

12/13' Find θ . If θ Terminates In Quadrant II And $\sin \theta = \frac{12}{13}$, Find $\cos \theta$.

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Understanding trigonometric functions and their properties is essential in mathematics, especially when working with right triangles and the unit circle. In this article, we will explore a specific problem: given that $\sin \theta = \frac{12}{13}$, the angle θ terminates in Quadrant II, and we need to find $\cos \theta$. This problem involves fundamental concepts of trigonometry, including the Pythagorean theorem, the signs of trigonometric functions in different quadrants, and how to determine the cosine value based on given information.

Understanding the Problem

Before diving into calculations, it is crucial to interpret what the problem states:

- The sine of the angle θ is $\frac{12}{13}$.
- The angle θ terminates in Quadrant II.
- The goal is to find $\cos \theta$.

This problem combines knowledge of the unit circle, the signs of trigonometric functions in various quadrants, and the Pythagorean identity.

Key Concepts in Trigonometry

The Unit Circle and Coordinates

The unit circle is a circle with a radius of 1 centered at the origin in the coordinate plane. Any point (x, y) on this circle satisfies:

$\sqrt{x^2 + y^2} = 1$

$$x^2 + y^2 = 1$$

\]

In trigonometry, the coordinate $((x, y))$ associated with an angle (θ) corresponds to:

$$- (x = \cos \theta)$$

$$- (y = \sin \theta)$$

Given $(\sin \theta = y)$, we can use this relationship to find $(\cos \theta)$.

Signs of Trigonometric Functions in Different Quadrants

The coordinate plane is divided into four quadrants:

- Quadrant I: $(0^\circ < \theta < 90^\circ)$ — both $(\sin \theta)$ and $(\cos \theta)$ are positive.
- Quadrant II: $(90^\circ < \theta < 180^\circ)$ — $(\sin \theta)$ is positive, $(\cos \theta)$ is negative.
- Quadrant III: $(180^\circ < \theta < 270^\circ)$ — both $(\sin \theta)$ and $(\cos \theta)$ are negative.
- Quadrant IV: $(270^\circ < \theta < 360^\circ)$ — $(\sin \theta)$ is negative, $(\cos \theta)$ is positive.

Since the problem states that (θ) terminates in Quadrant II, we know that:

\[

$$\sin \theta > 0 \quad \text{and} \quad \cos \theta < 0$$

\]

Step-by-Step Solution

Step 1: Recall the Pythagorean Identity

The fundamental identity in trigonometry is:

\[

$$\sin^2 \theta + \cos^2 \theta = 1$$

\]

Given $(\sin \theta = \frac{12}{13})$, substitute into the identity:

$$\left[\left(\frac{12}{13}\right)^2 + \cos^2 \theta = 1\right]$$

Calculate $(\sin^2 \theta)$:

$$\left[\frac{144}{169} + \cos^2 \theta = 1\right]$$

Step 2: Solve for $(\cos^2 \theta)$

Rearranged:

$$\left[\cos^2 \theta = 1 - \frac{144}{169}\right]$$

Express 1 as $(\frac{169}{169})$:

$$\left[\cos^2 \theta = \frac{169}{169} - \frac{144}{169} = \frac{25}{169}\right]$$

Step 3: Find $(\cos \theta)$

Taking the square root of both sides:

$$\left[\cos \theta = \pm \sqrt{\frac{25}{169}} = \pm \frac{5}{13}\right]$$

Since (θ) terminates in Quadrant II, where cosine is negative, we select the negative value:

$$\left[\boxed{\cos \theta = -\frac{5}{13}}\right]$$

Final Answer and Explanation

Based on the calculations and quadrant considerations, the cosine of (θ) is:

$$\boxed{\cos \theta = -\frac{5}{13}}$$

This result aligns with the properties of the unit circle, the signs of trigonometric functions in Quadrant II, and the Pythagorean identity.

