16. Points X, Y, And Z Are Collinear, And Y Isthe Midpoint Of XZ. Find The Value Of B.

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Introduction

Understanding the relationships between points on a line is a fundamental aspect of coordinate geometry. When three points are collinear—that is, they lie along the same straight line—their coordinates satisfy certain linear relationships. Specifically, when one point is the midpoint of a segment connecting two other points, this relationship becomes a key tool in problem-solving. In this article, we will explore the scenario where points X, Y, and Z are collinear, with Y as the midpoint of segment XZ, and aim to find the value of the unknown B, which appears as a coordinate component.

Understanding the Problem Statement

What Is Given?

- Points X, Y, and Z are collinear.
- Point Y is the midpoint of segment XZ.
- Coordinates of X, Y, and Z are either explicitly given or expressed in terms of variables, including B.

What Is To Be Found?

- The value of B, which is a coordinate component of either point X, Y, or Z.

Fundamentals of Collinearity and Midpoints

Collinearity of Points

Three points $X(x_1, y_1)$, $Y(x_2, y_2)$, and $Z(x_3, y_3)$ are collinear if the vectors $\vec{XY}$ and $\vec{YZ}$ are scalar multiples of each other or, equivalently, if the area of the triangle formed by them is zero. Mathematically, the collinearity condition can be tested via the slope method or the area method.

- Slope Method: The slope between X and Y is equal to the slope between Y and Z.

$$
\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}
$$

- Area Method: The area of triangle XYZ is zero:

$$
\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0
$$

Midpoint Definition

- The midpoint Y of segment XZ has coordinates:

$$
Y\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)
$$

Given that Y is the midpoint, the above relationship allows us to relate the coordinates of X and Z.

Setting Up Coordinates for the Points

Suppose the coordinates are as follows:

- $X(x_1, y_1)$
- $Y(x_2, y_2)$
- $Z(x_3, y_3)$

Given the problem involves an unknown B, it is typical that B appears as a coordinate component of one of these points. For clarity, assume the points are:

- $X(a, b)$
- $Y(c, d)$
- $Z(e, f)$

and that the coordinate b is the unknown B to be determined.

Expressing the Midpoint Condition

Since Y is the midpoint of $X Z$, then:

$$
c = \frac{a + e}{2}
$$
$$
d = \frac{b + f}{2}
$$

If the known coordinates of Y are given, these relationships can be rearranged to express the unknown B in terms of other known quantities.

Applying Collinearity Condition

For the three points to be collinear, the slope between X and Y must equal the slope between Y and Z:

$$
\frac{d - b}{c - a} = \frac{f - d}{e - c}
$$

Substituting the midpoint expressions:

$$
\frac{d - b}{c - a} = \frac{f - d}{e - c}
$$

Given that $c = \frac{a + e}{2}$ and $d = \frac{b + f}{2}$, this becomes:

$$
\frac{\frac{b + f}{2} - b}{\frac{a + e}{2} - a} = \frac{f - \frac{b + f}{2}}{e - \frac{a + e}{2}}
$$

Simplify numerator and denominator:

Left side numerator:

\[

$$
\frac{b + f}{2} - b = \frac{b + f - 2b}{2} = \frac{f - b}{2}
$$

Left side denominator:

$$
\frac{a + e}{2} - a = \frac{a + e - 2a}{2} = \frac{e - a}{2}
$$

So the left side slope:

$$
\frac{\frac{f - b}{2}}{\frac{e - a}{2}} = \frac{f - b}{e - a}
$$

Similarly, the right side numerator:

$$
f - \frac{b + f}{2} = \frac{2f - b - f}{2} = \frac{f - b}{2}
$$

Right side denominator:

$$
e - \frac{a + e}{2} = \frac{2e - a - e}{2} = \frac{e - a}{2}
$$

Hence, the right side slope:

$$
\frac{\frac{f - b}{2}}{\frac{e - a}{2}} = \frac{f - b}{e - a}
$$

Therefore, both sides are equal, confirming the collinearity condition reduces to an identity, which suggests we need to use the actual coordinates provided in the problem for a concrete calculation.

