7. How Many Ordered Triples Of Integers (x, Y, Z) Satisfy 36x + 100y +2252 = 12600?

7. How Many Ordered Triples Of Integers (x, y, z) Satisfy 36x + 100y + 2252 = 12600?

Understanding how many ordered triples of integers (x, y, z) satisfy a given linear Diophantine equation is a fundamental problem in number theory and combinatorics. In this article, we explore the specific equation:

$36x + 100y + 2252 = 12600$

Our goal is to determine all integer solutions (x, y, z) that satisfy this equation, with particular focus on the variables involved. Although the equation involves three variables, note that the variable z does not appear explicitly in the equation, which suggests that the solution set depends primarily on x and y. We will analyze the problem step-by-step, considering the constraints, and then extend the findings to understand the total number of solutions.

Understanding the Equation and Its Variables

Equation Breakdown

The given equation is:

```
36x + 100y + 2252 = 12600
```

Rearranged, it becomes:

```
36x + 100y = 12600 - 2252
```

Calculating the right side:

```
12600 - 2252 = 10348
```

Thus, the simplified form is:

```
36x + 100y = 10348
```

Variables and Constraints

Since x, y, and z are integers, and the equation involves only x and y explicitly, the variable z appears to be free unless additional constraints are specified. For the purpose of this problem, we assume z can be any integer, making the total number of solutions infinite, provided x and y satisfy the equation.

Therefore, the key task is to find all integer pairs (x, y) that satisfy:

$$36x + 100y = 10348$$

Once (x, y) are determined, for each such pair, z can be any integer, leading to an infinite number of solutions. To make the problem meaningful, we often consider the number of solutions in bounded ranges for x and y or restrict z to a certain set. However, for this analysis, we will focus on counting the number of solutions in the entire integer domain, noting that z is unrestricted.

Analyzing the Diophantine Equation: Solving for (x, y)

Reducing the Equation

Our core equation is:

$$36x + 100y = 10348$$

To analyze integer solutions, it is essential to determine whether the coefficients and the constant term are compatible in terms of divisibility and common factors.

Finding the Greatest Common Divisor (GCD)

Calculate the GCD of 36 and 100:

- $\gcd(36, 100) = 4$

Check whether 4 divides 10348:

- Divide 10348 by 4:

$$10348 \div 4 = 2587$$

Since 2587 is an integer, the divisibility condition is satisfied, and solutions exist.

Dividing the Equation by the GCD

Divide the entire equation by 4 to simplify:

```
(36/4)x + (100/4)y = 10348/4
```

```
9x + 25y = 2587
```

Solving the Simplified Equation $9x + 25y = 2587$

Checking for Integer Solutions

We now analyze the linear Diophantine equation:

```
9x + 25y = 2587
```

Since $\gcd(9, 25) = 1$, and 1 divides 2587, solutions exist.

Expressing y in terms of x

Rearranged, solve for y:

```
25y = 2587 - 9x
```

Thus:

```
y = (2587 - 9x)/25
```

For y to be an integer, the numerator must be divisible by 25:

```
2587 - 9x ≡ 0 (mod 25)
```

Modulo Analysis

Calculate 2587 mod 25:

```
25 × 103 = 2575
2587 - 2575 = 12
```

Thus:

```
2587 ≡ 12 (mod 25)
```

Our divisibility condition becomes:

```
12 - 9x ≡ 0 (mod 25)
```

which simplifies to:

```
9x ≡ 12 (mod 25)
```

Solving for x modulo 25

We need to solve:

```
9x ≡ 12 (mod 25)
```

Since 9 and 25 are coprime, 9 has a multiplicative inverse modulo 25.

Finding the Inverse of 9 mod 25

We seek an integer k such that:

```
9k ≡ 1 (mod 25)
```

Using the Extended Euclidean Algorithm or trial, check small multiples of 9:

- $9 \times 3 = 27 \equiv 2 \pmod{25}$
- $9 \times 13 = 117 \equiv 117 - 4 \times 25 = 117 - 100 = 17 \pmod{25}$

- 9 × 17 = 153 ≡ 153 - 6×25 = 153 - 150 = 3 (mod 25)
- 9 × 19 = 171 ≡ 171 - 6×25 = 171 - 150 = 21 (mod 25)
- 9 × 21 = 189 ≡ 189 - 7×25 = 189 - 175 = 14 (mod 25)
- 9 × 23 = 207 ≡ 207 - 8×25 = 207 - 200 = 7 (mod 25)
- 9 × 24 = 216 ≡ 216 - 8×25 = 216 - 200 = 16 (mod 25)

Continuing, check 9 × 14:

```
9 × 14 = 126 ≡ 126 - 5×25 = 126 - 125 = 1 (mod 25)
```

Thus, k = 14 is the inverse of 9 modulo 25.

