5. Determine The Value Of T4 In An Arithmetic Sequence Given That T = 11 And S9 = 243.

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Understanding how to find specific terms within an arithmetic sequence is a fundamental concept in algebra and mathematics. When given certain information about the sequence—such as the common difference, specific terms, or sums—it becomes possible to determine other unknown terms. In this article, we will explore how to determine the value of the fourth term, T4, in an arithmetic sequence, given that T = 11 and the sum of the first nine terms, S9, equals 243. We will walk through the necessary formulas, step-by-step calculations, and explanations to clarify this process.

Understanding the Basics of Arithmetic Sequences

Before diving into the problem, it's essential to understand what an arithmetic sequence is and the key properties involved.

What Is an Arithmetic Sequence?

An arithmetic sequence is a sequence of numbers where each term after the first is obtained by adding a fixed amount, called the common difference (d), to the previous term. The general form of an arithmetic sequence is:

$$T_n = T_1 + (n - 1)d$$

Where:
- T_n is the nth term,
- T_1 is the first term,
- d is the common difference,
- n is the term number.

Sum of the First n Terms (S_n)

The sum of the first n terms of an arithmetic sequence is given by:

$$S_n = \frac{n}{2} (T_1 + T_n)$$

Alternatively, since $T_n = T_1 + (n - 1)d$:

$$S_n = \frac{n}{2} [2T_1 + (n - 1)d]$$

Given Data and What Is Required

The problem provides:
- $T = 11$: This indicates that the first term T_1 is 11.
- $S_9 = 243$: The sum of the first nine terms is 243.

The goal:
- Find the value of T_4, the fourth term in the sequence.

Step-by-Step Solution Approach

To find T_4, we need to determine the common difference d, then compute the fourth term using the general formula for the nth term.

Step 1: Identify Known Values

- $T_1 = 11$
- $S_9 = 243$

Our unknown:
- d

Step 2: Use the Sum Formula for the First 9 Terms

Recall:

$$S_9 = \frac{9}{2} [2T_1 + (9 - 1)d]$$

Substituting known values:

$$243 = \frac{9}{2} [2 \times 11 + 8d]$$

Simplify:

$$243 = \frac{9}{2} [22 + 8d]$$

Multiply both sides by 2 to clear denominator:

$$486 = 9 [22 + 8d]$$

Divide both sides by 9:

$$54 = 22 + 8d$$

Step 3: Solve for the Common Difference d

Subtract 22 from both sides:

$$54 - 22 = 8d$$

$$32 = 8d$$

Divide both sides by 8:

$$d = 4$$

So, the common difference is $d = 4$.

Step 4: Find T_4 Using the General Term Formula

Recall:

$$T_n = T_1 + (n - 1)d$$

For T_4:

$$T_4 = T_1 + (4 - 1)d$$

$$T_4 = 11 + 3 \times 4$$

$$T_4 = 11 + 12$$

$$T_4 = 23$$

Final Answer: The Value of T4

Based on the calculations, the fourth term T_4 in the arithmetic sequence is 23.

Additional Considerations and Related Concepts

Understanding how to manipulate the sum and term formulas allows for solving various problems involving arithmetic sequences. Here are some related concepts:

Verifying the Sequence

- The sequence begins with $T_1 = 11$.
- The common difference is $d = 4$.
- The sequence progresses as: 11, 15, 19, 23, 27, 31, 35, 39, 43, ...

Calculating Other Terms

- $T_2 = T_1 + d = 11 + 4 = 15$
- $T_3 = T_1 + 2d = 11 + 8 = 19$
- $T_4 = 11 + 3 \times 4 = 23$ (consistent with previous result)

Sum of Terms in Practice

- Sum of first 5 terms: $S_5 = \frac{5}{2}(T_1 + T_5)$
- Or directly: $S_5 = 11 + 15 + 19 + 23 + 27 = 95$

Conclusion

In summary, determining the value of T_4 in an arithmetic sequence involves understanding the fundamental formulas and applying basic algebra. Given the first term $T_1 = 11$ and the sum of the first nine terms $S_9 = 243$, we derived the common difference $d = 4$. Then, using the formula for the nth term, we calculated $T_4 = 23$. Mastering these concepts enables solving a wide range of problems related to arithmetic sequences and their properties.