Additional Tips for Solving Similar Problems

- **Always identify the quadrant:** Knowing the quadrant helps determine the sign of the trigonometric function.
- **Use the Pythagorean identity:** It allows you to find missing side lengths or values of functions when you have one trigonometric ratio.
- **Express fractions in simplest form:** Simplify calculations and avoid errors by reducing fractions.
- **Remember the signs:** Keep in mind the signs of sine and cosine in each quadrant to select the correct root.
- **Practice with unit circle values:** Familiarity with common sine and cosine values at standard angles makes solving these problems more intuitive.

Real-World Applications of Finding $\cos \theta$

Understanding how to find $\cos \theta$ when given $\sin \theta$ has practical implications across various fields:

Engineering and Physics

- Calculating forces, velocities, and oscillations often requires resolving components using sine and cosine functions.

Navigation and Robotics

- Determining directions and angles based on known sine or cosine ratios helps in pathfinding algorithms.

Architecture and Design

- Designing structures that involve angles and slopes relies on trigonometric calculations.

Summary

In this article, we explored how to find $\cos \theta$ given $\sin \theta = \frac{12}{13}$ with θ terminating in Quadrant II. By applying the Pythagorean identity, recognizing the signs of trigonometric functions in different quadrants, and performing algebraic manipulations, we deduced:

$$\boxed{\cos \theta = -\frac{5}{13}}$$

This problem exemplifies core principles of trigonometry and highlights the importance of quadrant awareness when working with trigonometric functions.

Relevant Keywords for SEO: trigonometry, find cosine given sine, quadrant II trigonometry, Pythagorean identity, unit circle, solve for cosine, sine and cosine relationships, trigonometric functions, angle in Quadrant II, sine ratio, cosine ratio

Frequently Asked Questions

What is the value of $\sin \theta$ given that it terminates in Quadrant II and $\sin \theta = 12/13$?

The value of $\sin \theta$ is $12/13$.

Given $\sin \theta = 12/13$ and θ terminates in Quadrant II, how do you find $\cos \theta$?

Since $\sin \theta = 12/13$ and θ is in Quadrant II, $\cos \theta = -5/13$, using the Pythagorean identity.

Why is $\cos \theta$ negative in Quadrant II when $\sin \theta$ is positive?

In Quadrant II, sine is positive and cosine is negative, which is why $\cos \theta = -5/13$.

How do you verify that $\cos \theta = -5/13$ given $\sin \theta = 12/13$?

Use the Pythagorean theorem: $\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - (12/13)^2} = -\sqrt{1 - 144/169} = -\sqrt{25/169} = -5/13$.

What is the significance of the given ratio $12/13$ in trigonometry?

It represents the sine of an angle in a right triangle, with the opposite side being 12 and hypotenuse 13.

How does knowing the quadrant help in determining the sign of cosine and sine?

Quadrant II has positive sine and negative cosine, which helps determine the signs of these trigonometric functions.

Can you find $\tan \theta$ given $\sin \theta = 12/13$ and $\cos \theta = -5/13$?

Yes. $\tan \theta = \sin \theta / \cos \theta = (12/13) / (-5/13) = -12/5$.

Additional Resources

12 13' Find 9. If Terminates In Quadrant II And Sin Theta $\frac{12}{13}$, Find Cos Theta

Understanding the relationships between trigonometric functions is fundamental in solving a wide variety of mathematical problems, especially those involving right triangles and the unit circle. In this comprehensive guide, we will explore how to find $\cos \theta$ given specific conditions such as the sine value, the quadrant in which the terminal side of the angle terminates, and the Pythagorean identity. The problem statement, "12 13' Find 9. If Terminates In Quadrant II And Sin Theta $\frac{12}{13}$, Find Cos Theta", appears to contain some typographical or formatting ambiguities, but with careful interpretation, we can decode the intent and provide a detailed step-by-step solution.

Understanding the Problem

Before diving into calculations, it's essential to clarify what the problem entails:

- The angle θ terminates in Quadrant II.
- The sine of θ is given as $\frac{12}{13}$.
- The task is to find $\cos \theta$.

