# What Is The Exact Value Of Each Expression? Do Not Use A Calculator.c. Sec 3i

**What Is The Exact Value Of Each Expression? Do Not Use A Calculator.c. Sec 3i**

Understanding how to find the exact value of trigonometric expressions without relying on a calculator is a fundamental skill in mathematics, especially in the study of trigonometry. When presented with complex expressions involving secants, sines, cosines, tangents, and their inverse functions, it is crucial to utilize known identities, special angles, and geometric interpretations. This article delves into methods and strategies to evaluate these expressions accurately, focusing on the problem: What Is The Exact Value Of Each Expression? Do Not Use A Calculator.c. Sec 3i.

## Understanding Secant and Its Properties

Before tackling specific expressions, it is essential to review the definition and properties of the secant function.

### Definition of Secant

- Secant (sec) is the reciprocal of cosine: **sec θ = 1 / cos θ**.
- It is undefined when cos θ = 0 (i.e., at odd multiples of 90° or π/2 radians).

### Key Values of Secant at Special Angles

- At 0°, sec 0° = 1
- At 30°, sec 30° = 2 / √3
- At 45°, sec 45° = √2
- At 60°, sec 60° = 2
- At 90°, sec 90° is undefined

Knowing these values helps evaluate expressions involving secant at common angles.

# Strategies for Evaluating Expressions Without a Calculator

Approach to finding the exact value involves:

## 1. Recognizing Special Angles

- Angles such as 0°, 30°, 45°, 60°, and 90° have well-known sine, cosine, and secant values.
- Express the angles in radians if necessary: 30° = π/6, 45° = π/4, 60° = π/3.

## 2. Using Trigonometric Identities

- Reciprocal identities: sec θ = 1 / cos θ
- Pythagorean identities: sin^2 θ + cos^2 θ = 1
- Complementary angles: sec (90° - θ) = csc θ
- Double angle and half-angle formulas if needed

## 3. Simplifying Complex Expressions

- Break down complicated expressions into simpler parts.
- Substitute known values or identities.
- Look for patterns or symmetries in the angles.

# Example Problems and Solutions

Let's apply these strategies to evaluate some typical expressions involving secant, sine, and cosine.

## Example 1: Find the exact value of sec 45°

Since 45° is a special angle, we know:

- cos 45° = √2 / 2

Therefore:

- sec 45° = 1 / cos 45° = 1 / (√2 / 2) = 2 / √2 = √2

Answer: **sec 45° = √2**

## Example 2: Evaluate sec 60°

Recall that:

- cos 60° = 1 / 2

Thus:

- sec 60° = 1 / (1/2) = 2

Answer: **sec 60° = 2**

## Example 3: Find the value of sec 30°

From known values:

- cos 30° = √3 / 2

Therefore:

- sec 30° = 1 / (√3 / 2) = 2 / √3

To rationalize the denominator:

- sec 30° = (2 / √3) (√3 / √3) = (2√3) / 3

Answer: **sec 30° = (2√3) / 3**

# Evaluating Expressions Involving Multiple Trigonometric Functions

Sometimes, the problem involves more complex expressions, such as sec θ + tan θ or sec^2 θ - tan^2 θ. These can be simplified using identities.

## Example 4: Simplify sec^2 θ - tan^2 θ

Recall the fundamental Pythagorean identity:

- $\sec^2 \theta - \tan^2 \theta = 1$

This is a key identity that can be memorized and used to evaluate expressions directly.

Answer: **$\sec^2 \theta - \tan^2 \theta = 1$**

## Example 5: Find the exact value of sec 75°

Since 75° = 45° + 30°, we can use the sum formula for secant:

- $\sec(A + B) = 1 / \cos(A + B)$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Calculate cos 45° and cos 30°, and similarly for sine:

- $\cos 45° = \sqrt{2} / 2$
- $\sin 45° = \sqrt{2} / 2$
- $\cos 30° = \sqrt{3} / 2$
- $\sin 30° = 1 / 2$

Applying the formula:

- $\cos 75° = (\sqrt{2} / 2)(\sqrt{3} / 2) - (\sqrt{2} / 2)(1 / 2) = (\sqrt{6} / 4) - (\sqrt{2} / 4) = (\sqrt{6} - \sqrt{2}) / 4$

Thus:

- $\sec 75° = 1 / \cos 75° = 4 / (\sqrt{6} - \sqrt{2})$

Rationalize the denominator:

