance, physics, or engineering, the principles of geometric sequences provide valuable tools for analysis and problem-solving.

By mastering how to identify and compute the common ratio, students and professionals alike can unlock deeper insights into the patterns that govern various phenomena, making geometric sequences an essential component of mathematical literacy.

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