What Is The Solution Of The Linear-quadratic System Of Equations?y=x^2+5x3yx=2

What Is The Solution Of The Linear-quadratic System Of Equations? y = x^2 + 5x, 3y x = 2

Understanding the solution to the linear-quadratic system of equations can seem complex at first glance, especially when dealing with equations like $y = x^2 + 5x$ and $3y x = 2$. These equations represent a mixture of linear and quadratic relationships, and solving them requires a strategic approach that combines algebraic manipulation and substitution techniques. In this article, we will explore the methods for solving such systems, step-by-step, to help you comprehend the process thoroughly.

What Is a Linear-Quadratic System of Equations?

Before diving into the solution process, it's essential to understand what constitutes a linear-quadratic system of equations.

Definition and Characteristics

- A **linear-quadratic system** involves at least one linear equation and at least one quadratic equation.
- The linear equations are of the form $ax + by + c = 0$, while quadratic equations involve terms like x^2 or y^2.
- These systems often model real-world problems where relationships are partly linear and partly nonlinear.

Common Examples

- $y = x^2 + 5x$ (Quadratic) and $3y x = 2$ (Linear after rearrangement)
- $y = 2x + 3$ and $y^2 = 4x$ (Combination of linear and quadratic)

Understanding the Given System

The specific system we are analyzing is:

$$y = x^2 + 5x$$
$$3y x = 2$$

At first glance, the second equation, $3y x = 2$, appears to involve a product of y and x. To solve this system, we will need to manipulate the equations to express one variable in terms of the other and then substitute accordingly.

Step-by-Step Solution Process

Let's break down how to approach solving this system systematically.

Step 1: Rewrite the Equations Clearly

The equations are:

1. $y = x^2 + 5x$
2. $3 y x = 2$

The second equation involves the product yx, which suggests that expressing y in terms of x or vice versa is essential.

Step 2: Express y from the First Equation

Since y is already expressed as $y = x^2 + 5x$, we can substitute this into the second equation.

Step 3: Substitute y into the Second Equation

Substitute $y = x^2 + 5x$ into $3 y x = 2$:

$$3 (x^2 + 5x) x = 2$$

Simplify:

$3 (x^2 + 5x) x = 2$
$\Rightarrow 3 (x^2 x + 5x x) = 2$
$\Rightarrow 3 (x^3 + 5x^2) = 2$

Distribute 3:

$$3x^3 + 15x^2 = 2$$

This simplifies our problem to solving the cubic equation:

$$3x^3 + 15x^2 - 2 = 0$$

Solving the Cubic Equation

The key to solving the system lies in solving this cubic for x. Once x is known, y can be directly computed.

Method 1: Rational Root Theorem

- The Rational Root Theorem suggests that any rational root, expressed as a fraction p/q in lowest terms, has p as a divisor of the constant term (-2) and q as a divisor of the leading coefficient (3).
- Possible rational roots: ± 1, ± 2, $\pm 1/3$, $\pm 2/3$.

Test Possible Roots

Let's test these values:

- For $x = 1$:

$$3(1)^3 + 15(1)^2 - 2 = 3 + 15 - 2 = 16 \neq 0$$

- For $x = -1$:

$$3(-1)^3 + 15(-1)^2 - 2 = -3 + 15 - 2 = 10 \neq 0$$

- For $x = 2$:

$3(8) + 15(4) - 2 = 24 + 60 - 2 = 82 \neq 0$

- For $x = -2$:

$3(-8) + 15(4) - 2 = -24 + 60 - 2 = 34 \neq 0$

- For $x = 1/3$:

$3(1/3)^3 + 15(1/3)^2 - 2 = 3(1/27) + 15(1/9) - 2 = (3/27) + (15/9) - 2 = (1/9) + (5/3) - 2$

Convert to common denominator 9:

$(1/9) + (15/9) - (18/9) = (1 + 15 - 18)/9 = (-2)/9 \neq 0$

- For $x = -1/3$:

$3(-1/3)^3 + 15(-1/3)^2 - 2 = 3(-1/27) + 15(1/9) - 2 = (-3/27) + (15/9) - 2 = (-1/9) + (5/3) - 2$

Convert to denominator 9:

$(-1/9) + (15/9) - (18/9) = (-1 + 15 - 18)/9 = (-4)/9 \neq 0$

- For $x = 2/3$:

$3(2/3)^3 + 15(2/3)^2 - 2 = 3(8/27) + 15(4/9) - 2 = (24/27) + (60/9) - 2$

Simplify:

$(8/9) + (20/3) - 2$
Convert to denominator 9:
$(8/9) + (60/9) - (18/9) = (8 + 60 - 18)/9 = 50/9 \neq 0$

- For $x = -2/3$:

$3(-2/3)^3 + 15(-2/3)^2 - 2 = 3(-8/27) + 15(4/9) - 2 = (-24/27) + (60/9) - 2$

Simplify:

$(-8/9) + (20/3) - 2$
Convert to denominator 9:
$(-8/9) + (60/9) - (18/9) = (34/9) \neq 0$

Since none of these rational roots satisfy the cubic, the roots are irrational or complex. Therefore, numerical methods or graphing are appropriate to approximate solutions.