Working Through a Concrete Example

Suppose the points are given as:

- $X(2, b)$
- $Y(4, 3)$
- $Z(6, f)$

Given that Y is the midpoint of $X Z$:

\[

4 = \frac{2 + 6}{2} \Rightarrow 4 = 4
\]

which is already consistent. The \(y\)-coordinate of \(Y\):

\[
3 = \frac{b + f}{2}
\]

Rearranged:

\[
b + f = 6
\]

Now, the points are:

- \(X(2, b)\)
- \(Y(4, 3)\)
- \(Z(6, f)\)

The collinearity condition:

\[
\text{slope } XY = \text{slope } YZ
\]

Calculate:

\[
\text{slope } XY = \frac{3 - b}{4 - 2} = \frac{3 - b}{2}
\]
\[
\text{slope } YZ = \frac{f - 3}{6 - 4} = \frac{f - 3}{2}
\]

Set equal:

\[
\frac{3 - b}{2} = \frac{f - 3}{2}
\]

Multiply both sides by 2:

\[
3 - b = f - 3
\]

Rearranged:

\[

f = 6 - b
\]

Recall from earlier that:

\[
b + f = 6
\]

Substitute \(f = 6 - b\):

\[
b + (6 - b) = 6
\]
\[
6 = 6
\]

This identity indicates that the system is consistent for any \(b\) satisfying the earlier relationships, meaning \(b\) is not uniquely determined unless more information is provided.

General Approach to Find B in the Original Problem

In the original problem, the specific coordinates are essential. Usually, the problem provides the coordinates of at least two points explicitly, with one involving an unknown B. The approach involves:

1. Writing the midpoint condition:

\[
Y = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)
\]

2. Expressing the collinearity condition:

\[
\text{Slope } XY = \text{Slope } YZ
\]

3. Substituting known values and the midpoint expressions to relate B to known quantities.

4. Solving the resulting equation to find the numerical value of B.

Conclusion

The key to solving problems involving collinearity and midpoints lies in understanding the relationships between coordinates. When points are collinear and one is the midpoint of a segment, their coordinates satisfy specific linear relationships. Setting up the equations based on the midpoint formula and the slope condition allows

Frequently Asked Questions

In a line segment XYZ, if Y is the midpoint of XZ, and the coordinates of X and Z are known, how can we find the value of B if it represents the coordinate of Y?

Since Y is the midpoint of XZ, its coordinate B is the average of the coordinates of X and Z. Therefore, $B = (X + Z) / 2$.

What is the significance of Y being the midpoint of XZ in coordinate geometry problems?

Y being the midpoint implies that it divides the segment XZ into two equal parts, which allows us to find unknown coordinates like B by averaging the coordinates of X and Z.

If points X, Y, and Z are collinear and Y is the midpoint of XZ, how does this impact the calculation of B in a coordinate plane?

It means B (the coordinate of Y) can be calculated as the average of the coordinates of X and Z, i.e., $B = (X + Z) / 2$, assuming a one-dimensional coordinate system.

Given that points are collinear with Y as the midpoint of XZ, how can the concept of midpoint be applied in coordinate geometry to find B?

Use the midpoint formula: $B = (X + Z) / 2$, which ensures Y lies exactly halfway between X and Z on the line.

In a problem where B is the coordinate of the midpoint Y of segment XZ, and points are collinear, what formula should be used to find B?

Use the midpoint formula: B = (coordinate of X + coordinate of Z) / 2.

How does the collinearity of points X, Y, and Z confirm the method to find B when Y is the midpoint of segment XZ?

Collinearity confirms that Y lies on the line segment between X and Z, validating the use of the midpoint formula B = (X + Z) / 2 to find B.

Additional Resources

Points X, Y, And Z Are Collinear, And Y Is The Midpoint Of XZ. Find The Value Of B.
This problem is a classic example of coordinate geometry, where understanding the relationships between points on a line segment can lead to solving for unknown variables. The phrase "Points X, Y, And Z Are Collinear, And Y Is The Midpoint Of XZ" hints at fundamental geometric principles involving midpoints and collinearity, which are essential for solving numerous algebraic geometry problems. In this guide, we will explore the concepts step-by-step, analyze the problem thoroughly, and derive a comprehensive method to find the value of B.

Understanding the Problem

Before diving into calculations, it’s vital to clarify what the problem entails:

- Points X, Y, and Z are collinear: this means all three points lie on a single straight line.
- Y is the midpoint of segment XZ: Y divides the segment XZ into two equal parts.
- Find the value of B: B is an unknown variable, likely part of the coordinate expressions of the points.

Typical Setup

In coordinate geometry, points are represented as ordered pairs:
- $X = (x_1, y_1)$
- $Y = (x_2, y_2)$
- $Z = (x_3, y_3)$

The problem often involves giving the coordinates of two points and asking for the third or the unknown variable B, which could be part of these coordinates.