FAQs

Q1: Can this method be used to find any term in an arithmetic sequence?
A: Yes. Once you know the first term and common difference, you can find any term using $T_n = T_1 + (n - 1)d$.

Q2: What if the sum of the first n terms is given, but not T_1?
A: You may need additional information or equations to solve for both T_1 and d.

Q3: How does this approach change if the sequence is geometric instead of arithmetic?
A: The formulas differ significantly; geometric sequences involve multiplication by a common ratio instead of addition.

Q4: Are there real-world applications of arithmetic sequences?
A: Yes. Examples include calculating savings over time with fixed deposits, scheduling, and population growth models with constant increments.

This comprehensive guide provides a clear understanding of how to determine the value of a specific term in an arithmetic sequence, using given sums and initial terms, with step-by-step calculations and explanations.

Frequently Asked Questions

What is the first step to find T4 in an arithmetic sequence when T = 11 and S9 = 243?

The first step is to understand the given terms: T = 11 (which is the first term T1), and the sum of the first 9 terms, S9 = 243. Then, use the formulas for an arithmetic sequence to find the common difference and subsequent terms.

How do you express the sum of the first n terms (S_n) in an arithmetic sequence?

The sum of the first n terms is given by the formula S_n = n/2 (2T1 + (n - 1)d), where T1 is the first term and d is the common difference.

Given T1 = 11 and S9 = 243, how can you find the common difference d?

Substitute T1 = 11 and n = 9 into the sum formula: 243 = 9/2 (211 + (9 - 1)d). Simplify and solve for d to find its value.

What is the calculated common difference d in this sequence?

By solving 243 = (9/2) (22 + 8d), we get 243 = 4.5 (22 + 8d). Dividing both sides by 4.5 yields 54 = 22 + 8d, so 8d = 32, resulting in d = 4.

How do you find T4 once the common difference d is known?

Use the formula for the nth term of an arithmetic sequence: Tn = T1 + (n - 1)d. For T4, substitute T1 = 11, n = 4, and d = 4.

What is the value of T4 in this sequence?

T4 = 11 + (4 - 1) 4 = 11 + 3 4 = 11 + 12 = 23.

Can you verify the value of T4 by calculating the sum of the first 4 terms?

Yes. The first four terms are T1 = 11, T2 = 15, T3 = 19, T4 = 23. Their sum is 11 + 15 + 19 + 23 = 68, which aligns with the sequence's pattern and the total sum calculations.

Additional Resources

5. Determine The Value Of T4 In An Arithmetic Sequence Given That T = 11 And S9 = 243

Understanding sequences is fundamental in mathematics, especially when it comes to arithmetic progressions—where each term is obtained by adding a constant difference to the previous term. Today, we'll explore how to determine the specific term T4 in an arithmetic sequence, given two key pieces of information: the general term T = 11 and the sum of the first nine terms, S9 = 243. This problem exemplifies how the foundational concepts of sequences and series intertwine, offering a pathway to uncover unknown elements within a sequence.

Introduction to Arithmetic Sequences and Series

Before diving into calculations, it’s essential to understand what an arithmetic sequence and its associated series represent.

What Is an Arithmetic Sequence?

An arithmetic sequence is a list of numbers where each term increases or decreases by a fixed amount called the common difference (denoted as d). The general form of an arithmetic sequence is:

$$T_n = T_1 + (n - 1)d$$

where:
- T_n is the nth term,
- T_1 is the first term,
- d is the common difference,
- n is the position of the term in the sequence.

Sum of the First n Terms (Series)

The sum of the first n terms, denoted as S_n, can be calculated using the formula:

$$S_n = \frac{n}{2} [2T_1 + (n - 1)d]$$

Alternatively, since the sequence is arithmetic, the sum can also be written as:

$$S_n = \frac{n}{2}(T_1 + T_n)$$

which relies on knowing the first and nth terms.

Clarifying the Given Data and the Problem

In our problem, we are given:

- The general term value $T = 11$.
This indicates that the nth term, T_n, is 11. However, without explicit clarification, this is typically interpreted as the general term T_n equals 11, suggesting that the sequence’s formula involves this value.
- The sum of the first 9 terms: $S_9 = 243$.

Our task is to find the value of the 4th term, T_4.