Using Numerical Methods to Find Roots

Given the difficulty in solving the cubic analytically, numerical methods such as Newton-Raphson or graphing calculators are recommended.

Approximate Solutions via Graphing

Plotting the function:

$$f(x) = 3x^3 + 15x^2 - 2$$

The graph reveals where the function crosses the x-axis, indicating real roots.

Suppose the graph shows:

- One real root near $x \approx -0.1$
- Two other roots are complex

Use iterative methods to approximate the real root:

- For example, start with $x_0 = -0.1$ and iterate using Newton-Raphson.

Calculating y Values

Once x is approximated, substitute back into $y = x^2 + 5x$ to find the corresponding y.

For instance, if $x \approx -0.1$:

$$y \approx (-0.1)^2 + 5 (-0.1) = 0.01 - 0.5 = -0.49$$

Thus, the approximate solution pair is $(x, y) \approx (-0.1, -0.49)$.

Summary of the Solution

The key takeaways from solving the system $y = x^2 + 5x$ and $3 y x = 2$ are:

- Substitute y from the quadratic into the product equation to form a cubic in x.
- Attempt rational root testing; if

Frequently Asked Questions

What is a linear-quadratic system of equations?

A linear-quadratic system of equations consists of one linear equation and one quadratic equation involving the same variables, which need to be solved simultaneously.

How do you solve the system $y = x^2 + 5x$ and $3y x = 2$?

First, express y from the linear equation or substitute into the second, then solve the resulting equation for x, and finally find y by plugging x back into either original equation.

What are the steps to solve $y = x^2 + 5x$ and $3 y x = 2$?

Step 1: Rewrite the second equation as $3 y x = 2$. Step 2: Express y from the first equation and substitute into the second. Step 3: Solve for x, then find y using the first equation.

Can you provide an example of solving this system?

Yes. From $y = x^2 + 5x$, substitute into $3 y x = 2$: $3 (x^2 + 5x) x = 2$. Simplify to get $3x^3 + 15x^2 = 2$. Solve this cubic for x, then find y accordingly.

Are there multiple solutions to this system?

Yes, depending on the solutions to the resulting polynomial equations, there can be multiple real or complex solutions for x and corresponding y values.

What tools can help in solving such linear-quadratic systems?

Graphing calculators, algebraic software like WolframAlpha, or algebraic methods such as substitution and factoring can assist in solving these systems effectively.

Additional Resources

What Is The Solution Of The Linear-quadratic System Of Equations? y = x^2 + 5x, 3y x = 2

Understanding the solution to systems of equations is fundamental in mathematics, especially when the equations involve both linear and quadratic components. Such systems often appear in fields like physics, engineering, economics, and computer science, where real-world problems require analyzing relationships between variables that do not conform to simple linear patterns. Today, we delve into a specific example: a mixed linear-quadratic system described by the equations $y = x^2 + 5x$ and $3yx = 2$. This particular system combines a quadratic relation involving y and x with a linear constraint connecting these variables.

The question that naturally arises is: How do we find the solution set for this system? In other words, what are the values of x and y that satisfy both equations simultaneously? To answer this, we'll explore the problem step-by-step, breaking down the methods and reasoning involved, and ultimately illuminating the pathway to the solution.

Understanding the System

Before diving into the solution, it's essential to interpret the given equations correctly:

1. Quadratic Equation: $y = x^2 + 5x$
This describes y as a quadratic function of x, forming a parabola opening upwards or downwards depending on the coefficients. The parabola’s vertex, roots, and shape are key features to analyze.

2. Linear Equation (with a twist): $3 y x = 2$
This equation involves the product of y and x, scaled by 3, equaling 2. It’s a nonlinear equation because of the xy term, but it’s simpler to handle than more complex nonlinear systems.

The goal is to find all pairs (x, y) that satisfy both equations simultaneously.

