What Is The Solution To The System Of Equations Graphed Below Y=1/2x+2 Y=-2x+-3

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Understanding the solution to a system of equations is fundamental in algebra, especially when visualized through graphs. When you are presented with the equations y = 1/2 x + 2 and y = -2x - 3, determining their solution involves analyzing their graphical intersection point. This intersection point represents the values of x and y that satisfy both equations simultaneously. In this article, we will explore in detail what the solution to this particular system is, how to find it both graphically and algebraically, and why understanding these methods is essential in mathematics.

Understanding the System of Equations

Before delving into solutions, it is crucial to understand the structure and form of the equations involved.

Equations in Slope-Intercept Form

Both equations are written in the slope-intercept form y = mx + b, where:

- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

For the given equations:

1. y = 1/2 x + 2
- Slope (m): 1/2
- Y-intercept (b): 2

2. y = -2x - 3
- Slope (m): -2
- Y-intercept (b): -3

Understanding the slopes and intercepts helps visualize the lines and anticipate their intersection points.

Graphical Interpretation of the System

Plotting the equations on a coordinate plane provides a visual representation of their relationship.

Plotting $y = 1/2\ x + 2$

- The y-intercept is at (0, 2). This is the point where the line crosses the y-axis.
- The slope is 1/2, indicating that for every 2 units moved horizontally to the right, the line moves up 1 unit.

To plot:

- Mark the point (0, 2).
- From this point, move right 2 units and up 1 unit to mark the second point at (2, 3).
- Draw a straight line through these points.

Plotting $y = -2x - 3$

- The y-intercept is at (0, -3).
- The slope is -2, meaning that for every 1 unit moved to the right, the line moves down 2 units.

To plot:

- Mark the point (0, -3).
- From this point, move right 1 unit and down 2 units to mark (1, -5).
- Draw the line through these points.

Finding the Intersection

Once both lines are drawn, the intersection point is where they cross. This point indicates the solution to the system: the specific x and y values satisfying both equations simultaneously.

Algebraic Methods to Find the Solution

Graphing provides a visual approximation, but algebraic methods ensure precise solutions. The two primary algebraic methods are substitution and elimination.

Method 1: Substitution

Since both equations are solved for y, substitution involves setting the right-hand sides equal:

- Set $1/2\ x + 2$ equal to $-2x - 3$:

$$1/2\ x + 2 = -2x - 3$$

- Solve for x:

1/2 x + 2 = -2x - 3

Add 2x to both sides:

1/2 x + 2 + 2x = -3

Combine like terms:

(1/2 x + 2x) + 2 = -3

Convert 2x to 1/1 x:

(1/2 x + 2/1 x) + 2 = -3

Find common denominator:

(1/2 x + 4/2 x) + 2 = -3

Sum:

(5/2 x) + 2 = -3

- Subtract 2 from both sides:

5/2 x = -5

- Multiply both sides by 2/5:

x = (-5) (2/5) = -2

- Find y by substituting x = -2 into either original equation:

y = 1/2 (-2) + 2 = -1 + 2 = 1

Or check with the second equation:

y = -2 (-2) - 3 = 4 - 3 = 1

- Solution: x = -2, y = 1

Method 2: Elimination

Alternatively, the elimination method involves aligning equations to eliminate one variable:

- Write the equations:

1) y = 1/2 x + 2

2) y = -2x - 3

- Set equal to each other:

$1/2 x + 2 = -2x - 3$

- Multiply through to clear fractions (multiply entire equation by 2):

$x + 4 = -4x - 6$

- Add 4x to both sides:

$x + 4 + 4x = -6$

Combine like terms:

$5x + 4 = -6$

- Subtract 4 from both sides:

$5x = -10$

- Divide both sides by 5:

$x = -2$

- Plug into either original equation to find y:

$y = 1/2 (-2) + 2 = -1 + 2 = 1$

- Solution: $x = -2$, $y = 1$

This confirms the intersection point found graphically.

Implications of the Solution

The intersection point $(-2, 1)$ is the unique solution to the system, meaning it is the only point where both lines cross. This point satisfies both equations simultaneously, confirming the correctness of the algebraic and graphical approaches.