Calculating x

Multiplying both sides of the congruence by 14:

```
x ≡ 12 × 14 (mod 25)
```

```
12 × 14 = 168
```

Now, 168 mod 25:

```
25 × 6 = 150
168 - 150 = 18
```

Therefore:

```
x ≡ 18 (mod 25)
```

Expressing the General Solution for x and y

Since x ≡ 18 (mod 25), x can be written as:

```
x = 18 + 25k, where k ∈ ℤ
```

Correspondingly, y is given by:

$$y = (2587 - 9x)/25$$

Substituting x:

$$y = (2587 - 9(18 + 25k))/25 = (2587 - 162 - 225k)/25 = (2425 - 225k)/25$$

Dividing numerator by 25:

$$y = 97 - 9k$$

Summary of the Solution Set

Combining the above results, the solutions (x, y) are:

- $x = 18 + 25k$
- $y = 97 - 9k$

where $k \in \mathbb{Z}$ (any integer). Since z is unrestricted, for each such pair, z can be any integer, leading to an infinite number of solutions.

Counting the Number of Solutions

Unbounded Solutions

Because there

Frequently Asked Questions

How can we determine the number of integer solutions to the equation $36x + 100y + 2252z = 12600$?

To find the number of integer solutions, we can first isolate one variable, analyze the divisibility conditions, and then count the solutions systematically, often by considering the equation modulo certain numbers or using the Euclidean algorithm to understand the constraints on x, y, and z.

What is the significance of the coefficients 36, 100, and 2252 in the equation regarding the number of solutions?

The coefficients determine the divisibility and potential for integer solutions. Since 36, 100, and 2252 are multiples of 4, analyzing their greatest common divisors (GCDs) helps identify whether solutions exist and how many solutions are possible, based on divisibility of the constant term 12600.

Can we use modular arithmetic to simplify counting solutions to 36x + 100y + 2252z = 12600?

Yes, modular arithmetic allows us to reduce the equation modulo certain numbers to find necessary conditions for solutions. For example, analyzing the equation modulo the GCD of the coefficients can help determine the possible values of z and subsequently x and y.

What approach can be taken to count the number of integer solutions for the given equation?

One approach is to fix one variable, such as z, and then solve the resulting two-variable linear Diophantine equation for x and y. By iterating over all possible integer z values that satisfy the constraints, we can count the total number of solutions.

Are there any constraints on x, y, and z for the solutions to exist in the equation 36x + 100y + 2252z = 12600?

Yes, solutions exist only when the right-hand side, 12600, is divisible by the greatest common divisor of 36, 100, and 2252. Additionally, for each fixed z, the remaining equation must be solvable in integers, which imposes conditions on the possible z values.

Additional Resources

How Many Ordered Triples Of Integers (x, y, z) Satisfy 36x + 100y + 2252 = 12600?

Introduction

Mathematics often presents us with intriguing puzzles that challenge our understanding of number theory and algebra. One such problem involves determining the number of integer solutions to a linear equation with multiple variables. Specifically, this article investigates the question:

How many ordered triples of integers (x, y, z) satisfy the equation
\[36x + 100y + 2252 = 12600?\]

While the initial appearance of the problem seems straightforward, uncovering the complete set of solutions involves a detailed analysis of divisibility, parameterization, and the structure of the solution space. This exploration not only deepens our understanding of linear Diophantine equations but also illuminates methods applicable across various domains in mathematics and related fields.

Restating and Simplifying the Problem

The core problem can be rewritten for clarity:

Find the number of integer solutions (x, y, z) such that:

$$
36x + 100y + 2252 = 12600.
$$

Note that the variable z appears in the problem statement, but as the equation involves only x and y, the role of z must be clarified.

Key Observation:
The original problem specifies "ordered triples (x, y, z)." However, the equation involves only x and y. This suggests that z might be unconstrained, or perhaps the problem intends to consider z as a parameter or an additional variable not involved in the original equation. Given the information, the logical assumption is that the variable z is free and can take any integer value once x and y satisfy the equation.

Therefore, the problem reduces to:

- Find the number of integer solutions (x, y) satisfying the linear equation.
- For each such solution, the variable z can be any integer.

Consequently, the total number of solutions (x, y, z) becomes:

$$
\text{Number of solutions for } (x, y) \times \text{Number of solutions for } z,
$$

which is infinite unless constraints on z are specified.

Assumption:
Since the problem explicitly asks for the number of solutions, it is most reasonable to interpret the question as:

> How many solutions (x, y) satisfy the equation? And considering z as an independent variable, the total number of solutions is infinite.

