What Is The Smallest Integer Greater Than 800 That Is Relatively Prime To 10!

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When exploring the fascinating world of number theory, one intriguing question arises: *What is the smallest integer greater than 800 that is relatively prime to 10!* This problem involves understanding factorials, prime factors, and the concept of coprimality. In this article, we will delve into the details to identify such an integer, exploring the mathematical principles behind it and providing a comprehensive explanation suitable for enthusiasts and students alike.

Understanding the Basics: What Is 10! and What Does It Mean to Be Relatively Prime?

What Is 10!?

The notation 10! (read as "10 factorial") represents the product of all positive integers from 1 to 10:

- 10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1
- Calculating 10! gives: 3,628,800

This large number encompasses all prime factors involved in the numbers from 1 to 10.

What Does It Mean to Be Relatively Prime?

Two integers are said to be relatively prime or coprime if their greatest common divisor (GCD) is 1. That is, they share no common positive prime factors.

For example:

- 7 and 15 are coprime because GCD(7, 15) = 1.
- 12 and 18 are not coprime because GCD(12, 18) = 6.

In the context of our problem, we need to find the smallest integer greater than 800 that shares no common prime factors with 10!.

Prime Factors of 10! and Their Significance

Prime Factorization of 10!

Since 10! includes all numbers from 1 to 10, its prime factors are all primes less than or equal to 10:

- 2, 3, 5, 7

The exponents of these primes in 10! can be calculated using Legendre's formula, but for our purpose, the key point is that 10! is divisible by all these primes.

Implication for Coprimality

Any integer that shares a common prime factor with 10! must be divisible by at least one of 2, 3, 5, or 7. To be coprime with 10!, the number must not be divisible by any of these primes.

Thus, the problem reduces to identifying the smallest integer greater than 800 that is not divisible by 2, 3, 5, or 7.

Strategy to Find the Smallest Integer Greater Than 800 That Is Relatively Prime to 10!

Step 1: Understand Which Numbers Are Not Allowed

Any number divisible by 2, 3, 5, or 7 is not coprime with 10!. Therefore, these numbers are excluded.

Step 2: Find the Next Candidate After 800

Start from 801 and check each number sequentially:

- Is the number divisible by 2, 3, 5, or 7?
- If yes, discard and move to the next number.
- If no, this number is coprime with 10! and is our candidate.

Step 3: Use Modular Arithmetic to Speed Up the Search

Instead of testing each number individually, analyze the pattern of numbers coprime to these primes.

Analyzing the Pattern of Numbers Coprime to 2, 3, 5, and 7

Residue Classes Modulo the Product of the Primes

The least common multiple (LCM) of 2, 3, 5, and 7 is:

- LCM = 2 × 3 × 5 × 7 = 210

Any number coprime to all four primes must not be divisible by any of them. Therefore, within each cycle of 210, the numbers coprime to 10! are those that are not divisible by 2, 3, 5, or 7.

Finding Numbers Coprime to 10! in Each Cycle

The pattern repeats every 210 numbers. To find the smallest number greater than 800 that is coprime to 10!, we proceed as follows:

- Determine the cycle in which 800 lies.
- Identify the numbers coprime to 210 within that cycle.
- Find the smallest such number greater than 800.

Step-by-Step Solution: Finding the Smallest Integer Greater Than 800 Coprime to 10!

Step 1: Find the Cycle of 800

Divide 800 by 210:

- 800 ÷ 210 ≈ 3.809...

- So, 800 is in the cycle starting at 3 × 210 = 630

The next cycle starts at $4 \times 210 = 840$.

Step 2: List the Numbers in the Cycle Starting at 841

Numbers from 841 to 1050 (which is 5×210) form the next cycle.

Since 800 is less than 840, the next candidate numbers are in the cycle starting at 841.

Step 3: Identify Numbers in the Cycle Coprime to 210

Numbers coprime to 210 are those that are not divisible by 2, 3, 5, or 7.

The numbers coprime to 210 between 841 and 1050 are:

- 841 (which is 29^2, and since 29 is prime, check if it divides by 2, 3, 5, or 7): 29 is prime and not divisible by 2, 3, 5, or 7, so 841 is coprime.

Similarly, check subsequent numbers:

- 842: even → divisible by 2 → exclude.
- 843: sum of digits 8+4+3=15 divisible by 3 → exclude.
- 844: even → exclude.
- 845: ends with 5 → divisible by 5 → exclude.
- 846: even → exclude.
- 847: $847 = 7 \times 121$ → divisible by 7 → exclude.
- 848: even → exclude.
- 849: sum digits 8+4+9=21 divisible by 3 → exclude.
- 850: ends with 0 or 5 → divisible by 5 → exclude.
- 851: check divisibility:
- Not divisible by 2 (odd).
- Sum of digits 8+5+1=14, not divisible by 3.
- Not ending with 0 or 5, so not divisible by 5.
- $7 \times 121=847$, $851-847=4$, so not divisible by 7.
- Check prime factors:
- $851 \div 11=77.36...$ no.
- $13 \times 65=845$, no.
- $23 \times 37=851$ → divisible by 23 and 37.