Why Is This Important?

- Understanding the solution to systems of equations is critical in fields such as engineering, physics, economics, and computer science.
- It helps in solving real-world problems involving multiple variables and constraints.
- Mastery of graphing and algebraic techniques enhances problem-solving skills and mathematical literacy.

Applications of Solving Systems of Equations

Systems of equations appear frequently in various real-life scenarios:

- **Business and Economics:** Determining the equilibrium point where supply equals demand.
- **Physics:** Calculating intersection points of paths or trajectories.
- **Engineering:** Finding points where different forces or signals intersect.
- **Computer Graphics:** Rendering scenes by calculating intersections of lines and shapes.

In all these cases, accurately solving the system ensures effective decision-making and analysis.

Summary

To summarize, the solution to the system of equations $y = 1/2\, x + 2$ and $y = -2x - 3$ is the point where both lines intersect, which algebraically is at $(x, y) = (-2, 1)$. Whether approached through graphical methods or algebraic techniques such as substitution or elimination, understanding how to find this intersection point is a fundamental skill in algebra. It not only reinforces comprehension of linear relationships but also provides a foundation for tackling more complex systems involving multiple variables and equations.

Final Thoughts

Mastering the process of solving systems of equations both graphically and algebraically is essential for students and professionals alike. It enhances analytical thinking, improves problem-solving capabilities, and deepens understanding of linear relationships. The solution to the system of $y = 1/2\, x + 2$ and $y = -2x - 3$ exemplifies how different methods converge to the same result, demonstrating the consistency and reliability of mathematical techniques. As you continue to explore systems of equations, remember that practice and visualization are key to developing proficiency and confidence in solving such problems efficiently.

Frequently Asked Questions

What is the point of intersection for the lines y = 1/2 x + 2 and y = -2 x - 3?

The point of intersection is $(-2, 1)$.

How do you find the solution to the system of equations graphically?

By plotting both lines on a graph and identifying the point where they intersect.

Are the lines y = 1/2 x + 2 and y = -2 x - 3 parallel, intersecting, or coincident?

They intersect at a single point, so they are intersecting lines.

What is the method to solve these equations algebraically?

Set the equations equal to each other: (1/2 x + 2) = (-2 x - 3), then solve for x.

What is the solution to the system of equations y = 1/2 x + 2 and y = -2 x - 3?

The solution is x = -2, y = 1.

How does the slope of each line affect their intersection point?

The slopes (1/2 and -2) are different, so the lines will intersect at exactly one point.

Can these lines be parallel or coincident? Why or why not?

No, because their slopes are different; parallel lines have equal slopes, and coincident lines have identical equations.

What is the significance of the intercepts in these equations?

The y-intercepts are 2 and -3 respectively, which help in graphing and understanding where the lines cross the y-axis.

How can understanding the solution to this system help in real-world applications?

It can help in scenarios like finding equilibrium points, optimizing resources, or analyzing intersecting trends in data.

Additional Resources

Solution to the System of Equations: Graphed Insights and Analytical Approach

When delving into the world of algebra, particularly systems of equations, understanding how to find their solutions is fundamental. The equations Y = (1/2)x + 2 and Y = -2x - 3 serve as classic examples illustrating the intersection point of two lines in a coordinate plane. This point of intersection embodies the solution to the system—an elegant visual and algebraic harmony. In this comprehensive analysis, we will explore the solution from multiple perspectives, including graphing principles, algebraic methods, and practical applications, ensuring a thorough grasp of how to determine the precise point where these two lines meet.

Understanding the System of Equations

Before jumping into solving, it’s essential to understand what the system represents.

The Equations in Context

- Y = (1/2)x + 2: This is a linear equation representing a line with a slope of 1/2 and a y-intercept at (0, 2).
- Y = -2x - 3: This line has a slope of -2 and a y-intercept at (0, -3).

These two lines are plotted in the Cartesian plane, and their points of intersection reveal the solution to the system.

Graphical Significance of the Equations

- The slope indicates the steepness and direction of each line.
- The y-intercept indicates where each line crosses the y-axis.
- The point of intersection (if any) visually confirms the solution.