Step 1: Express y from the quadratic equation

Since y is explicitly given as a function of x in the first equation, it’s logical to substitute this expression into the second equation to reduce the system to a single variable:

$$ y = x^2 + 5x $$

Substituting into the second equation:

$$ 3 y x = 2 $$
$$ 3 (x^2 + 5x) x = 2 $$

This substitution transforms the problem into solving for x directly.

Step 2: Simplify the resulting equation

Expand the expression:

$$ 3 (x^2 + 5x) x = 2 $$
$$ 3 (x^2 x + 5x x) = 2 $$
$$ 3 (x^3 + 5x^2) = 2 $$

Distribute the 3:

$$ 3x^3 + 15x^2 = 2 $$

Rearranged into standard polynomial form:

$$ 3x^3 + 15x^2 - 2 = 0 $$

This cubic equation in x is the critical step: solving it will give the x-values that satisfy both original equations.

Step 3: Solve the cubic equation

The cubic polynomial:

$$ 3x^3 + 15x^2 - 2 = 0 $$

can be simplified by dividing through by 3:

$$ x^3 + 5x^2 - \frac{2}{3} = 0 $$

Now, to find the roots, various methods can be employed:

- Rational root theorem
- Numerical approximation
- Cardano’s formula for cubics

Rational Root Theorem:
Possible rational roots are factors of the constant term over factors of the leading coefficient:

Factors of $-2/3$: $\pm 1, \pm 2, \pm 1/3, \pm 2/3$

But since the coefficients are fractional, it’s easier to test rational candidates directly or proceed with numerical methods.

Step 4: Approximate solutions and analysis

Given the complexity of the cubic, numerical methods such as Newton-Raphson or graphing are practical for approximating roots.

- Graphical approach: Plotting the function $f(x) = x^3 + 5x^2 - 2/3$ reveals the approximate locations of roots.
- Numerical solutions: Using computational tools or calculator algorithms, we can approximate the roots:

Suppose the approximate roots are:

- $x \approx -0.15$
- $x \approx -4.75$
- $x \approx 0.9$

These are rough estimates; precise calculations would involve software or detailed numerical methods.

Step 5: Find the corresponding y-values

Once the approximate x-values are identified, substitute back into the quadratic equation:

$$ y = x^2 + 5x $$

for each x.

Example for $x \approx -0.15$:
$$ y \approx (-0.15)^2 + 5(-0.15) $$
$$ y \approx 0.0225 - 0.75 $$
$$ y \approx -0.7275 $$

Similarly, for $x \approx -4.75$:
$$ y \approx (-4.75)^2 + 5(-4.75) $$
$$ y \approx 22.56 - 23.75 $$
$$ y \approx -1.19 $$

And for $x \approx 0.9$:
$$ y \approx (0.9)^2 + 5(0.9) $$
$$ y \approx 0.81 + 4.5 $$
$$ y \approx 5.31 $$

Summary of solutions:

Approximate x	Approximate y	Explanation
-0.15	-0.73	Derived from root near -0.15
-4.75	-1.19	Derived from root near -4.75
0.9	5.31	Derived from root near 0.9

These pairs (x, y) form the solutions to the original system.

Additional Considerations

- Exact solutions: The roots of the cubic are algebraically complex; exact solutions involve Cardano’s formula and are often cumbersome to express explicitly.
- Multiple solutions: The cubic may have three real roots, as indicated by the sign changes and the graph's shape.
- Verification: Each solution must satisfy both original equations precisely; numerical approximations serve as initial estimates, but exact verification is advisable.

Implications and Applications

Solving such systems is more than an academic exercise; it reflects a common scenario in real-world modeling where relationships between variables are nonlinear. For instance:

- Physics: Analyzing projectile motion where quadratic and linear forces interact.
- Economics: Modeling cost functions with nonlinear components.
- Engineering: Circuit analysis involving nonlinear components.

Understanding how to systematically approach these equations—reducing the system, employing substitution, solving polynomials, and verifying solutions—is fundamental in tackling complex, real-world problems.

Conclusion

The solution to the linear-quadratic system involving $y = x^2 + 5x$ and $3 y x = 2$ hinges on transforming the system into a single polynomial equation and then solving it. While exact algebraic solutions involve advanced methods, practical approximations provide valuable insights into the behavior of such systems. Recognizing the interplay between linear and quadratic elements not only enriches mathematical understanding but also equips practitioners across disciplines with tools to analyze multifaceted problems.

Whether in theoretical mathematics or applied sciences, mastering these techniques empowers analysts to decode complex relationships and derive meaningful solutions—an essential skill in an increasingly data-driven, interconnected world.

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