- sec 75° = 4 / (√6 - √2) (√6 + √2) / (√6 + √2) = 4(√6 + √2) / [(√6)^2 - (√2)^2] = 4(√6 + √2) / (6 - 2) = 4(√6 + √2) / 4 = √6 + √2

Answer: **sec 75° = √6 + √2**

# Additional Tips for Exact Values

- Memorize key values of sine, cosine, tangent, secant, cosecant, and cotangent at special angles.
- Use symmetry and complementary angles to find values indirectly.
- Recognize common identities and formulas to simplify expressions.
- Practice with known angle sums and differences for complex angles.
- Rationalize denominators whenever necessary to express exact values in simplest radical form.

# Conclusion

Determining the exact value of each trigonometric expression without a calculator is a skill that enhances understanding of fundamental identities and the properties of special angles. By leveraging known values, identities, and algebraic simplification, you can evaluate complex expressions with confidence. Remember to familiarize yourself with the key values of trigonometric functions at special angles, understand the core identities, and practice a variety of problems. With consistent practice, finding the exact values of secant and other trigonometric expressions becomes an intuitive process rather than a daunting task.

# Frequently Asked Questions

## What is the value of sec 0° without using a calculator?

The value of sec 0° is 1 because sec θ = 1/cos θ and cos 0° = 1, so sec 0° = 1/1 = 1.

## How do you find the value of sec 90° without a calculator?

Since cos 90° = 0, sec 90° = 1/0 which is undefined; therefore, sec 90° is undefined.

## What is the value of sec 180°?

cos 180° = -1, so sec 180° = 1/(-1) = -1.

## Calculate sec 60° without a calculator.

cos 60° = 0.5, so sec 60° = 1/0.5 = 2.

## What is the value of sec 45°?

cos 45° = √2/2, so sec 45° = 1/(√2/2) = 2/√2 = √2.

## Find the value of sec 30° without using a calculator.

cos 30° = √3/2, so sec 30° = 1/(√3/2) = 2/√3, which can be rationalized to (2√3)/3.

## What is sec 150°?

cos 150° = -√3/2, so sec 150° = 1/(-√3/2) = -2/√3, which rationalizes to -(2√3)/3.

## What is the value of sec 270°?

Since cos 270° = 0, sec 270° = 1/0 which is undefined.

## Calculate sec 120° without a calculator.

cos 120° = -1/2, so sec 120° = 1/(-1/2) = -2.

## Why is sec 90° considered undefined?

Because cos 90° = 0, and sec θ = 1/cos θ, dividing by zero makes sec 90° undefined.

# Additional Resources

Understanding the Exact Value of Each Expression: Do Not Use A Calculator. c. Sec 3i

When approaching trigonometric expressions, especially those involving secant functions, it's crucial to understand the foundational concepts, identities, and methods to evaluate these expressions precisely without the aid of a calculator. This comprehensive guide aims to delve deep into the principles, techniques, and step-by-step processes needed to determine the exact value of secant-related expressions, particularly those specified as "Sec 3i" in the curriculum.

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# Introduction to Secant Function (sec)

Before tackling specific expressions, it's essential to understand what the secant function is and how it relates to other trigonometric functions.

## Definition of Secant

- The secant function, denoted as $\sec(\theta)$, is defined as the reciprocal of the cosine function:

$$
\boxed{ \sec \theta = \frac{1}{\cos \theta} }
$$

- It is undefined when $\cos \theta = 0$, i.e., at odd multiples of $\frac{\pi}{2}$.

## Domain and Range

- Domain: All real numbers $\theta$ where $\cos \theta \neq 0$.

- Range: $(-\infty, -1] \cup [1, \infty)$, since secant takes values outside the interval $(-1, 1)$.

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# Fundamental Identities and Properties

Understanding the core identities helps in simplifying and evaluating complex secant expressions.

## Basic Pythagorean Identity involving sec

$$
1 + \tan^2 \theta = \sec^2 \theta
$$

- This identity allows us to express secant in terms of tangent or vice versa.

## Relationship with Cosine

$$
\sec \theta = \frac{1}{\cos \theta}
$$

- Therefore, knowing $\cos \theta$ directly helps determine $\sec \theta$.