Since 23 is prime, 851 is divisible by 23, so exclude.

- 852: even → exclude.
- 853:
- Not divisible by 2, 3, 5, or 7.
- 853 is a prime number (verified through primality tests).
- Since prime and not divisible by 2, 3, 5, or 7, 853 is coprime.

Conclusion: The first number greater than 800 in this cycle coprime to 10! is 841, and the next is 853.

Compare 841 and 853:

- 841 is less than 800? No, it's greater than 800.
- 853 is also greater than 800 and coprime to 10!.

Between these, 841 was in the previous cycle (since 841 > 800), but it's important to confirm whether 841 is greater than 800. Yes, 841 > 800.

Is 841 coprime to 10!?
Yes, since $841 = 29^2$, and 29 is prime and not among the primes dividing 10! (2, 3, 5, 7), 29 does not divide 10!, so 841 is coprime.

Is 841 the smallest?
Let's check the number just below 841:

- 840: divisible by 2, 3, 5, 7, so not coprime.

Therefore, 841 is the smallest integer greater than 800 that is coprime to 10!.

Final Answer and Summary

The smallest integer greater than 800 that is

Frequently Asked Questions

What is the smallest integer greater than 800 that is relatively prime to 10!?

The smallest integer greater than 800 that is relatively prime to 10! is 811.

Why does the number 811 qualify as the smallest integer greater than 800 and coprime to 10!?

Because 811 is greater than 800 and does not share any prime factors with 10!, which factors include all primes up to 10; 811 is not divisible by 2, 3, 5, or 7.

What is 10! and why is it relevant to determining the coprimality of numbers?

10! is the factorial of 10, equal to 3,628,800, and it includes all prime factors up to 10. A number is coprime to 10! if it does not share any of these prime factors.

How can I find numbers that are coprime to 10!?

Numbers coprime to 10! are those not divisible by any prime factors of 10!, namely 2, 3, 5, or 7. To find such numbers greater than a certain value, check for divisibility by these primes.

Are there other numbers just above 800 that are coprime to 10!?

Yes, but the smallest such number greater than 800 that is coprime to 10! is 811; others above 800 may not be coprime if they share factors 2, 3, 5, or 7.

What is the significance of identifying numbers coprime to 10!?

Identifying such numbers is important in number theory, combinatorics, and cryptography, especially for understanding coprimality and primitive roots related to factorials.

Could 809 be the smallest integer greater than 800 and coprime to 10!?

No, because 809 is less than 811, but 809 shares a common factor with 10! (since it's prime), or it may not be coprime; in this case, 809 is prime and not divisible by 2, 3, 5, or 7, so it is coprime to 10!, but since it is less than 800, it doesn't meet the 'greater than 800' criterion.

How do prime factors of 10! affect the coprimality of larger numbers?

Any number sharing a prime factor with 10! (which includes 2, 3, 5, 7) will not be coprime to 10!. To be coprime, the number must not be divisible by any of these primes.

What steps can I take to verify if a number is coprime to 10!?

Factor the number or check divisibility by 2, 3, 5, and 7. If it is not divisible by any of these primes, then it is coprime to 10!.

Why is 811 the smallest integer greater than 800 that is coprime to 10!?

Because 811 is the next prime number after 809 that is greater than 800, and it is not divisible by 2, 3, 5, or 7, making it coprime to 10! and the smallest such number above 800.

Additional Resources

What Is The Smallest Integer Greater Than 800 That Is Relatively Prime To 10!

Understanding the problem of identifying integers that are relatively prime to certain numbers involves exploring key concepts in number theory, particularly prime factors and divisibility. In this article, we focus on the specific question: What is the smallest integer greater than 800 that is relatively prime to 10!? This exploration not only deepens our understanding of coprimality but also demonstrates practical problem-solving strategies in mathematics, with applications that extend to cryptography, algorithm design, and advanced number theory.

Introduction to the Problem and Fundamental Concepts

Before delving into the detailed analysis, it is essential to clarify the key terms and concepts involved in this problem.

What Does "Relatively Prime" Mean?

Two integers are said to be relatively prime (or coprime) if their greatest common divisor (GCD) is 1. In other words, they share no common prime factors. For two numbers a and b:

$$
\gcd(a, b) = 1
$$

This property indicates that the two numbers are "independent" in terms of their prime factorizations, sharing no common divisors other than 1.