Graphing the Equations: Visual Intuition

Graphing provides a tangible method for visualizing solutions.

Step-by-Step Graphing Process

1. Plot the Y-intercept:
- For Y = (1/2)x + 2, plot at (0, 2).
- For Y = -2x - 3, plot at (0, -3).

2. Use the slope to find additional points:
- For Y = (1/2)x + 2:
- Slope = 1/2 means rise over run = 1/2.
- From (0, 2), move 1 unit up and 2 units right to get (2, 3).
- For Y = -2x - 3:
- Slope = -2 means for each 1 unit increase in x, y decreases by 2.
- From (0, -3), move 1 unit right and 2 units down to get (1, -5).

3. Draw the lines:
- Connect the points smoothly, extending the lines across the graph.

Visual Analysis

- The lines will cross at a single point, indicating a unique solution.
- The intersection point's coordinates can be approximated visually, but for exactness, algebraic methods are preferred.

Algebraic Solution: Finding the Exact Point of Intersection

While graphing offers a visual, algebra provides precision. Here, we’ll explore methods such as substitution and elimination to find the exact solution.

Method 1: Substitution

Since both equations are solved for Y, substitution is straightforward.

Step 1: Set the two equations equal to each other:

\[
(1/2) x + 2 = -2x - 3
\]

Step 2: Solve for x:

\[
(1/2) x + 2 = -2x - 3
\]

Bring all x terms to one side:

\[
(1/2) x + 2x = -3 - 2
\]

Combine like terms:

\[
(1/2 + 2) x = -5
\]

Express 2 as 4/2 to combine:

\[

(1/2 + 4/2) x = -5
\]

\[
(5/2) x = -5
\]

Multiply both sides by 2 to clear the fraction:

\[
5x = -10
\]

Divide both sides by 5:

\[
x = -2
\]

Step 3: Find Y by substituting x = -2 into either original equation:

Using Y = (1/2)x + 2:

\[
Y = (1/2)(-2) + 2 = -1 + 2 = 1
\]

Alternatively, verify with Y = -2x - 3:

\[
Y = -2(-2) - 3 = 4 - 3 = 1
\]

Both yield Y = 1.

Solution:

\[
\boxed{
\text{The lines intersect at } (-2, 1)
}
\]

Verification and Practical Significance

Verification ensures the solution is accurate, and understanding its practical implications demonstrates the importance of solving such systems.

Verification

Plug $x = -2$ and $Y = 1$ into both original equations:

- $Y = (1/2)x + 2$:

$$
1 = (1/2)(-2) + 2 = -1 + 2 = 1 \quad \checkmark
$$

- $Y = -2x - 3$:

$$
1 = -2(-2) - 3 = 4 - 3 = 1 \quad \checkmark
$$

Both verify the point (-2, 1) as the exact intersection.

Practical Applications and Relevance

Understanding the solution to this system isn’t just an academic exercise; it’s pivotal in numerous real-world scenarios:

- Economics: Equilibrium points where supply and demand curves intersect.
- Physics: When two objects move along different paths, their point of intersection indicates collision points.
- Engineering: Structural analysis where different force vectors or load lines intersect.

In all these cases, precise solutions allow for accurate planning, safety margins, and optimized design.

Summary of Key Takeaways

- The solution to the system $Y = (1/2)x + 2$ and $Y = -2x - 3$ is (-2, 1).
- Graphing provides a visual confirmation but lacks the exactness needed for precise applications.
- Algebraic methods, especially substitution, can efficiently and reliably find the exact intersection point.
- Verification ensures the solution satisfies both original equations.

Conclusion: Mastering System Solutions for Broader Success

Mastering the solution to systems of equations like these enhances both your algebraic intuition and your capacity to apply mathematics practically. Whether visualizing the problem through graphing or employing algebraic techniques, understanding how to find and verify solutions fosters a deeper appreciation for the interconnectedness of mathematical concepts. The intersection point (-2, 1) marks the harmony of the two lines, embodying both a mathematical solution and a metaphor for problem-solving—finding a common ground through logical, systematic approaches. As you continue exploring more complex systems, remember that the principles demonstrated here serve as foundational tools for tackling diverse challenges across scientific, technological, and analytical domains.

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