Alternatively, if the problem intends to consider only solutions where z is fixed or constrained, more information would be necessary.

For the purposes of this analysis, we focus on the solutions for (x, y). The total solutions in (x, y, z) form a set with infinitely many elements, given that z is unrestricted.

Analyzing the Equation: Reducing to a Diophantine Equation

The core of the problem lies in solving:

$$
36x + 100y = 12600 - 2252.
$$

Calculating the right side:

$$
12600 - 2252 = 10348.
$$

Therefore, the simplified equation is:

$$
36x + 100y = 10348.
$$

This is a linear Diophantine equation in two variables. The key to solving it is understanding the divisibility conditions and parametrizing solutions.

Divisibility and the Greatest Common Divisor (GCD)

To determine whether solutions exist, we examine the coefficients' greatest common divisor (gcd).

Calculate:

$$
\gcd(36, 100).
$$

Prime factorization:

- $36 = 2^2 \times 3^2$,
- $100 = 2^2 \times 5^2$.

Thus,

$$
\gcd(36, 100) = 2^2 = 4.
$$

Next, check whether 4 divides 10348:

$$
10348 \div 4 = 2587.
$$

Since 2587 is an integer, solutions exist for x, y.

Conclusion:

- The equation $36x + 100y = 10348$ has integer solutions.

Parametric Solution of the Diophantine Equation

Given that $\gcd(36, 100) = 4$ divides the RHS, solutions exist and can be characterized.

Method:

Divide the entire equation by 4:

$$
\frac{36}{4}x + \frac{100}{4}y = \frac{10348}{4}.
$$

Simplify:

$$

$$
9x + 25y = 2587.
$$

Now, the problem reduces to finding all integer solutions of:

$$
9x + 25y = 2587.
$$

Solving the Simplified Equation: $9x + 25y = 2587$

This is a linear Diophantine equation in standard form.

Step 1: Find particular solution

We need integers x_0, y_0 satisfying:

$$
9x_0 + 25y_0 = 2587.
$$

To find such a solution, we can use the Extended Euclidean Algorithm.

Extended Euclidean Algorithm

Find integers a, b such that:

$$
9a + 25b = \gcd(9, 25) = 1.
$$

Then, multiply both sides by 2587 to get a particular solution.

Applying the Algorithm:

- $25 = 9 \times 2 + 7$
- $9 = 7 \times 1 + 2$
- $7 = 2 \times 3 + 1$
- $2 = 1 \times 2 + 0$

Back substitution:

- $1 = 7 - 2 \times 3$

But since $2 = 9 - 7$:

- $1 = 7 - (9 - 7) \times 3 = 7 - 9 \times 3 + 7 \times 3 = 7 \times 4 - 9 \times 3$.

Recall $7 = 25 - 9 \times 2$:

- $1 = (25 - 9 \times 2) \times 4 - 9 \times 3 = 25 \times 4 - 9 \times 8 - 9 \times 3 = 25 \times 4 - 9 \times 11$.

Thus,

$$
1 = 25 \times 4 - 9 \times 11.
$$

Multiply both sides by 2587:

$$
2587 = 25 \times (4 \times 2587) - 9 \times (11 \times 2587).
$$

So, a particular solution (x_0, y_0) is:

$$
x_0 = -11 \times 2587,
$$

$$
y_0 = 4 \times 2587.
$$

Calculating:

- $y_0 = 4 \times 2587 = 10348$,
- $x_0 = -11 \times 2587 = -28457$.

Particular solution:

$$
x = -28457, \quad y = 10348.
$$

General Solution Set

The general solution for the linear Diophantine equation:

$$
9x + 25y = 2587,
$$

is given by:

$$
x = x_0 + k \times \frac{b}{\gcd(a, b)} = -28457 + 25k,
$$

$$
y = y_0 - k \times \frac{a}{\gcd(a, b)} = 10348 - 9k,
$$

where $k \in \mathbb{Z}$.

Note: The signs are consistent with the coefficients; the negative sign in the y term comes from the derivation during the algorithm.

Reintroduction to the Original Variables

Recall that:

$$
x, y
$$

are related to the original variables via the scaling:

$$
36x + 100y = 10348,
$$

which was derived from the initial problem.

The solutions for x and y are:

$$
x = -28457 + 25k,
$$

$$
y = 10348 - 9k,
$$

for all integers k.

Role of z in the Solution

Since the original equation involves only x and y, and z does not appear, z is unrestricted—for each solution (x, y), the variable z can be any integer.

Therefore, the total number of solutions (x, y, z) is infinite.

Counting Solutions: When Is It Finite?

If the problem intended to ask for the number of solutions with additional constraints, such as bounded variables or specific ranges, the counting could be finite.

For example:

- If x and y

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