The phrase "12 13'" can be interpreted as the ratio $\frac{12}{13}$, which suggests a reference to a right triangle with sides of lengths 12 and 13. The phrase "Find 9" is somewhat ambiguous but may be an extraneous element or a formatting error. For the purposes of this guide, we will focus on the core problem:

Given:

- $\sin \theta = \frac{12}{13}$
- θ terminates in Quadrant II

Find:

- $\cos \theta$

Fundamental Concepts and Tools

The Pythagorean Identity

The primary relation connecting sine and cosine is:

$$\sin^2 \theta + \cos^2 \theta = 1$$

This identity holds true for all angles θ , regardless of the quadrant.

The Unit Circle and Quadrants

- Quadrant I: Both sine and cosine are positive.
- Quadrant II: Sine is positive; cosine is negative.
- Quadrant III: Both sine and cosine are negative.
- Quadrant IV: Sine is negative; cosine is positive.

Since θ terminates in Quadrant II, we know:

- $\sin \theta > 0$
- $\cos \theta < 0$

Triangle Ratios

Given the ratio $\sin \theta = 12/13$, this corresponds to a right triangle where:

- Opposite side (opposite to angle θ): 12 units
- Hypotenuse: 13 units

Using the Pythagorean theorem, the adjacent side (next to θ) can be calculated as:

$$\sqrt{\text{Hypotenuse}^2 - \text{Opposite}^2}$$

Step-by-Step Solution

Step 1: Confirm the Triangle Sides

Given:

- Opposite side = 12
- Hypotenuse = 13

Calculate the adjacent side:

$$\sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$$

Step 2: Determine the Sign of Cos θ

Since θ terminates in Quadrant II, $\cos \theta$ is negative.

Step 3: Find Cos θ Using the Definition

Cosine is:

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{5}{13}$$

Considering the quadrant, the cosine value is negative:

$$\boxed{\cos \theta = -\frac{5}{13}}$$

Step 4: Verify Using the Pythagorean Identity

Check if the values satisfy the identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(\frac{12}{13}\right)^2 + \left(-\frac{5}{13}\right)^2 = \frac{144}{169} + \frac{25}{169} = \frac{169}{169} = 1$$

The identity holds, confirming our calculations.

Additional Insights and Related Concepts

Understanding the Significance of Quadrant

Knowing the quadrant helps determine the sign of the trigonometric functions, which is crucial in accurate calculations.

Real-World Applications

- Physics: Analyzing wave oscillations where phase angles are in different quadrants.
- Engineering: Signal processing involving phase angles.
- Navigation: Calculating directions based on angles in different quadrants.

Common Mistakes to Avoid

- Confusing the signs of sine and cosine in different quadrants.
- Miscalculating the adjacent side when using the Pythagorean theorem.
- Overlooking the importance of the angle's quadrant in determining the sign of the trigonometric functions.

Summary of the Key Steps

Step	Action	Result
1	Recognize the given sine ratio and interpret the triangle	Opposite = 12, Hypotenuse = 13
2	Calculate the adjacent side using Pythagoras	Adjacent = 5
3	Determine the sign of cosine based on the quadrant	Negative in Quadrant II
4	Compute cosine	$\cos \theta = -\frac{5}{13}$
5	Verify with Pythagorean identity	Identity holds true

Final Thoughts

The problem of finding $\cos \theta$ given $\sin \theta = 12/13$ and the angle terminating in Quadrant II exemplifies how fundamental identities and geometric interpretations allow us to solve trigonometric problems efficiently. By understanding the core principles—such as the Pythagorean theorem, the signs in different quadrants, and the relationships on the unit circle—we can confidently determine the missing function values and deepen our grasp of trigonometry.

Whether you're preparing for exams, tackling engineering problems, or simply exploring the beauty of mathematics, mastering these foundational techniques will serve you well in a variety of contexts. Remember, always pay attention to the quadrant, verify your results, and use identities as reliable tools in your problem-solving toolkit.

Happy problem-solving!

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