Step 1: Recall the Midpoint Formula

The key principle here is the midpoint formula, which states:

$$
Y = \text{Midpoint of } XZ = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)
$$

This means that the coordinates of Y are the averages of the coordinates of points X and Z.

Step 2: Applying the Midpoint Formula to Find B

Suppose the points are given as:

- $X = (x_1, y_1)$
- $Y = (x_2, y_2)$
- $Z = (x_3, y_3)$

And further, assume that B appears in the coordinate of either point X, Y, or Z. For example, suppose:

- $X = (a, b)$
- $Y = (c, d)$
- $Z = (e, f)$

Given that Y is the midpoint of XZ, then:

$$
c = \frac{a + e}{2} \quad \text{and} \quad d = \frac{b + f}{2}
$$

If B is part of one of these coordinates (say, B = b or B = f), then these equations can help us solve for B once other values are known.

Step 3: Analyzing the Collinearity Condition

Since the points are collinear, the slope of XY must be equal to the slope of YZ:

$$
\text{Slope } XY = \text{Slope } YZ
$$

For points:

$$
\text{Slope } XY = \frac{d - c}{b - a}
$$

$$
\text{Slope } YZ = \frac{f - d}{e - c}
$$

Setting these equal:

$$
\frac{d - c}{b - a} = \frac{f - d}{e - c}
$$

This equation can be used to find B if some coordinates are known or expressed in terms of B.

Step 4: Step-by-Step Example with Specific Coordinates

Let’s illustrate with an example where the problem is explicitly defined:

Suppose the points are:

- $X = (x, y)$
- $Y = (x_m, y_m)$
- $Z = (x', y')$

Given that:

- Y is the midpoint of XZ,
- The coordinates of X and Z are known, but the coordinate involving B is unknown.

Example Data:

- $X = (2, 3)$
- $Y = (5, B)$
- $Z = (8, 7)$

Question: Find the value of B.

Solution:

Using the midpoint formula for the y-coordinate:

$$
y_m = \frac{y_x + y_z}{2}
$$

Plug in the known values:

$$
B = \frac{3 + 7}{2} = \frac{10}{2} = 5
$$

Answer: $B = 5$

Step 5: Generalized Approach to Find B

In more complex problems, the points may be expressed algebraically, for example:

- $X = (x, B)$
- $Z = (x', y')$

And the midpoint $Y = (x_m, y_m)$ is given or expressed in terms of B.

General steps:

1. Write down the midpoint equations:

$$
x_m = \frac{x + x'}{2}
$$
$$
y_m = \frac{B + y'}{2}
$$

2. Use the given information to substitute known values or relations.

3. Set up the collinearity condition via slope equality or vector cross product:

- Slope method:

$$
\frac{y_m - y_x}{x_m - x} = \frac{y_z - y_m}{x' - x_m}
$$

- Cross product method:

For points A, B, C, they are collinear if the area of triangle ABC is zero:

$$
(x_B - x_A)(y_C - y_A) - (y_B - y_A)(x_C - x_A) = 0
$$

4. Solve the resulting equation for B.

Step 6: Additional Tips and Common Pitfalls

- Ensure correctness of coordinates: Double-check the points’ coordinates before calculating.
- Watch for coordinate axes: Sometimes B may be part of an x- or y-coordinate; handle accordingly.
- Use symmetry: The midpoint divides the segment equally; leverage this to find unknowns.
- Collinearity tests: Always verify that the points are collinear using the slope or cross product method.

Summary

To find the value of B when points X, Y, and Z are collinear with Y as the midpoint of XZ:

- Use the midpoint formula to relate the coordinates.
- Apply the collinearity condition (slope equality or cross product).

- Substitute known values and solve algebraically for B.

This approach combines fundamental geometry principles with algebraic manipulation, enabling you to solve a wide variety of related problems efficiently.

Final Thoughts

Understanding the relationship between midpoints and collinearity is a powerful tool in coordinate geometry. Whether you're tackling simple problems with specific coordinates or more complex algebraic expressions, the key lies in carefully applying the midpoint and collinearity formulas. Practice with different configurations and data sets to strengthen your problem-solving skills and become proficient in deriving unknown variables like B in geometric contexts.

If you have any specific values or a particular problem statement involving points X, Y, Z, and B, feel free to share it, and I can guide you through a tailored solution!

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