What Is The Value Of X When The Function F(x)=2x 2 +8x12, Has A Slope Of 4 ?

What Is The Value Of X When The Function F(x)=2x^2 + 8x + 12 Has A Slope Of 4?

Understanding how to find the value of x when a function's slope reaches a specific value is a fundamental concept in calculus. In this article, we will explore how to determine the value of x for the function F(x) = 2x^2 + 8x + 12 when its slope is equal to 4. Whether you're a student studying derivatives for the first time or someone brushing up on calculus principles, this guide aims to clarify the process with detailed explanations and step-by-step instructions.

Fundamentals of Derivatives and Slope

What Is the Derivative of a Function?

The derivative of a function provides us with the rate at which the function's value changes concerning x. In geometric terms, the derivative at a point gives the slope of the tangent line to the curve at that specific point.

Why Is the Derivative Important for Finding Slope?

Knowing the derivative of a function allows us to determine the slope of the function at any given x-value. If we want to find where the slope equals a particular value (such as 4 in this case), we need to first find the derivative and then solve for the x-values that satisfy the slope condition.

Calculating the Derivative of F(x)=2x^2 + 8x + 12

Step-by-Step Derivative Calculation

Given the function:

F(x) = 2x^2 + 8x + 12

The derivative, denoted as F'(x), is calculated as follows:

- Derivative of 2x^2: 2 2x = 4x
- Derivative of 8x: 8
- Derivative of 12: 0 (since it's a constant)

Thus,

F'(x) = 4x + 8

Interpreting the Derivative

This derivative function, F'(x) = 4x + 8, represents the slope of the original function at any point x. Our goal is to find the specific x-values where this slope equals 4.

Finding the x-Values When Slope Equals 4

Setting Up the Equation

To find when the slope is 4, we set the derivative equal to 4:

$F'(x) = 4$

Substituting the derivative expression:

$4x + 8 = 4$

Solving for x

Rearranging the equation:

$4x + 8 = 4$

Subtract 8 from both sides:

$4x = 4 - 8$

$4x = -4$

Divide both sides by 4:

$x = -1$

This indicates that when $x = -1$, the slope of the function is 4.

Verifying the Result and Understanding Its Significance

Confirming the Slope at x = -1

Plug $x = -1$ back into the derivative:

$F'(-1) = 4(-1) + 8 = -4 + 8 = 4$

As expected, the slope at $x = -1$ is indeed 4.

Calculating the Function Value at x = -1

To understand the point on the curve where the slope is 4, compute $F(-1)$:

F(-1) = 2(-1)^2 + 8(-1) + 12

$= 2(1) - 8 + 12$

$= 2 - 8 + 12$

$= 6$

Therefore, the point on the graph where the slope is 4 is at (-1, 6).

Applications and Implications of the Calculation

Real-World Contexts

Understanding where a function has a particular slope can be useful in various fields:

- Physics: Determining when a vehicle's acceleration (rate of change of velocity) reaches a specific value.
- Economics: Finding the level of output where the rate of profit change matches a target.
- Engineering: Locating points on a curve where the rate of change meets design specifications.

Designing and Optimizing Functions

Knowing the x-values corresponding to certain slopes helps in optimizing functions, such as maximizing profit or minimizing cost, by analyzing the slopes at critical points.

Summary and Key Takeaways

- The derivative of the function $F(x) = 2x^2 + 8x + 12$ is $F'(x) = 4x + 8$.
- Setting $F'(x)$ equal to 4 allows us to find the x-value where the slope is 4.
- Solving $4x + 8 = 4$ yields $x = -1$.
- At $x = -1$, the function value is $F(-1) = 6$, and the slope is confirmed to be 4.

Conclusion

Determining the value of x when a function's slope reaches a specific value is a fundamental calculus skill. By calculating the derivative and solving the resulting equation, we find that for the function $F(x) = 2x^2 + 8x + 12$, the slope is 4 at $x = -1$. Understanding this process enhances your ability to analyze the behavior of functions and their rates of change, which is essential across many scientific and engineering disciplines.

If you want to deepen your understanding of derivatives, slopes, and quadratic functions, consider exploring additional topics such as critical points, concavity, and optimization techniques. Mastering these concepts provides a strong foundation for advanced calculus and mathematical problem-solving.

Frequently Asked Questions

What is the function f(x) given in the problem?

The function is $f(x) = 2x^2 + 8x + 12$.

How do you find the slope of the function $f(x)$?

The slope is given by the derivative of $f(x)$, which for a quadratic function is $f'(x)$.

What is the derivative of $f(x) = 2x^2 + 8x + 12$?

The derivative is $f'(x) = 4x + 8$.

How do you determine the value of x when the slope is 4?

Set the derivative equal to 4 and solve for x: $4x + 8 = 4$.

What is the solution to $4x + 8 = 4$?

Subtract 8 from both sides: $4x = -4$, then divide both sides by 4: $x = -1$.

What does the value $x = -1$ represent in this context?

It is the x-value at which the slope of the function $f(x)$ is 4.

Is the value of $x = -1$ the only point where the slope is 4?

Yes, since the derivative is linear and equal to $4x + 8$, it equals 4 at only $x = -1$.

Can you verify that the slope at $x = -1$ is indeed 4?

Yes, substitute $x = -1$ into $f'(x)$: $4(-1) + 8 = -4 + 8 = 4$, confirming the slope.

What is the significance of finding x when the slope is 4 in the function?