Interpreting the Given Data

The critical initial step is to interpret what "T = 11" signifies. Two common interpretations are:

1. T = 11 refers to a specific term in the sequence, such as $T_n = 11$ for some n.
2. T = 11 refers to the general term formula, i.e., the sequence's nth term has a formula involving T, or T denotes a known value in the sequence.

Given the context, it is most reasonable to assume that T = 11 means the 11th term of the sequence is T11 = 11. This assumption aligns with the common notation where Tn denotes the nth term.

Therefore, the key pieces of information are:

- $T_{11} = 11$
- $S_9 = 243$

Our goal: Determine T_4.

Step 1: Expressing Known Terms Using Sequence Formulas

1. General term:

$$T_n = T_1 + (n - 1)d$$

2. 11th term:

$$T_{11} = T_1 + (11 - 1)d = T_1 + 10d$$

Given:

$$T_{11} = 11$$

Thus:

$$T_1 + 10d = 11 \quad \quad (1)$$

3. Sum of first 9 terms:

$$S_9 = \frac{9}{2} [2T_1 + (9 - 1)d] = \frac{9}{2} [2T_1 + 8d]$$

Given:

$$S_9 = 243$$

So:

$$243 = \frac{9}{2} (2T_1 + 8d)$$

Multiply both sides by 2 to clear denominator:

$$486 = 9(2T_1 + 8d)$$

Divide both sides by 9:

$$\frac{486}{9} = 2T_1 + 8d$$

$$54 = 2T_1 + 8d \quad \quad (2)$$

Step 2: Solving for T_1 and d

From equation (1):

$$T_1 = 11 - 10d$$

Substitute into equation (2):

$$54 = 2(11 - 10d) + 8d$$

Simplify:

$$54 = 22 - 20d + 8d$$

$$54 = 22 - 12d$$

Bring all to one side:

$$54 - 22 = -12d$$

$$32 = -12d$$

Solve for d:

$$d = -\frac{32}{12} = -\frac{8}{3}$$

Now, find T_1:

$$T_1 = 11 - 10 \times \left(-\frac{8}{3}\right)$$

$$T_1 = 11 + \frac{80}{3}$$

Express 11 as a fraction:

$$T_1 = \frac{33}{3} + \frac{80}{3} = \frac{113}{3}$$

Step 3: Find T_4

Recall the general term:

$$T_n = T_1 + (n - 1)d$$

Substitute $T_1 = \frac{113}{3}$ and $d = -\frac{8}{3}$:

$$T_4 = \frac{113}{3} + (4 - 1) \times \left(-\frac{8}{3}\right)$$

$$T_4 = \frac{113}{3} + 3 \times \left(-\frac{8}{3}\right)$$

$$T_4 = \frac{113}{3} - 3 \times \frac{8}{3}$$

$$T_4 = \frac{113}{3} - \frac{24}{3}$$

$$T_4 = \frac{113 - 24}{3} = \frac{89}{3}$$

Expressed as a decimal:

$$T_4 \approx 29.67$$

Final Result:

The value of the 4th term in the sequence is:

$$\boxed{\frac{89}{3} \text{ or approximately } 29.67}$$

Additional Insights: Why This Matters

Understanding how to derive specific terms in an arithmetic sequence from sum information and particular term values is a vital skill in mathematics. Such techniques are applicable in various real-world contexts—be it financial calculations, predicting trends, or analyzing data patterns. This problem emphasizes the importance of translating word problems into algebraic equations and systematically solving for unknowns.

Summary of the Methodology

1. Identify the knowns: Recognize what the problem states explicitly—here, the 11th term and the sum of the first 9 terms.
2. Set up the formulas: Use the formulas for the nth term and the sum of the first n terms.
3. Express unknowns: Write the first term and common difference in terms of knowns.
4. Solve the system: Use substitution and algebraic manipulation to find the unknowns.
5. Calculate the desired term: Plug the known values into the general term formula for the specific position, in this case, the 4th term.

Conclusion

Determining specific terms within an arithmetic sequence involves a combination of understanding the sequence's structure and applying algebraic techniques to solve for unknowns. Given the data $T_{11} = 11$ and $S_9 = 243$, we've successfully calculated that the 4th term T_4 is $\frac{89}{3}$, or approximately 29.67. This process underscores the interconnectedness of sequence formulas and series sums, showcasing how mathematicians decode complex data to reveal individual components within a structured numerical pattern.

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