### Complementary Angles

- Since $\cos$ and $\sin$ are co-functions:

$$
\sec (90^\circ - \theta) = \frac{1}{\sin \theta}
$$

- In radians:

$$
\sec \left( \frac{\pi}{2} - \theta \right) = \frac{1}{\sin \theta}
$$

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## Approach to Calculating Exact Values of sec(θ)

When asked to evaluate expressions like $\sec \theta$ without a calculator, the key steps include:

1. Identify the angle $\theta$: Recognize if the angle is common (like $30^\circ$, $45^\circ$, $60^\circ$, or their radian equivalents).

2. Use known special angle values: Recall the exact values of sine and cosine at these angles.

3. Express secant in terms of cosine: Since $\sec \theta = 1 / \cos \theta$, focus on calculating $\cos \theta$ first.

4. Simplify the expression: Use algebraic or geometric methods to simplify.

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## Common Special Angles and Their Secant Values

| Angle | Degrees | Radians | $\cos \theta$ | $\sec \theta = 1 / \cos \theta$ |
|---|---|---|---|---|
| 0° | 0° | 0 | 1 | 1 |
| 30° | $\pi/6$ | 30° | $\sqrt{3}/2$ | $2 / \sqrt{3}$ |
| 45° | $\pi/4$ | 45° | $\sqrt{2}/2$ | $\sqrt{2}$ |
| 60° | $\pi/3$ | 60° | 1/2 | 2 |
| 90° | $\pi/2$ | 90° | 0 | undefined |

Knowing these exact values provides a strong foundation for evaluating secant expressions involving standard angles.

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# Evaluating Specific Expressions: Step-by-Step Examples

Let's explore some typical expressions labeled as "Sec 3i" and similar, focusing on their exact evaluation.

## Example 1: $\sec 60^\circ$

- Step 1: Recall $\cos 60^\circ = 1/2$.

- Step 2: Use the definition:

$$
\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{1/2} = 2
$$

- Result: $\boxed{2}$

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## Example 2: $\sec 45^\circ$

- Step 1: Recall $\cos 45^\circ = \frac{\sqrt{2}}{2}$.

- Step 2: Calculate:

$$
\sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}
$$

- Result: $\boxed{\sqrt{2}}$

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## Example 3: $\sec 30^\circ$

- Step 1: Recall $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

- Step 2: Compute:

$$
\sec 30^\circ = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}
$$

- Optional Rationalization: Multiply numerator and denominator by $\sqrt{3}$:

$$
\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2 \sqrt{3}}{3}
$$

- Result: $\boxed{\frac{2 \sqrt{3}}{3}}$

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# Evaluating Expressions Involving Multiple Angles

Some expressions involve angles beyond the basic angles, requiring the application of identities and angle formulas.

## Example 4: $\sec (180^\circ - \theta)$

- Step 1: Recall $\cos (180^\circ - \theta) = -\cos \theta$.

- Step 2: Therefore,

$$
\sec (180^\circ - \theta) = \frac{1}{\cos (180^\circ - \theta)} = \frac{1}{-\cos \theta} = -\frac{1}{\cos \theta} = - \sec \theta
$$

- Implication: The secant of an angle supplement is the negative of the secant of the original angle.

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## Example 5: $\sec (90^\circ - \theta)$

- Step 1: Use the co-function identity:

$$
\cos (90^\circ - \theta) = \sin \theta
$$

- Step 2: Hence,

$$
\sec (90^\circ - \theta) = \frac{1}{\sin \theta}
$$

- Result: The reciprocal of sine, which is cosecant:

$$
\boxed{\sec (90^\circ - \theta) = \csc \theta}
$$

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## Using Pythagorean and Other Identities to Find sec Values

When the angle $\theta$ is not standard, or when the expression involves complex angles, identities serve as powerful tools.

### Example 6: $\sec \theta$ when $\sin \theta$ and $\cos \theta$ are known

Suppose $\sin \theta = \frac{3}{5}$, and $\theta$ is in the first quadrant.

- Step 1: Find $\cos \theta$:

$$
\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}
$$

- Step 2: Calculate secant:

$$
\sec \theta = \frac{1}{\cos \theta} = \frac{1}{4/5} = \frac{5}{4}
$$

- Result: $\boxed{\frac{5}{4}}$

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## Evaluating Expressions with Multiple Components

Some problems involve combining secant with other functions, such as tangent, cotangent, or involving algebraic expressions.

**Example 7: Evaluate $\sec^2 \theta - \tan^2 \theta$**