81x^2-49=0Square Root, Factoring, Completing The Square, Or The Quadratic Formula

81x^2-49=0Square Root, Factoring, Completing The Square, Or The Quadratic Formula is a fundamental quadratic equation that provides an excellent opportunity to explore various methods of solving quadratic equations. Whether you're a student learning algebra or someone brushing up on mathematical techniques, understanding how to approach such an equation is essential. In this article, we will delve into each method—Square Root Property, Factoring, Completing the Square, and the Quadratic Formula—explaining when and how to use them, and demonstrating their application step-by-step.

Understanding the Equation: 81x^2 - 49 = 0

Before diving into solution methods, let's analyze the given quadratic equation:

$$81x^2 - 49 = 0$$

This equation is a quadratic because it contains an x^2 term, and it is set equal to zero, making it suitable for various solving techniques. Notice that the coefficients suggest a perfect square structure, which can simplify the solving process.

Method 1: Square Root Property

The Square Root Property is often the simplest method when the quadratic is set equal to zero and can be expressed as a perfect square.

Step 1: Recognize a Perfect Square

Observe that:

- $81x^2$ is a perfect square because $81x^2 = (9x)^2$.
- 49 is a perfect square because $49 = 7^2$.

Thus, the equation can be rewritten as:

$$(9x)^2 - 7^2 = 0$$

This is a difference of squares, which has a specific factorization.

Step 2: Apply the Difference of Squares Formula

Recall that:

$$a^2 - b^2 = (a - b)(a + b)$$

Applying this to our equation:

$$(9x - 7)(9x + 7) = 0$$

Step 3: Solve for x

Set each factor equal to zero:

1. $9x - 7 = 0 \Rightarrow 9x = 7 \Rightarrow x = \frac{7}{9}$
2. $9x + 7 = 0 \Rightarrow 9x = -7 \Rightarrow x = -\frac{7}{9}$

Solution via Square Root Property:

$$
x = \pm \frac{7}{9}
$$

This method is efficient because it leverages the difference of squares and the perfect square nature of the coefficients.

Method 2: Factoring

Factoring is a common algebraic technique to solve quadratics, especially when the quadratic can be written as a product of binomials.

Step 1: Recognize the quadratic form

The equation:

$$81x^2 - 49 = 0$$

can be viewed as a quadratic in terms of x^2:

$$(9x)^2 - 7^2 = 0$$

which, as previously shown, is a difference of squares.

Step 2: Rewrite as a factorization

Using the difference of squares:

$$(9x - 7)(9x + 7) = 0$$

Step 3: Solve each factor

Set each factor equal to zero:

- $9x - 7 = 0 \Rightarrow x = \frac{7}{9}$
- $9x + 7 = 0 \Rightarrow x = -\frac{7}{9}$

Note: This approach is essentially the same as the square root method but emphasizes the factorization process, demonstrating the importance of recognizing special products in quadratic equations.

Method 3: Completing the Square

Completing the square is a versatile method, particularly useful for quadratics not easily factorable or when deriving the quadratic formula.

Step 1: Isolate the quadratic term

The equation is already in standard form:

$$81x^2 - 49 = 0$$

Add 49 to both sides:

$$81x^2 = 49$$

Step 2: Divide both sides by the coefficient of x^2

Divide both sides by 81:

$$x^2 = \frac{49}{81}$$

Step 3: Take the square root of both sides

Applying the square root:

$$x = \pm \sqrt{\frac{49}{81}}$$

which simplifies to:

$$x = \pm \frac{\sqrt{49}}{\sqrt{81}} = \pm \frac{7}{9}$$

Note: Although this method involves taking roots rather than explicitly completing the square as a process, it aligns with the square root property. For more complex quadratics, completing the square involves creating a perfect square trinomial.

Method 4: The Quadratic Formula

The quadratic formula provides a universal solution method for any quadratic equation of the form $ax^2 + bx + c = 0$:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Step 1: Write the equation in standard form

Our equation:

$$81x^2 - 49 = 0$$

can be written as:

$$81x^2 + 0x - 49 = 0$$

where:

- $a = 81$
- $b = 0$
- $c = -49$