It helps identify the point on the curve where the rate of change (slope) is exactly 4, which is useful in tangent line calculations and understanding the function's behavior.

Additional Resources

What Is The Value Of X When The Function $F(x) = 2x^2 + 8x + 12$, Has A Slope Of 4?

Introduction

Understanding the relationship between a function and its slope is fundamental in calculus, with wide-ranging applications in physics, engineering, economics, and beyond. One common question posed in mathematical analysis involves determining the specific value(s) of the variable x at which a given function exhibits a particular slope. In this comprehensive investigation, we will explore the problem:

> What is the value of x when the function $F(x) = 2x^2 + 8x + 12$ has a slope of 4?

This question requires an understanding of derivatives, as they provide the slope of the tangent line to the function at any point. We will methodically dissect the problem, starting with the fundamentals of derivatives, then apply them step by step to find the specific x values that satisfy the given condition.

Background: The Role of Derivatives in Finding Slopes

In calculus, the derivative of a function $f(x)$, denoted as $f'(x)$ or $\frac{df}{dx}$, measures the instantaneous rate of change of the function at a point x. Geometrically, this is the slope of the tangent line to the curve at that point.

Key points:

- The derivative provides the slope at any point x.
- To find where the function has a certain slope m, we solve the equation $f'(x) = m$.

In our case, the function $f(x) = 2x^2 + 8x + 12$ is quadratic, and its derivative will be a linear function, making the process straightforward.

Step 1: Calculating the Derivative of $f(x)$

Given:

$$
f(x) = 2x^2 + 8x + 12
$$

The derivative, using basic differentiation rules, is:

$$
f'(x) = \frac{d}{dx} (2x^2) + \frac{d}{dx} (8x) + \frac{d}{dx} (12)
$$

Applying the power rule ($\frac{d}{dx} x^n = n x^{n-1}$):

$$
f'(x) = 2 \times 2x^{2-1} + 8 \times 1 + 0 = 4x + 8
$$

Thus, the derivative, which represents the slope at any point x, is:

$$
f'(x) = 4x + 8
$$

Step 2: Setting the Derivative Equal to the Given Slope

The problem states:

> When the slope of the function is 4, what is the corresponding x?

Mathematically, we solve:

$$
f'(x) = 4
$$

Substituting the derivative:

\[

$$
4x + 8 = 4
$$

Step 3: Solving for x

Isolating x:

$$
4x = 4 - 8
$$

$$
4x = -4
$$

$$
x = -1
$$

Result:

$$
\boxed{
x = -1
}
$$

This indicates that at $x = -1$, the function $F(x)$ has a slope of 4.

Deep Dive: Verifying and Interpreting the Result

Confirming the Calculation

- The derivative $F'(x) = 4x + 8$ is linear.
- Setting the derivative equal to 4 yields $x = -1$.
- Plugging $x = -1$ back into $F'(x)$:

$$
F'(-1) = 4 \times (-1) + 8 = -4 + 8 = 4
$$

which confirms the calculation.

Geometric Interpretation

At $x = -1$:

- The tangent line to the graph of $F(x)$ has a slope of 4.
- The specific point on the curve can be found by evaluating $F(-1)$:

$$
F(-1) = 2(-1)^2 + 8(-1) + 12 = 2(1) - 8 + 12 = 2 - 8 + 12 = 6
$$

So, the point of tangency is $(-1, 6)$.

The tangent line at this point has a slope of 4, indicating a relatively steep ascent at $x = -1$.

Broader Implications and Related Concepts

Critical Points and Extrema

While not directly asked, understanding where the slope is zero or where the function reaches maxima or minima is valuable. For completeness:

- Set $F'(x) = 0$:

$$
4x + 8 = 0 \Rightarrow x = -2
$$

- At $x = -2$:

$$
F(-2) = 2(-2)^2 + 8(-2) + 12 = 2(4) - 16 + 12 = 8 - 16 + 12 = 4
$$

- Since the second derivative $F''(x) = 4$ (a positive constant), the point at $x = -2$ is a local minimum.

Understanding the Function's Behavior

- The function is quadratic with a positive leading coefficient, opening upward.
- The slope function $F'(x) = 4x + 8$ increases linearly.
- The point $x = -1$, where the slope is 4, occurs to the right of the vertex at $x = -2$.

Summary and Practical Applications

- The value of x at which the function $F(x) = 2x^2 + 8x + 12$ has a slope of 4 is $x = -1$.
- The point on the graph corresponding to this x is $(-1, 6)$.
- The tangent line at this point has an equation:

$$
\text{Tangent line at } x = -1: \quad y - 6 = 4(x + 1)
$$

which simplifies to:

$$
y = 4x + 10
$$

Practical significance:

This analysis can be applied in physical contexts where the rate of change (e.g., velocity, growth rate) is specified, and the corresponding position or state must be determined.

Final Remarks

This investigation exemplifies the power of calculus in solving real-world problems involving rates of change.

By differentiating a quadratic function, setting the derivative equal to a specified value, and solving for the variable, we obtain precise insights into the behavior of the function at particular points.

Understanding these principles is essential for professionals and students alike, fostering a deeper grasp of the interplay between a function's shape and its rate of change. Whether analyzing motion, optimizing systems, or modeling phenomena, the ability to connect slopes to specific values of x remains a cornerstone of mathematical literacy.

In conclusion:

> The value of x when the function $f(x) = 2x^2 + 8x + 12$ has a slope of 4 is $\boxed{-1}$.

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