Understanding 10! (Factorial of 10)

The notation $n!$ represents the factorial of n, which is the product of all positive integers up to n:

$$
10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1
$$

Calculating $10!$:

$$
10! = 3,628,800
$$

\]

The prime factorization of $10!$ reveals the prime factors that are included:

$$
10! = 2^8 \times 3^4 \times 5^2 \times 7 \times \text{(other primes)}
$$

However, since 10! includes all primes less than or equal to 10 (i.e., 2, 3, 5, 7), any number coprime to 10! must not be divisible by these primes.

Prime Factors of 10! and Their Significance

To determine whether a number is coprime to 10!, it is crucial to understand the prime factors contained within 10!. As established, 10! includes the primes 2, 3, 5, and 7, with multiplicities reflecting their exponents in the factorial.

Prime Factors of 10!

- 2: Since 2 appears in the factors 2, 4, 6, 8, and 10, it appears with high multiplicity, specifically 2^8.
- 3: Appears in 3, 6, 9, and 12 (though 12 exceeds 10, so only up to 9), with exponent 3^4.
- 5: Appears in 5 and 10, with exponent 5^2.
- 7: Appears in 7, with exponent 1.

Any number that shares a prime factor with $10!$ must be divisible by at least one of 2, 3, 5, or 7 to not be coprime with $10!$.

Implication:
To be coprime with $10!$, a candidate number must not be divisible by 2, 3, 5, or 7.

Restating the Problem in Mathematical Terms

The core problem reduces to:

> Find the smallest integer $n > 800$ such that:

$$
\gcd(n, 10!) = 1
$$

Given the prime factors of \(10! \), this is equivalent to:

\[
\text{n is not divisible by } 2, 3, 5, \text{ or } 7
\]

This simplifies the problem to finding the smallest integer greater than 800 that is coprime to 2, 3, 5, and 7.

Approach to Finding the Smallest Integer Greater Than 800 That Is Coprime to 10!

The approach involves systematic elimination of numbers based on divisibility rules, understanding of modular arithmetic, and pattern recognition.

Step 1: Identify the Prime Factors to Avoid

Any candidate number must not be divisible by:

- 2 (even numbers)
- 3
- 5
- 7

Thus, the candidate must be coprime to 2, 3, 5, and 7.

Step 2: Search for Candidates Above 800

Given the constraints, candidates must be:

- Odd (not divisible by 2)
- Not divisible by 3, 5, or 7

Because checking divisibility for all numbers above 800 would be impractical, modular arithmetic provides an efficient means to identify candidate numbers.

Step 3: Modular Pattern Analysis

Numbers not divisible by 2, 3, 5, or 7 follow a certain pattern modulo the product of these primes.

- The product:

$$
2 \times 3 \times 5 \times 7 = 210
$$

- All numbers coprime to 2, 3, 5, and 7 are congruent to certain residues modulo 210.

> Key insight:
> All such numbers are congruent to residues that are coprime to 210.

Residue sets coprime to 210 are numbers between 1 and 209 that are not divisible by 2, 3, 5, or 7.

Identifying Residues and the Pattern of Coprimality

Residue Classes of Coprime Numbers Modulo 210

The numbers coprime to 210 within 1 to 210 are:

$$
\{1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97, 101, 103, 107, 109, 113, 121, 123, 127, 131, 137, 139, 143, 149, 151, 157, 163, 167, 169, 173, 179, 181, 187, 191, 193, 197, 199, 209\}
$$

(These are the residues less than 210 that are coprime to 210.)

Note:
In practice, the residues repeat every 210, meaning that any number n with:

$$
n \equiv r \pmod{210}
$$

where r is coprime to 210, will not be divisible by 2, 3, 5, or 7.

Step 4: Find the Candidate Greater Than 800

Since the pattern repeats every 210, to find the smallest number greater than 800 coprime to 10!, we:

1. Find the largest multiple of 210 less than 800:

$$
\left\lfloor \frac{800}{210} \right\rfloor = 3
$$

$$
3 \times 210 = 630
$$

2. The next multiple:

$$
4 \times 210 = 840
$$

3. Now, examine residues above 800, i.e., numbers between 801 and 840.

- Find the residue of 801 modulo 210:

$$
801 \mod 210 = 801 - 3 \times 210 = 801 - 630 = 171
$$

- Similarly, 802:

$$
802 - 3 \times 210 = 802 - 630 = 172
$$

and so on, up to 840.

Explicit Search for the Smallest Suitable Number > 800

Now, for each candidate:

- 801: 171 mod 210 → Check if 171 is in the coprime set.
- 802: 172
- 803: 173
- 804: 174
- ...

We look for the smallest among these residues that are coprime to 210.

From the residues:

- 171: Is 171 coprime to 210?

171 factors as \(3 \times 57 \), so divisible by 3 → No.
- 172: factors include 2 → No.
- 173: Check if coprime to 210:
- 173 is prime, and not divisible by 2, 3, 5, or 7 → Yes.

Since 173 is coprime to 210, the corresponding number:

\[
\text{Number}

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